

06.11.2014

Теория на игрите - консултация
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5

Матрична игра 2×2 - първата има оптимална чиста стратегия $\Rightarrow$ втората има оптимална чиста стратегия

$$p(x, y) = a_{11}x_1y_1 + a_{12}x_1y_2 + a_{21}x_2y_1 + a_{22}x_2y_2$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$p(x^*, y^*)$$

$$p(x, y^*) \leq p(x^*, y^*) \leq p(x^*, y) \quad \forall x \in X \quad \forall y \in Y$$

$$x^* = (1, 0)$$

$$a_{11}y_1^* + a_{12}y_2^* \leq a_{11}y_1 + a_{12}y_2$$

$$f(y) = a_{11}y_1 + a_{12}y_2$$

$$\min f(y)$$

$$y_1 + y_2 = 1 \quad y_2 = 1 - y_1$$

$$\min_{y_1 \in [0, 1]} f(y) = \min_{y_1 \in [0, 1]} a_{11}y_1 + (1 - y_1)a_{12} = (a_{11} - a_{12})y_1 + a_{12}$$

$$\text{I сл. } a_{11} - a_{12} > 0 \Rightarrow y_1^* = 0$$

$$\text{II сл. } a_{11} - a_{12} < 0 \Rightarrow y_1^* = 1$$

$$\text{III сл. } a_{11} - a_{12} = 0; \{a_{11} = a_{12}\} = V$$

$$\begin{pmatrix} v & v \\ a & b \end{pmatrix} \begin{pmatrix} v \\ a \end{pmatrix}$$

$$\text{Целта } a \leq b$$

$$\bar{x} = (0, 1)$$

$$p(\bar{x}, y^*) = a y_1^* + b y_2^* \leq v (y_1^* + y_2^*) = v$$

$$v = a(y_1^* + y_2^*)$$

6 игра

матрица $A = \{a_{ij}\}_{2009 \times 2009}$

$$a_{ij} = i - j \text{ за } \forall i, j$$

~~и~~ оптимальна стратегия на I е оптимальна и за II

$$a_{ji} = j - i = -a_{ij}$$

$$A^T = -A$$

антагонистични игри

$$p(x, y) = x^T A y$$

$$(p(x, y))^T = (x^T A y)^T = y^T A^T x = -y^T A x$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_{2009} \end{pmatrix}$$

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_{2009} \end{pmatrix}$$

$$p(x, x) = x^T A x$$

$$p(x, x) = -x^T A x$$

$$2x^T A x = 0$$

$$x^T A x = 0 = p(x, x)$$

$$\max_{x \in X} (\min_{y \in Y} p(x, y))$$

$$\min_{y \in Y} p(x, y) \leq 0 = p(x, x) \leq \max_{y \in Y} p(x, y)$$

$$\left. \begin{array}{l} \max_{x \in X} v(x) \leq 0 \\ v_{II} \geq 0 \end{array} \right\} \Rightarrow v = 0 \quad \text{За оптимальни: } v_I = v_{II} \Rightarrow v = 0$$

$$p(i, y^*) \leq 0 \leq p(x^*, j)$$

$$i = 1 \dots 2009$$

$$j = 1 \dots 2009$$

$$x^*$$

$$p(x^*, i) \geq 0 \Leftrightarrow p(i, x^*) \leq 0$$

$$\sum_{j=1}^{2009} a_{ji} x_i^* \geq 0 \Leftrightarrow \sum_{j=1}^{2009} a_{ij} x_j^* \geq 0$$

$$\sum_{j=1}^{2009} -a_{ij} x_j^* \geq 0$$

$$\sum_{j=1}^{2009} a_{ij} x_j^* \leq 0$$

7 игра от матрица $A = \begin{pmatrix} 4 & -3 \\ -1 & 3 \\ 0 & -1 \\ -3 & 0 \end{pmatrix}$

$\min(x, y)$

$\begin{pmatrix} 4 & -3 \\ -1 & 3 \\ 0 & -1 \\ -3 & 0 \end{pmatrix}$

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$\max(\min(x, y))$

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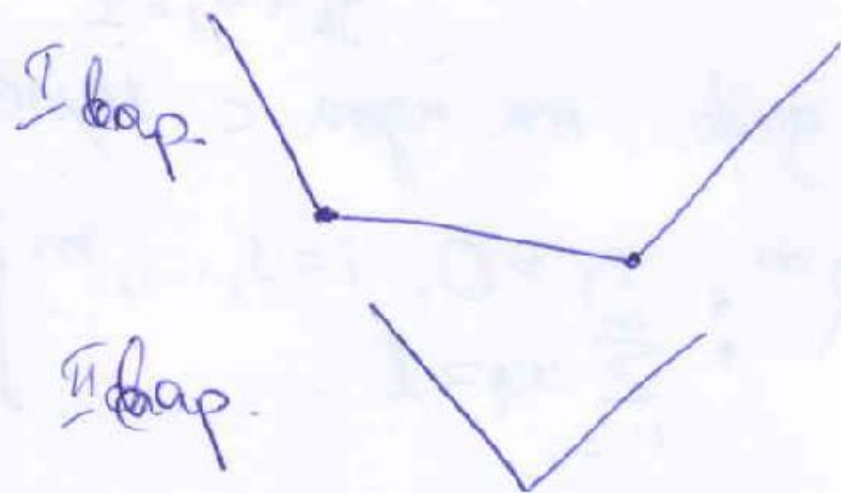
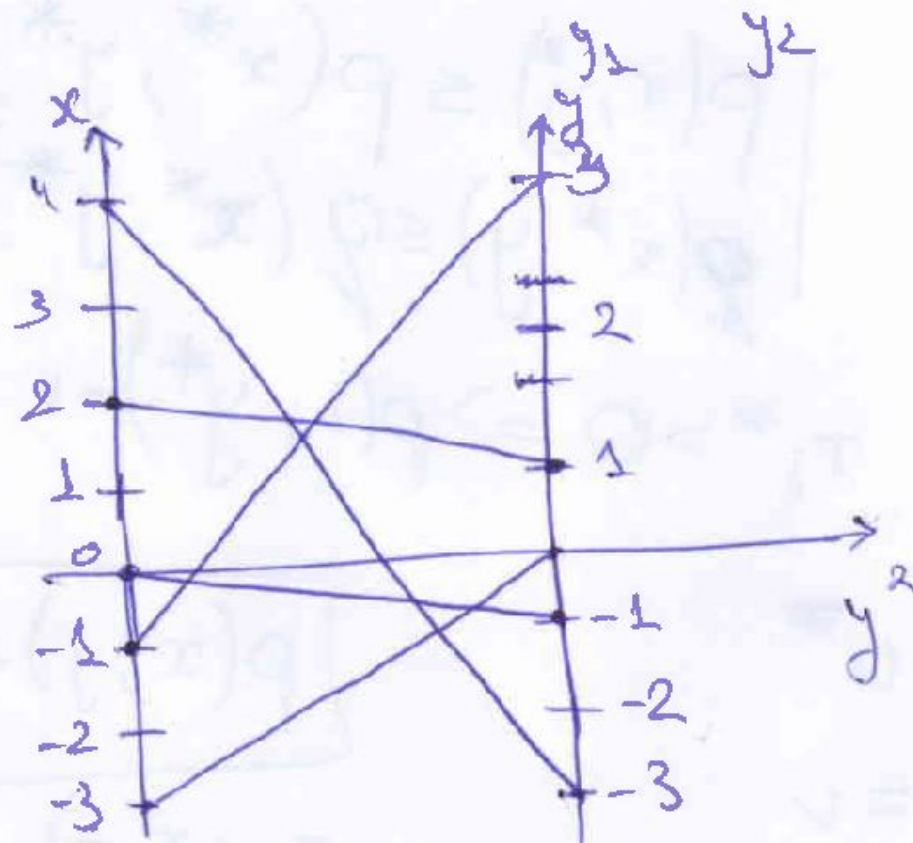
$\max(\min(x, y))$

$$y_1^* > 0$$

$$y_2^* > 0$$

$$x^* (0, \frac{4}{5}, \frac{1}{5}, 0, 0)$$

$$y^* (\frac{2}{5}, \frac{3}{5})$$



за $y \rightarrow \max$ траектория
в най-ниската точка

за $x \rightarrow \min$ траектория
в най-високата точка



$$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

$$\begin{cases} 2x_1 - x_2 = v \\ x_1 + 3x_2 = v \\ x_1 + x_2 = 1 \end{cases}$$

$$\begin{cases} x_1 - 4x_2 \geq 0 \\ x_1 + x_2 \geq 1 \end{cases}$$

$$\begin{cases} x_1^* = \frac{4}{5} \\ x_2^* = \frac{1}{5} \end{cases} \quad v = \frac{7}{5}$$

$$\begin{cases} 2y_1 + y_2 = v \\ -y_1 + 3y_2 = v \\ y_1 + y_2 = 1 \end{cases}$$

$$3y_1 - 2y_2 \geq 0$$

$$\begin{cases} y_1^* = \frac{2}{5} \\ y_2^* = \frac{3}{5} \end{cases}$$

8

$$A = \begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$$

max игра
Игра
 u^*

$$B = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$

max игра
Игра
 v^*

x_1	4	2
x_2	5	3

y_1 y_2

равновесие по Неш

$$u_i(x_i, y^*) \leq u_i(x_i^*, y^*)$$

$$\forall x_i \in X$$

$$u_i(x, y) = a_{ij}$$

$$x^* = (0, 1)$$

$$y^* = (0, 1)$$

$$\begin{cases} p(x, y^*) \leq p(x^*, y^*) \\ q(x^*, y) \leq q(x^*, y^*) \end{cases}$$

$$x_i^* > 0 \Rightarrow p(i; y^*) = v$$

$$p(1, y^*) = p(2, y^*) = u^*$$

$$q(x^*, 1) = q(x^*, 2) = v$$

$$p(x; j) = \sum_{i=1}^m a_{ij} x_i$$

$$x_1 + x_2 = 1$$

$$y_1 + y_2 = 1$$

9 $p(x, y) + q(x, y) = 0$ - геоф. на игра с нулевой суммой

$$X = \{(x_1, x_2, \dots, x_n) \in R^m : x_i \geq 0; i=1, \dots, m; \sum_{i=1}^m x_i = 1\}$$

$$Y = \{(y_1, y_2, \dots, y_n) \in R^n : y_j \geq 0; j=1, \dots, n; \sum_{j=1}^n y_j = 1\}$$

$$p(x, y^*) \leq p(x^*, y^*)$$

$$q(x^*, y) \leq q(x^*, y^*)$$

$$-p(x^*, y) \leq -p(x^*, y^*)$$

$$p(x, y^*) \leq p(x^*, y^*) \leq p(x^*, y)$$

$$p(x^*, y^*) = v \quad \forall I = \max_{x \in X} (\min_{y \in Y} p(x, y))$$

$$\parallel \quad \forall II = \min_{y \in Y} (\max_{x \in X} p(x, y))$$

$$\parallel \quad v$$

9



Бинарична игра

$$A = p(x, y) = \sum_{i=1}^m \sum_{j=1}^n a_{ij} x_i y_j$$

$$B = q(x, y) = \sum_{i=1}^m \sum_{j=1}^n b_{ij} x_i y_j$$

) Огавани пеганоси

$$F: X \times Y \rightarrow X \times Y$$

$$F(x, y) \rightarrow (\bar{x}, \bar{y})$$

$$x = (x_1, x_2, \dots, x_m)$$

$$y = (y_1, y_2, \dots, y_n)$$

$$\bar{x}_i = \frac{x_i + \max\{0, p(i, y) - p(x, y)\}}{1 + \sum_{k=1}^m \max\{0, p(k, y) - p(x, y)\}}$$

$$\bar{y}_j = \frac{y_j + \max\{0, q(x, j) - q(x, y)\}}{1 + \sum_{l=1}^n \max\{0, q(x, l) - q(x, y)\}}$$

$$p(x, y^*) \leq p(x^*, y^*)$$

$$q(x^*, y) \leq q(x^*, y^*) - \text{неподвижната точка}$$

11) 4 от 6-те аксиомы на Нейш

$$(S, (u^*, v^*)) \rightarrow (\bar{u}, \bar{v})$$

u^*, v^* - индивидуални максимум
 S - таа, ако си сътрудничим

1. $(\bar{u}, \bar{v}) \in S$

2. $\bar{u} \geq u^*$
 $\bar{v} \geq v^*$

3. $\nexists$ вектор $(\tilde{u}, \tilde{v}) \in S$
 $\tilde{u} \geq \bar{u}$ или $\tilde{u} > \bar{u}$
 $\tilde{v} \geq \bar{v}$ или $\tilde{v} > \bar{v}$

4. $(S, (u^*, v^*)) \rightarrow (\bar{u}, \bar{v}) \in T$ - изникнало, ~~ограничено~~, затворено

$$(T, (u^*, v^*)) \rightarrow (\bar{u}, \bar{v})$$

5. $L = \begin{cases} u = \alpha_1 u + \beta_1 \\ v = \alpha_2 u + \beta_2 \end{cases} \quad \alpha_1, \alpha_2 > 0$
линейн оператор

6. Ако от $(u, v) \in S \Rightarrow (v, u) \in S \Rightarrow$
 $u^* = v^*$

$$(S, (u^*, u^*)) \rightarrow (\bar{v}, \bar{v})$$

$$(L(S), L(u^*, v^*)) \rightarrow L(\bar{u}, \bar{v})$$

$$\begin{cases} \max_{(u,v) \in S} g(u, v) \\ g(u, v) \geq (u - u^*)(v - v^*) \\ u \geq u^* \\ v \geq v^* \end{cases}$$

12

$$N = \{1, 2, \dots, n\}$$

$$V: 2^N \rightarrow \mathbb{R}$$

$$|2^N| = 2^n$$

$$1. v(\emptyset) = 0$$

$$2. v(S \cup T) \geq v(S) + v(T) \text{ за } S \cap T = \emptyset > \text{характеристика попарно независимости (коалиций)}$$

$$x = (x_1, x_2, \dots, x_n) \text{ — вектор } x \text{ — игра}$$

$$1. x_i \geq v(\{i\})$$

$$2. \sum_{i=1}^n x_i = v(N)$$

13 $y = (y_1, y_2, \dots, y_n)$

$$y \geq x$$

$$1. y_i \geq x_i, i \in S$$

$$2. \sum_{i \in S} y_i \leq v(S)$$

14 $a = (330, 490, 180)$

$$b = (330, 500, 190)$$

$$v(\{1\}) = 200$$

$$v(\{2\}) = 300$$

$$v(\{3\}) = 0$$

$$v(\{1, 2\}) = 800$$

$$v(\{2, 3\}) = 650$$

$$v(\{1, 3\}) = 500$$

$$v(\{1, 2, 3\}) = 1000$$

$$v(\{1\}) + v(\{3\}) < v(\{1, 3\})$$

— характеристическая игра

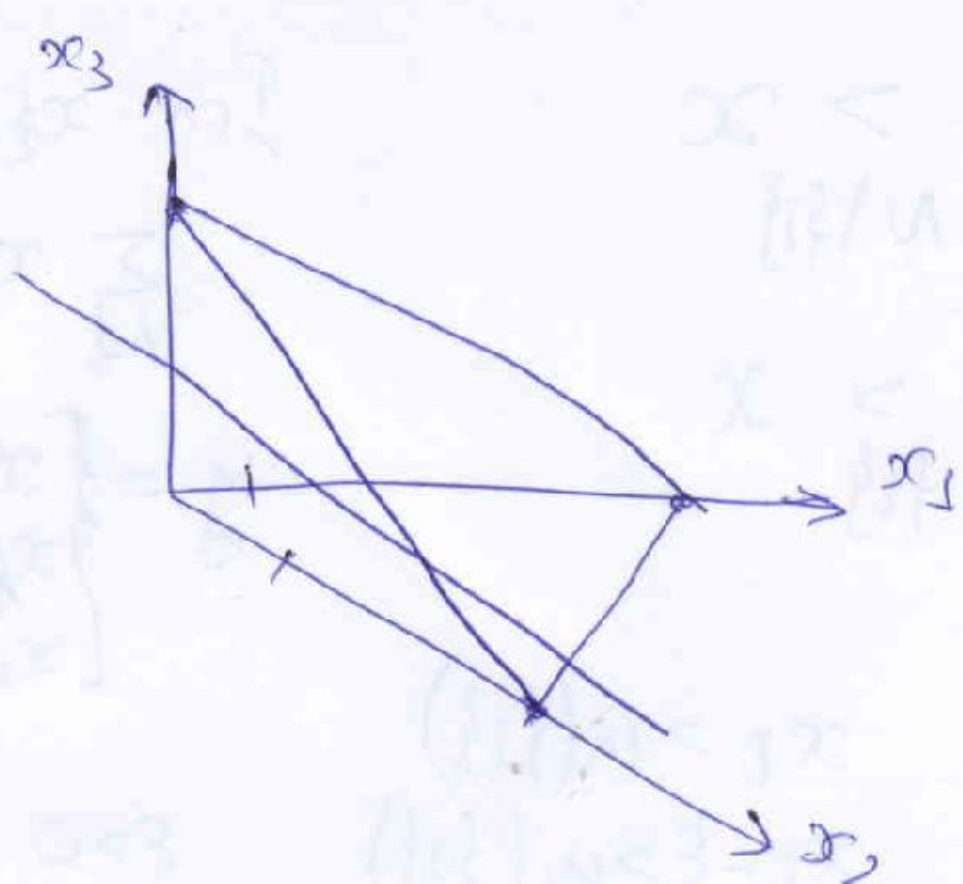
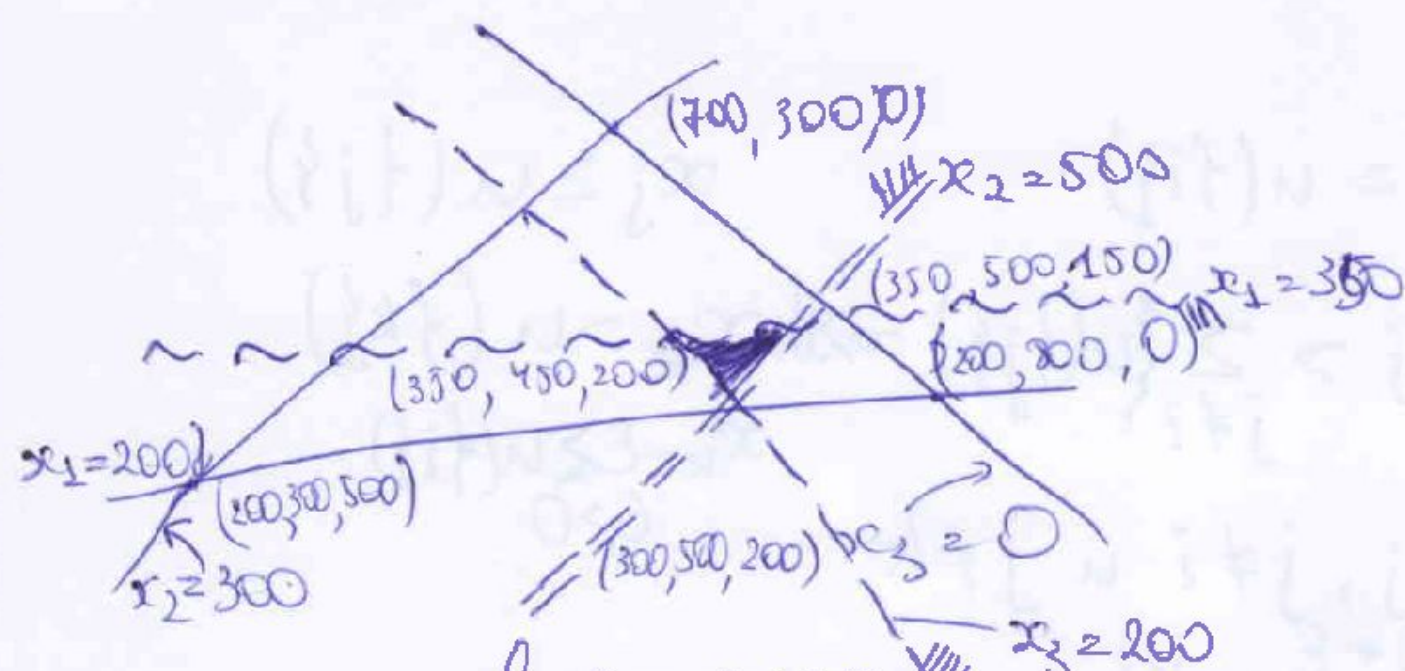
$$x \in C(v) \Leftrightarrow \begin{cases} \sum_{i \in S} x_i \geq v(S) \\ \sum_{i=1}^n x_i = v(N) \end{cases}$$

$$N = \{1, 2, 3\}$$

$$b \notin C(v)$$

$$b_1 + b_2 + b_3 = 1020 > 1000 = v(\{1, 2, 3\})$$

6



за да е в играта

$$\sum_{i \in S} x_i \geq v(S)$$

$$\begin{cases} x_1 + x_2 \geq 800 \\ x_2 + x_3 \geq 650 \\ x_1 + x_3 \geq 500 \end{cases} \iff \begin{cases} x_3 \leq 200 \\ x_1 \leq 350 \\ x_2 \leq 500 \end{cases}$$

a, b, c

$$C(u) = \left\{ x \in \mathbb{R}^n : \begin{aligned} &x = d_1 a + d_2 b + d_3 c \\ &d_1 \geq 0 \\ &d_2 \geq 0 \\ &d_3 \geq 0 \\ &d_1 + d_2 + d_3 = 1 \end{aligned} \right\}$$

15 a) $\sum_{i=1}^n u(\{i\}) < u(N)$ $N = \{1, \dots, n\}$

b) $u(N) = u(\{i\}) + u(N \setminus \{i\})$

$C(u) = \emptyset$

Допускаме ~~не~~ противното, т.е. $\exists x \in C(u)$ $\begin{cases} \sum_{i \in S} x_i \geq u(S) \\ \sum_{i=1}^n x_i = u(N) \end{cases}$

$\Rightarrow \sum_{i=1}^n x_i \geq \sum_{i=1}^n u(\{i\})$

$x_i \geq u(\{i\})$

$u(N) = \sum_{i=1}^n x_i > \sum_{i=1}^n u(\{i\}) < u(N)$

$u(\{i\}) + u(N \setminus \{i\}) > \sum_{j=1}^n u(\{j\})$

$u(N \setminus \{j\}) > \sum_{j=1}^n u(\{j\})$

$$y > x$$

$$N \setminus \{i\}$$

$$\text{con. } x_i \geq u(\{i\})$$

$$x_j \geq u(\{j\})$$

$$\sum_{j \neq i} x_j > \sum_{j \neq i} u(\{j\}) \Rightarrow \exists k: x_k > u(\{k\})$$

$$x_k - \varepsilon \geq u(\{i\})$$

$$\varepsilon > 0$$

$$y = \begin{cases} x_j, & j \neq i \text{ u } j \neq k \\ x_k + \varepsilon, & j = i \\ x_k - \varepsilon, & j = k \end{cases}$$

$$\text{con. } x_i \geq u(\{i\})$$

$$x_i - \varepsilon > u(\{i\}) \quad \varepsilon > 0$$

$$y_j = \begin{cases} x_i - \varepsilon, & j = i \\ x_j + \frac{\varepsilon}{|N \setminus \{i\}|}, & j \neq i \end{cases}$$

$$y > x$$

$$N \setminus \{i\}$$

$$\sum_{j \neq i} y_j \leq u(N \setminus \{i\})$$

$$\sum x_i \mid |x_i - u(\{i\})| > \varepsilon$$

$$\sum_{j \neq i} x_j + \varepsilon \leq u(N \setminus \{i\})$$

16) бинарный черн; равновесие на Нем

$$A = \begin{pmatrix} 2 & 4 & 5 \\ 4 & 2 & 1 \end{pmatrix}$$

$$x_1 \geq 0 \quad y_3 \geq 0$$

$$(x^*, y^*)$$

$$B = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 2 & 3 \end{pmatrix}$$

$$y_1 \quad y_2 \quad y_3$$

$$p(x, y) = \sum \sum a_{ij} x_i y_j$$

$$q(x, y) = \sum \sum b_{ij} x_i y_j$$

$$p(x, y^*) \leq p(x^*, y^*)$$

$$q(x^*, y) \leq q(x^*, y^*)$$

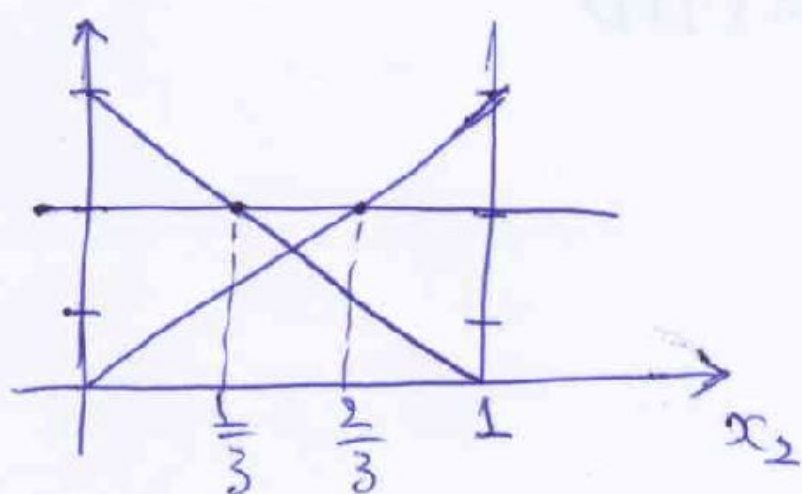
$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$x_1 \geq 0$$

$$x_2 > 0$$

$$\Rightarrow p(1, y^*) = p(2, y^*) = 1$$

$$\begin{cases} 2y_1^* + 4y_2^* + 5y_3^* = 1 \\ 4y_1^* + 2y_2^* + y_3^* = 1 \end{cases} \quad y_1 + y_2 + y_3 = 1$$



$$\text{con. } x_2 = \frac{1}{3}$$

$$x_1 = \frac{2}{3} \Rightarrow y_3 = 0$$

$$\text{con. } x_2 = \frac{2}{3}$$

$$x_1 = \frac{1}{3} \Rightarrow y_1 = 0$$

$$y_2 > 0; y_1 > 0$$

$$V_1$$