Equivalent Ellipsoid as an Interpretation Tool of Extended Current Distributions in Biomagnetic Inverse Problems

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***Abstract*—The paper presents an equivalent ellipsoid approach for interpretation and visualization of extended current distributions in biomagnetic inverse problems. The example of simulations performed with physical thorax phantom is also given.**

***Index Terms*—Biomagnetics, inverse problems, modeling, visualization.**

I. INTRODUCTION

IN the last decade the number of biomagnetic measurement systems available in clinical centers is rapidly growing up.

Biomagnetic measurements can provide information of the actual behavior of electric active organs, like heart or brain, in noninvasive way. The interpretation of measured, extremely weak ($10^{-10} \ldots 10^{-15}$ T), magnetic fields generated by human organs requires the application of special algorithms for localization/reconstruction of the sources. One class of methods for finding a distribution of extended current sources is based on a minimum norm approach. It is assumed that the total length of the vector that represents the best fitting current dipoles is minimized. Minimum norm solutions of the inverse problems in biomagnetism have been usually presented as color coded current density maps or current dipoles sets on brain/heart surfaces or in other defined regions like planes, volumes, etc. [1] These representations are usually interpreted as activation maps/volumes and the data analysis is often broken at this point. However, for statistical data analysis a method is needed which enables us to compare current density distributions for different formulations/hypotheses and within groups of patients or volunteers. To achieve this goal we propose to use a new technique based on a parameterization of current density distributions by means of *equivalent ellipsoids*. In this paper we would like to define the equivalent ellipsoid technique and to concentrate on the visualization aspects of proposed method. The results received during simulations with the models of extended sources placed in a realistically shaped torso-phantom are used as an illustration of the method.

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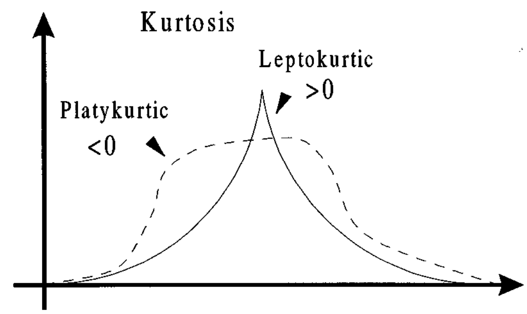

Fig. 1. Kurtosis values for different 1D distributions.

II. DATA VALIDATION PROCEDURES

A. Statistical Description of Data

For a statistical description of reconstructed extended current distributions we have used well known parameters [2] such as: *mean value*, which enables to estimate the value around which central clustering occurs, *variance*, a measure of "variability" around mean value, *standard deviation* σ—a measure of "width" of distribution around the mean value, and also less popular like: *skewness* (third central moment), which characterizes the degree of asymmetry of the distribution around its mean and *kurtosis*—a measure of the relative peakness/flatness of the distribution, defined below as (Fig. 1):

$$\mathrm{Kurt}(x_1, \ldots, x_N) = \left\{ \frac{1}{N} \sum_{i=1}^{N} \left[\frac{x_i - \bar{x}}{\sigma} \right]^4 \right\} - 3. \qquad (1)$$

The above parameters are used as the first step in validations of different extended current distributions. For more precise comparisons, we introduce an equivalent ellipsoid approach which enables not only a fast visualization but also a better interpretation of reconstructed currents.

B. Equivalent Ellipsoid

The equivalent ellipsoid is defined as an object connected with a certain concentration of current dipoles representing the current density distribution respectively to a predefined threshold. It is chosen for its geometrical simplicity (easy to draw) and straightforward interpretation of axes connected with

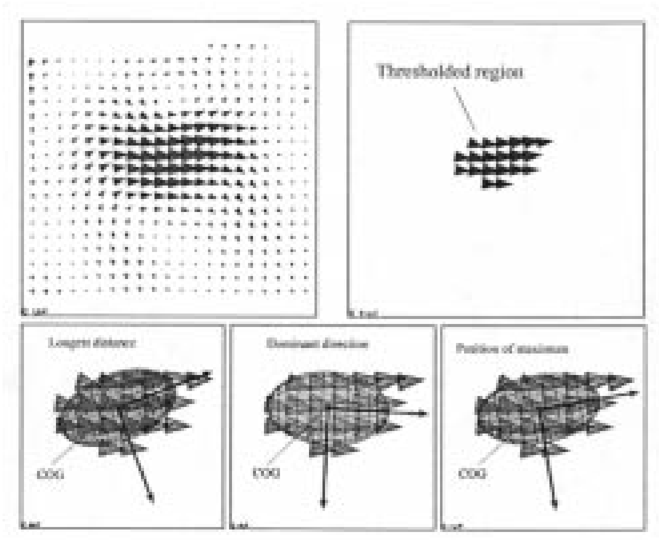

Fig. 2. Construction of equivalent ellipsoids: input current dipoles distribution (*upper left*), trimmed off region (*upper right*), found ellipsoids put together transparently with marked region for different strategies of founding main axis (*below*). COG denotes center of gravity of marked region.

dominant directions of current distribution. The problem of threshold choosing can be solved in certain cases by calculating a standard deviation of the distribution and then using it as a first threshold value. Usually, as we can see it later, the equivalent ellipsoid in that case is underestimated but it is also the only reasonable choice when we do not have any information about reconstructed sources. In the previous paper [3], we have described a simplified algorithm for equivalent ellipsoid construction when the extended current distribution is located in a source space in the form of the regular grid of points. The algorithm formulated below eliminates that restriction and it can be applied to any form of the source space—regular or nonregular one.

Construction of equivalent ellipsoid (Fig. 2):

1) Define threshold or use standard deviation as a selection guide for definition of active region.
2) Find center of gravity of marked region $(\mathbf{P}_{\rm COG})$.
3) Find direction of main axis of equivalent ellipsoid (a) using position of dipole with the longest distance from $\mathbf{P}_{\rm COG}$, (b) using position of the maximum of current density, (c) using an average direction for threshold area, and assign it as $\mathbf{a} = \mathbf{P}_1 - \mathbf{P}_{\rm COG}$.
4) Translate the distribution to the center of gravity $\mathbf{P}_{\rm COG}$.
5) Rotate the distribution—the vector a should lie on the z-axis of the new coordinate system.
6) Calculate an average length of a-axis of the equivalent ellipsoid using the current density modules and the z-coordinate of transformed distribution.
7) Find point $(\mathbf{P}_2)$ with the maximum distance from $\mathbf{P}_{\rm COG}$ projected on the new $x - y$ plane and use it as a direction of second axis of equivalent ellipsoid $(\mathbf{b} = \mathbf{P}_2 - \mathbf{P}_{{\rm COG}\,|\,x-y})$,
8) Find an average length of b-axis of equivalent ellipsoid using the current density modules and the distance from $\mathbf{P}_{\rm COG}$ calculated on $x - y$ plane.
9) Determine a direction of third axis as $\mathbf{c} = \mathbf{a} \times \mathbf{b}$.

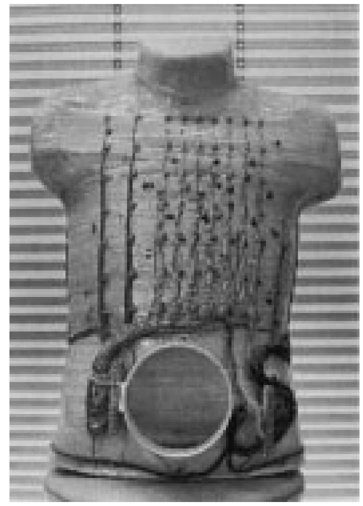

Fig. 3. Realistically shaped thorax phantom.

10) Rotate distribution once more, so that now the vector $\mathbf{c}$ points in the z-axis direction.
11) Find in the new coordinate system an average z-distance which gives a length of third axis $\mathbf{c}$.

To verify the equivalent ellipsoid approach we have formulated a test problem. The demand of precisely defined environment of experiments caused that we have chosen to that a reconstruction of a certain model of extended sources in a realistically shaped thorax phantom. In that case, the position and shape of the original source are well known and enable us to quantify the quality of reconstruction.

III. Test Example—“Butterfly” Source in Torso Phantom

A. Model Description

In the test study we have used a realistic thorax phantom [4] which was built as a tank-like model with a hollow space comparable to the body surface of a young male volunteer subject (Fig. 3).

The phantom model has been filled with 0.9% NaCl solution which corresponds to 14.4 mS/cm conductivity. Inside the tank, a model of extended source (*butterfly* source) has been installed (Fig. 4).

The complete setup of the model can be found on the WWW page under the following link: http://jenameg10.meg.uni-jena.de/romeo.htm. In the current paper, we present the results for the case of horizontal position of *butterfly* source. The measurements have been performed in a magnetically shielded room at the Biomagnetic Center in Jena, Germany [5]. The magnetic field has been recorded with a twin dewar biomagnetometer system with 2×31 channels.

The source reconstruction has been calculated using the CURRY™ software. The BEM [6] model was created from magnetic resonance imaging (MRI) data and it has been consisted of 8796 triangular elements with the average length of edges equals to 8 mm. We have used all methods available in CURRY™ system to reconstruct distributed current sources, but in the paper, we present only results for the most popular of them, i.e. minimum norm least squares (L2) [7], minimum

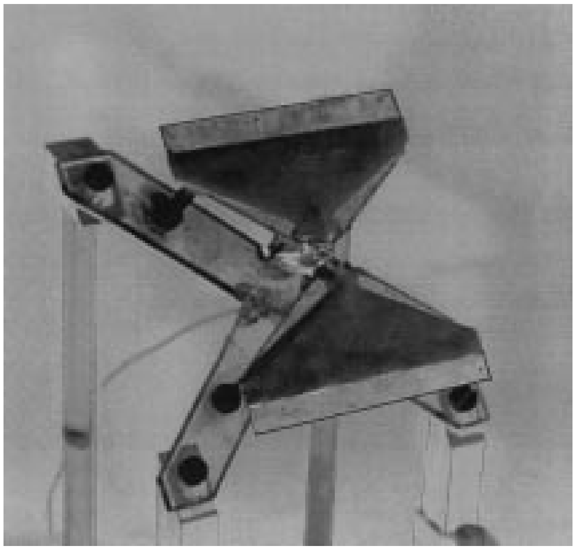

Fig. 4. *Butterfly* extended source model installed in thorax phantom.

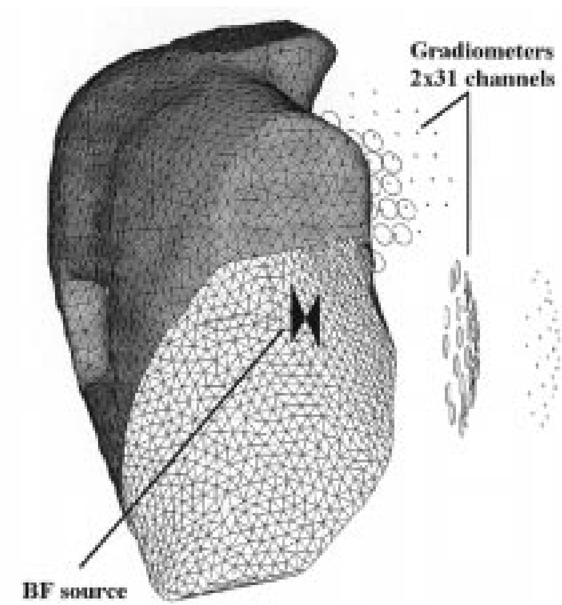

Fig. 5. BEM model used in test calculations (8796 triangles, 8 mm—average length of edges).

L_1-norm (L1) [8], and low resolution tomography, LORETA, [9] (LOR1, LOR2) applied to the source space defined as a regular grid positioned on the plane containing the original *butterfly* model. Fig. 5 shows cross-section of BEM model with the butterfly source and gradiometers positions.

B. Results of Simulations

Reconstruction of distributed sources has been performed by minimization an extended variance Δ^2 using minimum norm least squares (L_2) or minimum L_1-norm :

$$\Delta^2 = ||\mathbf{Lj} - \mathbf{m}||^p + \lambda||\mathbf{Wj}||^p, \quad p = 1, 2, \tag{2}$$

where $\mathbf{L}$ denotes a lead field matrix linking the current density vector $\mathbf{j}$ with the forward calculated data, $\mathbf{m}$ is a measured data vector and $\mathbf{W}$ is a diagonal location weighting matrix.

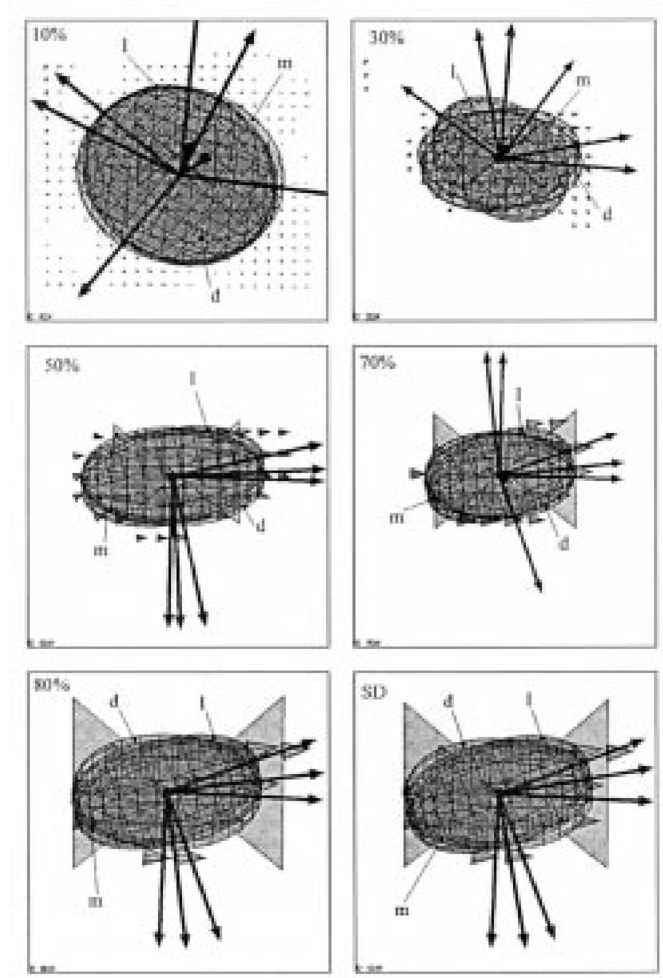

Fig. 6. Equivalent ellipsoids found using different strategies of main axis determination (l—longest distance, d—dominant direction, m—maximum position) for different threshold levels (10%–80%) and threshold estimated on the basis of standard deviation (SD) (L_2-norm extended source reconstruction).

Fig. 6 shows the equivalent ellipsoids calculated for various threshold levels together with the *butterfly* source region located in the background.

The threshold level has been defined with reference to the maximum of current dipole moment found in the input distribution and it has been chosen in an arbitrary way from the range between 10% and 80%. The resulting ellipsoids calculated on the basis of standard deviation for given distribution have been also shown. For every threshold level the ellipsoids corresponding to the three strategies for estimation of ellipsoid main axes have been calculated (using position of current dipole with the longest distance from COG of trimmed off distribution, using dominant direction estimated for marked region, and using position of dipole with the maximum of current dipole moment).

Comparing different strategies of estimation of main axis in equivalent ellipsoid, we can observe that calculated ellipsoids for every threshold level are not far removed from each other. However, this is only valid when the reconstructed distribution is restricted to the plane. In other cases, e.g. 3D-regular grids, free surface distributions, the best results can be achieved when

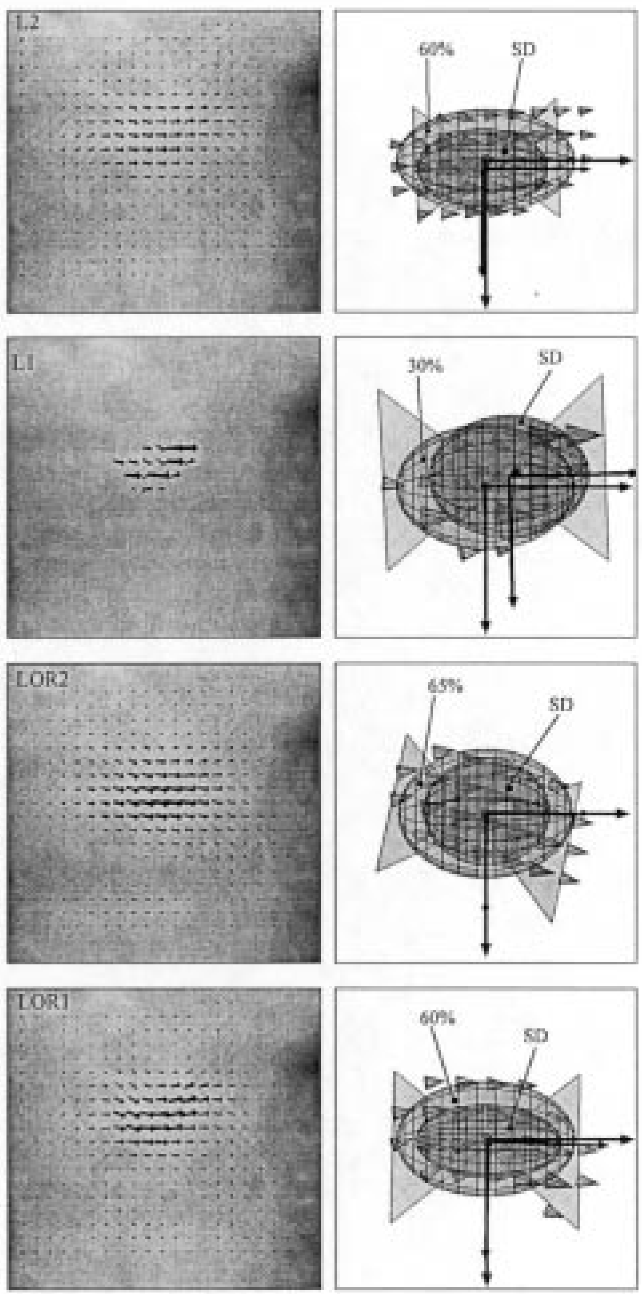

Fig. 7. Reconstructed current density distributions for different methods in the form of current dipoles sets accompanied by equivalent ellipsoids found using threshold based on standard deviation and selected one to cover original *butterfly* source area.

the strategy based on the dipole with longest distance from COG is applied.

Fig. 7 shows extended current distributions in the form of current dipoles sets reconstructed from the measured magnetic field produced by *butterfly* model. The following different approaches have been applied: minimum norm least squares (L2), minimum L_1-norm (L1), and low resolution tomography with two different norms (L_2, L_1) for data and model terms (2). The equivalent ellipsoids have been calculated using dominant direction technique. The smaller ellipsoids represent equivalent ellipsoids found when the standard deviation has been used for definition of threshold level (SD). The other ellipsoids have been estimated individually for every case to cover the *butterfly* source area. As we can see, in all cases, the standard deviation approach gives underestimated ellipsoid, but nevertheless, the characteristic features of every method, i.e. spread solution for L_2-norm methods and more focal solution for L_1-norm localizations, are proper found.

IV. Conclusion

The equivalent ellipsoid approach enables to overcome the difficulties of visualization and interpretation of large amount of data which usually appears during reconstruction of distributed sources over the regular and nonregular grids. It is a fast and reliable tool for extracting the most important features of found current distributions and it gives a good starting point to statistical comparison of various methods.

References

[1] M. Fuchs, H.-A. Wischmann, and O. Dössel, "Overlay of biomagnetic current reconstructions and morphological MR images," in *Proc. of IEEE Symposium on Neuroscience and Technology*, Lyon, 1992, pp. 56–59.

[2] W. H. Press, S. A. Teukolsky, W. T. Veterling, and B. P. Flannery, *Numerical Recipes in C*: Cambridge University Press, 1992.

[3] M. Ziolkowski, J. Haueisen, H. Nowak, and H. Brauer, "Visualization of current density distributions in biomagnetic inverse problems," presented at the Proc. of 9th International Symposium on Electromagnetic Fields in Electrical Engineering ISEF'99, Pavia, Italy, Sept. 1999, pp. 23–25.

[4] U. Tenner, J. Haueisen, H. Nowak, U. Leder, and H. Brauer, "Source localization in an inhomogeneous physical thorax phantom," *Phys. Med. Biol.*, vol. 44, pp. 1969–1981, 1999.

[5] *Magnetism in Medicine—A Handbook*, W. Andrä and H. Nowak, Eds., Wiley-VCH, Berlin, 1998, p. 117.

[6] M. Fuchs, R. Drenckhahn, H.-A. Wischmann, and M. Wagner, "An improved boundary element method for realistic volume-conductor modeling," *IEEE Trans. Biomed. Eng.*, vol. BME-45, no. 8, pp. 980–997, 1998.

[7] J. T. Nenonen, M. S. Hämäläinen, and R. J. Ilmoniemi, "Minimum-norm estimation in a boundary-element torso model," *Medical & Biol. Engineering & Computing*, pp. 43–48, Jan. 1994.

[8] M. Wagner, M. Fuchs, H.-A. Wischmann, R. Drenckhahn, and T. Kohler, "Current density reconstructions using the L1 norm," in *Advances in Biomagnetism Reserch: Biomag96*, New York, 1998, Springer-Verlag.

[9] R. D. Pascual-Marqui, "LORETA in 3D solution space," *ISBET Newsletter*, vol. 3, pp. 22–28, 1995.