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10.

# Inheritance

# Hierarchy and inheritance

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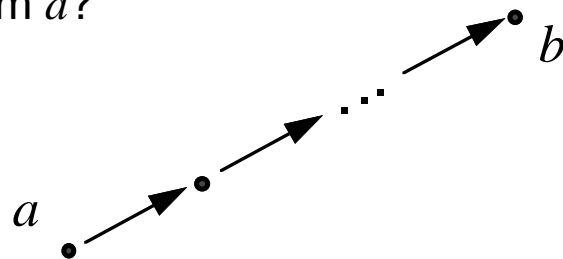
As we noticed with both frames and description logics, hierarchy or taxonomy is a natural way to view the world

importance of *abstraction* in remembering and reasoning

- groups of things share properties in the world
- do not have to repeat representations  
e.g. sufficient to say that “elephants are mammals” to know a lot about them

Inheritance is the result of transitivity reasoning over paths in a network

- for strict networks, *modus ponens* (if-then reasoning) in graphical form
- “does  $a$  inherit from  $b$ ?” is the same as “is  $b$  in the transitive closure of **:IS-A** (or subsumption) from  $a$ ?”



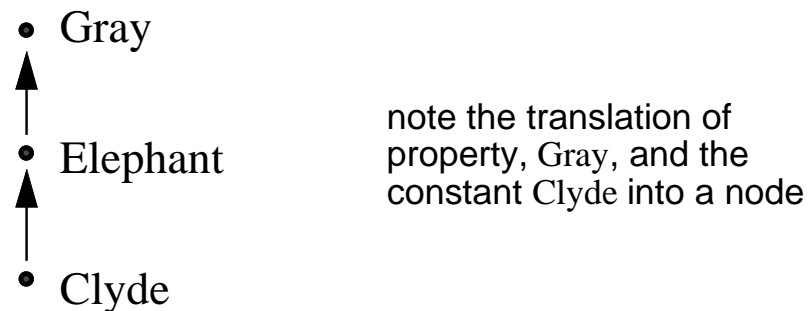
graphically, is there a path of **:IS-A** connections from  $a$  to  $b$ ?

# Path-based reasoning

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## Focus just on inheritance and transitivity

- many interesting considerations in looking just at where information comes from in a network representation
- abstract frames/descriptions, and properties into nodes in graphs, and just look at reasoning with paths and the conclusions they lead us to

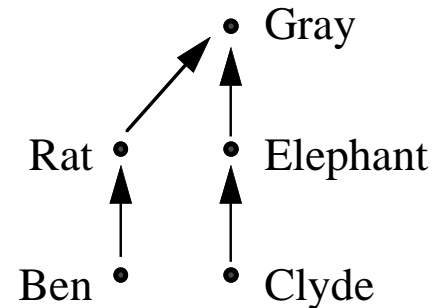


- edges in the network: Clyde·Elephant, Elephant·Gray
- paths included in this network: edges plus {Clyde·Elephant·Gray}  
in general, a path is a sequence of 1 or more edges
- conclusions supported by the paths:  
Clyde → Elephant; Elephant → Gray; Clyde → Gray

# Inheritance networks

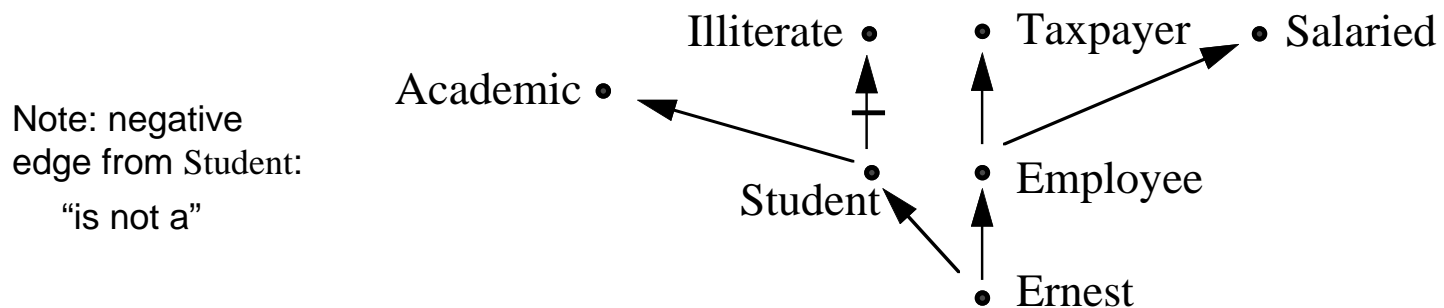
## (1) Strict inheritance in trees

- as in description logics
- conclusions produced by complete transitive closure on all paths (any traversal procedure will do); all reachable nodes are implied



## (2) Strict inheritance in DAGs

- as in DL's with multiple AND parents (= multiple inheritance)
- same as above: all conclusions you can reach by any paths are supported

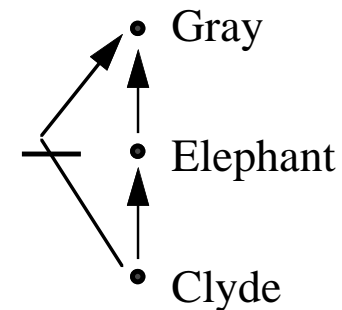


# Inheritance with defeasibility

## (3) Defeasible inheritance

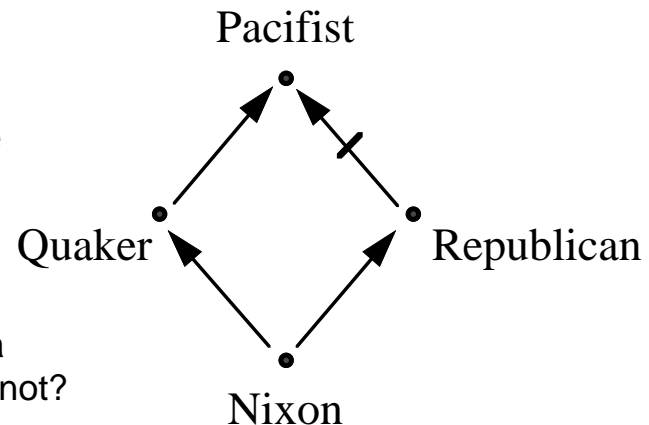
- as in frame systems
- inherited properties do not always hold, and can be *overridden* (defeated)
- conclusions determined by searching upward from “focus node” and selecting first version of property you want

while elephants in general are gray, Clyde is not



## A key problem: *ambiguity*

- *credulous* accounts choose arbitrarily
- *skeptical* accounts are more conservative



# Shortest path heuristic

## Defeasible inheritance in DAGs

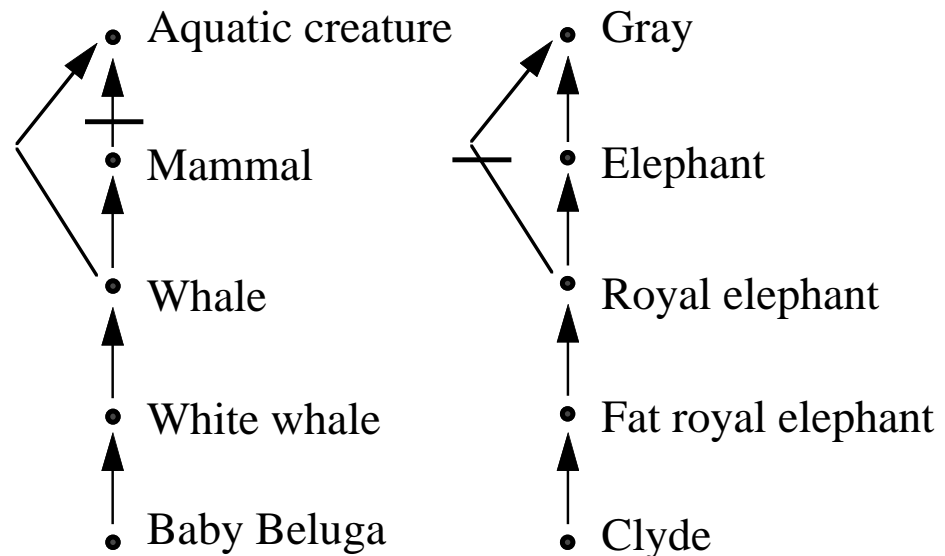
- links have *polarity* (positive or negative)
- use shortest path heuristic to determine which polarity counts

Intuition: inherit from the most specific subsuming class

- as a result, not all paths count in generating conclusions
  - some are “preempted”
  - but some are “admissible”

think of paths as *arguments* in support of conclusions

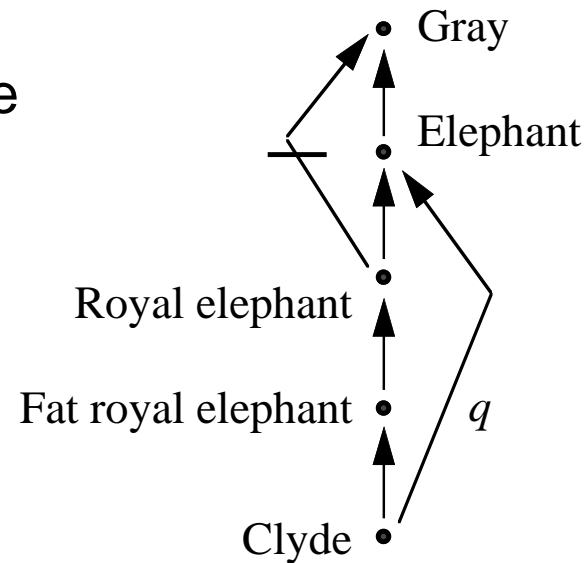
⇒ the inheritance problem = what are the admissible conclusions?



# Problems with shortest path

1. Shortest path heuristic produces incorrect answers in the presence of redundant edges (which are already implied!)

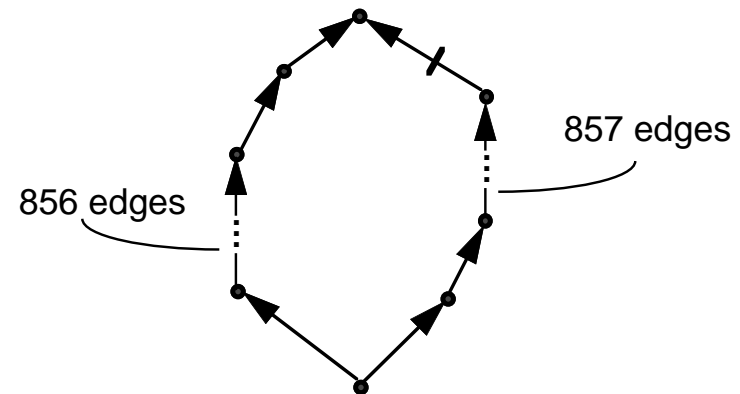
the redundant edge  $q$ ,  
expressing that Clyde is an  
Elephant changes polarity of  
conclusion about color



2. Anomalous behavior with ambiguity

adding 2 edges to the  
left side changes the  
conclusion!

Why should length be a factor?  
This network should be ambiguous...



# Specificity criteria

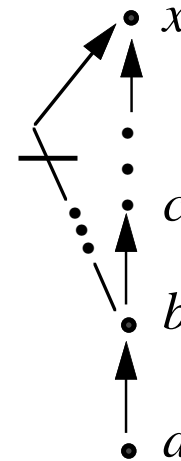
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Shortest path is a specificity criterion (sometimes called a preemption strategy) which allows us to make admissibility choices among competing paths

- It's not the only possible one
- Consider “*inferential distance*”:  
not linear distance, but topologically based
  - a node  $a$  is nearer to node  $b$  than to node  $c$   
if there is a path from  $a$  to  $c$  through  $b$
  - idea: conclusions from  $b$  preempt those from  $c$

This handles  $\text{Clyde} \rightarrow \neg\text{Gray}$  just fine,  
as well as redundant links

- But what if path from  $b$  to  $c$  has some of its  
edges preempted? what if some are redundant?





# A formalization (Stein)

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An inheritance hierarchy  $\Gamma = \langle V, E \rangle$  is a directed, acyclic graph (DAG) with positive and negative edges, intended to denote “(normally) is-a” and “(normally) is-not-a”, respectively.

- positive edges are written  $a \cdot x$
- negative edges are written  $a \cdot \neg x$

A sequence of edges is a path:

- a positive path is a sequence of one or more positive edges  $a \cdot \dots \cdot x$
- a negative path is a sequence of positive edges followed by a single negative edge  $a \cdot \dots \cdot v \cdot \neg x$

Note: there are no paths with more than 1 negative edge.

Also: there might be 0 positive edges.

A path (or argument) supports a conclusion:

- $a \cdot \dots \cdot x$  supports the conclusion  $a \rightarrow x$  ( $a$  is an  $x$ )
- $a \cdot \dots \cdot \neg x$  supports  $a \not\rightarrow x$  ( $a$  is not an  $x$ )

Note: a conclusion may be supported by many arguments

However: not all arguments are equally believable...

# Support and admissibility

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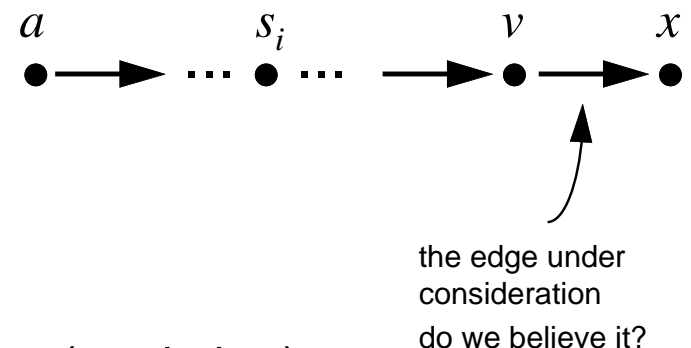
$\Gamma$  supports a path  $a \cdot s_1 \cdot \dots \cdot s_n \cdot (\neg)x$  if the corresponding set of edges  $\{a \cdot s_1, \dots, s_n \cdot (\neg)x\}$  is in  $E$ , and the path is admissible according to specificity (see below).

the hierarchy supports a conclusion  $a \rightarrow x$  (or  $a \nrightarrow x$ )  
if it supports some corresponding path

A path is admissible if every edge in it is admissible.

An edge  $v \cdot x$  is admissible in  $\Gamma$  wrt  $a$  if there is a positive path  $a \cdot s_1 \dots s_n \cdot v$  ( $n \geq 0$ ) in  $E$  and

1. each edge in  $a \cdot s_1 \dots s_n \cdot v$  is admissible in  $\Gamma$  wrt  $a$  (recursively);
2. no edge in  $a \cdot s_1 \dots s_n \cdot v$  is redundant in  $\Gamma$  wrt  $a$  (see below);
3. no intermediate node  $a, s_1, \dots, s_n$  is a preemptor of  $v \cdot x$  wrt  $a$  (see below).

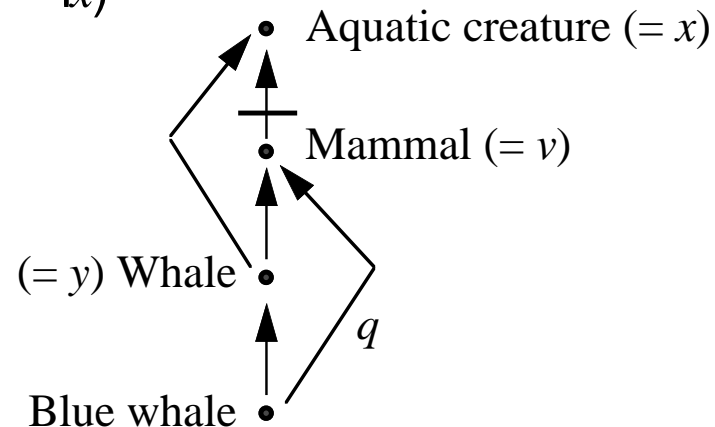


A negative edge  $v \cdot \neg x$  is handled analogously.

# Preemption and redundancy

A node  $y$  along path  $a \cdot \dots \cdot y \cdot \dots \cdot v$  is a preemptor of the edge  $v \cdot x$  wrt  $a$  if  $y \cdot \neg x \in E$  (and analogously for  $v \cdot \neg x$ )

for example, in this figure  
the node Whale preempts  
the negative edge from  
Mammal to Aquatic creature  
wrt both Whale and Blue whale



A positive edge  $b \cdot w$  is redundant in  $\Gamma$  wrt node  $a$  if there is some positive path  $b \cdot t_1 \cdot \dots \cdot t_m \cdot w \in E$  ( $m \geq 1$ ), for which

1. each edge in  $b \cdot t_1 \cdot \dots \cdot t_m$  is admissible in  $\Gamma$  wrt  $a$ ;
2. there are no  $c$  and  $i$  such that  $c \cdot \neg t_i$  is admissible in  $\Gamma$  wrt  $a$ ;
3. there is no  $c$  such that  $c \cdot \neg w$  is admissible in  $\Gamma$  wrt  $a$ .

The edge labelled  $q$  above is redundant

The definition for a negative edge  $b \cdot \neg w$  is analogous

# Credulous extensions

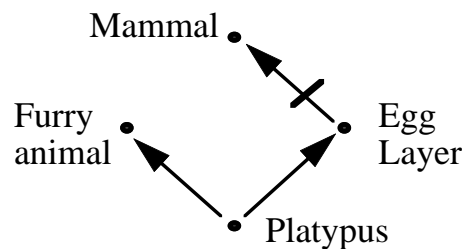
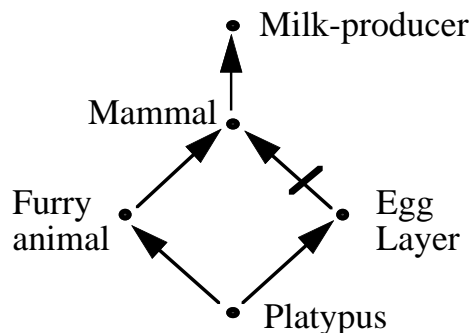
$\Gamma$  is  $a$ -connected iff for every node  $x$  in  $\Gamma$ , there is a path from  $a$  to  $x$ , and for every edge  $v \cdot (\neg) x$  in  $\Gamma$ , there is a *positive* path from  $a$  to  $v$ .

In other words, every node and edge is reachable from  $a$

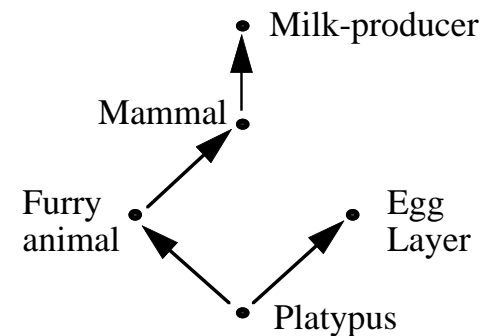
$\Gamma$  is (potentially) ambiguous wrt a node  $a$  if there is some node  $x \in V$  such that both  $a \cdot s_1 \dots s_n \cdot x$  and  $a \cdot t_1 \dots t_m \cdot \neg x$  are paths in  $\Gamma$

A credulous extension of  $\Gamma$  wrt node  $a$  is a maximal unambiguous  $a$ -connected subhierarchy of  $\Gamma$  wrt  $a$

If  $X$  is a credulous extension of  $\Gamma$ , then adding an edge of  $\Gamma$  to  $X$  makes  $X$  either ambiguous or not  $a$ -connected



**Extension 1**



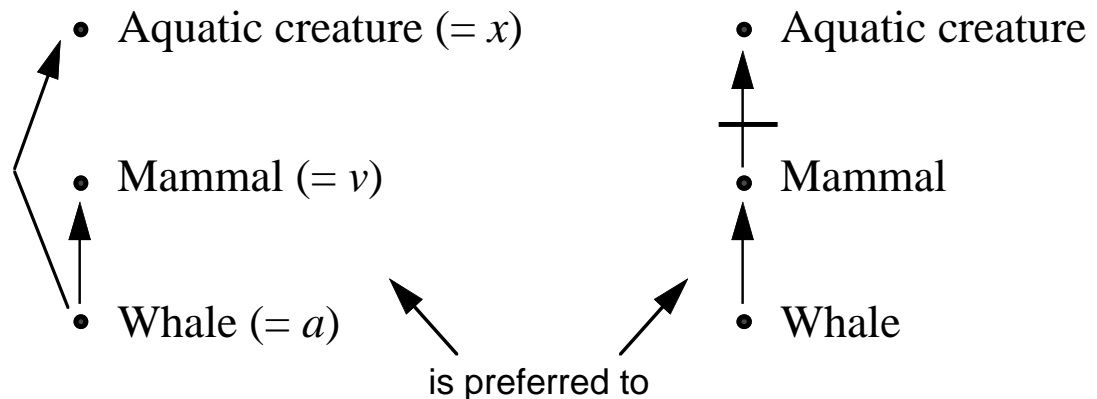
**Extension 2**

# Preferred extensions

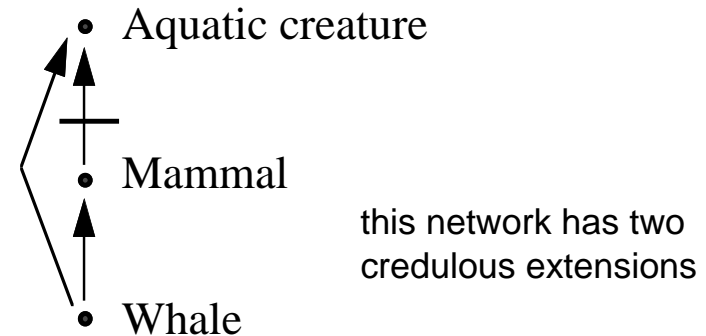
Credulous extensions do not incorporate any notion of admissibility or preemption.

Let  $X$  and  $Y$  be credulous extensions of  $\Gamma$  wrt node  $a$ .  $X$  is preferred to  $Y$  iff there are nodes  $v$  and  $x$  such that:

- $X$  and  $Y$  agree on all edges whose endpoints precede  $v$  topologically,
- there is an edge  $v \cdot x$  (or  $v \cdot \neg x$ ) that is *inadmissible* in  $\Gamma$ ,
- this edge is in  $Y$ , but not in  $X$ .



A credulous extension is a preferred extension if there is no other extension that is preferred to it.



# Subtleties

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## What to believe?

- “credulous” reasoning: choose a preferred extension and believe all the conclusions supported
- “skeptical” reasoning: believe the conclusions from any path that is supported by all preferred extensions
- “ideally skeptical” reasoning: believe the conclusions that are supported by all preferred extensions

note: ideally skeptical reasoning cannot be computed in a path-based way (conclusions may be supported by different paths in each extension)

## We’ve been doing “upwards” reasoning

- start at a node and see what can be inherited from its ancestor nodes
- there are many variations on this definition; none has emerged as the agreed upon, or “correct” one
- an alternative looks from the top and sees what propagates down  
upwards is more efficient