Елементи на Делоне́: Лема 6

Лема 6. Декартовите координати на $J^*(x_1, x_2, x_3)$ се изразяват чрез шесте елиптични елемента $(a, e, i, l, g+\theta, \theta)$ по формулата

$$
\begin{aligned}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos g & -\sin g & 0 \\ \sin g & \cos g & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a\,(\cos u - e) \\ a\sqrt{1-e^2}\sin u \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta & -\sin\theta\cos i & \sin\theta\sin i \\ \sin\theta & \cos\theta\cos i & -\cos\theta\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} \cos g & -\sin g & 0 \\ \sin g & \cos g & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a\,(\cos u - e) \\ a\,\sqrt{1-e^2}\sin u \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta\cos g - \sin\theta\cos i\sin g & -\cos\theta\sin g - \sin\theta\cos i\cos g & \sin\theta\sin i \\ \sin\theta\cos g + \cos\theta\cos i\sin g & -\sin\theta\sin g + \cos\theta\cos i\cos g & -\cos\theta\sin i \\ \sin i\sin g & \sin i\cos g & \cos i \end{pmatrix} \times \\
&\quad \times \begin{pmatrix} a\,(\cos u - e) \\ a\,\sqrt{1-e^2}\sin u \\ 0 \end{pmatrix}
\end{aligned}
$$

По координатно,

$$
\begin{aligned}
x_1 &= a\,(\cos u - e)(\cos\theta\cos g - \sin\theta\cos i\sin g) - a\sqrt{1-e^2}\sin u\,(\cos\theta\sin g - \cos\theta\cos i\cos g) \\
x_2 &= a\,(\cos u - e)(\sin\theta\cos g + \cos\theta\cos i\sin g) - a\sqrt{1-e^2}\sin u\,(\sin\theta\sin g - \cos\theta\cos i\cos g) \\
x_3 &= a\,(\cos u - e)\sin i\sin g + a\sqrt{1-e^2}\sin u\sin i\cos g\,.
\end{aligned}
$$

Доказателство. Ако $\theta = 0$ то

$$
\begin{aligned}
x_1 &= r\cos\psi\,, \\
x_2 &= r\sin\psi\cos i\,, \\
x_3 &= r\sin\psi\sin i\,.
\end{aligned}
$$

За $\theta \neq 0$ трябва да завъртим равнината на ъгъл θ, а от тук следва

$$
\begin{aligned}
x_1 &= r\cos\psi\cos\theta - r\sin\psi\cos i\sin\theta, \\
x_2 &= r\cos\psi\sin\theta + r\sin\psi\cos i\cos\theta, \\
x_3 &= r\sin\psi\sin i.
\end{aligned}
$$

В Лема 5 доказахме, че $g = \psi - v$, т. е.

$$\psi \;=\; g + v \;,$$

както и

$$\begin{aligned} r\cos v &= a\,(\cos u - e) \\ r\sin v &= a\sqrt{1-e^2}\,\sin u\,. \end{aligned}$$

Използвайки горните формули, пресмятаме

$$\begin{aligned} x_1 &= r\,\cos(g+v)\,\cos\theta - r\,\sin(g+v)\,\cos i\,\sin\theta \\ &= r\,(\cos g\cos v - \sin g\sin v)\,\cos\theta - r\,(\sin g\cos v + \cos g\sin v)\,\cos i\,\sin\theta \\ &= r\,\cos v\,\cos g - r\,\sin v\,\sin g\,\cos\theta - r\,\cos v\,\sin g\,\cos i\,\sin\theta - r\,\sin v\,\cos g\,\cos i\,\sin\theta \\ &= a\,(\cos u - e)\,\cos g\,\cos\theta - a\sqrt{1-e^2}\,\sin u\,\sin g\,\cos\theta \\ &\quad -a\,(\cos u - e)\,\sin g\,\cos i\,\sin\theta - a\sqrt{1-e^2}\,\sin u\,(\sin g\cos\theta + \cos g\cos i\sin\theta) \\ &= a\,(\cos u - e)\,(\cos g\cos\theta - \sin g\cos i\cos\theta) \\ &\quad -a\sqrt{1-e^2}\,\sin u\,(\sin g\cos\theta + \cos g\cos i\sin\theta). \end{aligned}$$

Аналогично,

$$\begin{aligned} x_2 &= r\,\cos(g+v)\,\sin\theta + r\,\sin(g+v)\,\cos i\,\cos\theta \\ &= a\,(\cos u - e)\,(\cos g\sin\theta + \sin g\cos\theta\cos i) \\ &\quad +\, a\sqrt{1-e^2}\,\sin u\,(-\sin g\,\sin\theta + \cos g\,\cos\theta\,\cos i), \\ x_3 &= r\sin(g+v)\sin i \\ &= a\,(\cos u - e)\,\sin g\,\sin i + a\,\sqrt{1-e^2}\,\sin u\,\cos g\,\sin i\,. \end{aligned}$$

Лема 6 е доказана.

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