ТВ лекции 18.03.15

X	x_1	x_2	x_3	...	x_n	...
P	p_1	p_2	p_3	...	p_n	...

$\sum p_i = 1$

Ф-ята на разпределение $F(x) = P(X \leq x)$

$P(X = x_i) = p_i, \; i = \overline{1, \infty}$

Деф

Математическо очакване $EX \stackrel{def}{=} \sum_{i=1}^{\infty} x_i p_i = const$

Дискретна равномерна

X	x_1	...	x_n
P	$\frac{1}{n}$	...	$1/n$

$EX = x_1 \frac{1}{n} + x_2 \frac{1}{n} + \dots + x_n \frac{1}{n} =$

$\frac{1}{n}(x_1 + \dots x_n)$ стохастичен модел

Момент на случайната величина

EX^k - k-ти начален момент

$E(X - X^k)$ - k-ти централен момент

$E(X - EX)^2 = DX$ - дисперсия $= Var(X)$

Втори начален момент !!! Той служи като мярка на разсейване

Стандартно отклонение ~~e~~ $\sigma = \sqrt{DX}$

Св-ва на EX

1) $EC = C$ $(C - const)$
2) $E(CX) = CEX$
3) $E(X \pm Y) = EX \pm EY$
4) $E(XY) = EXE(Y)$ при условие че $X \perp Y$ т.е X и Y са независими

$$DX = E(X-EX)^2 = E\{X^2 - 2XEX + (EX)^2\} = E(X^2) - 2EXEX + (EX)^2 = \underline{E(X^2) - (EX)^2}$$

Развита формула на дисперсията

Биномно разпределение

$$P(X=k) = \binom{n}{k} p^k q^{n-k}, \quad k = \overline{0,n}$$

$$EX = \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k} = \sum_{k=1}^{n} k \frac{n!}{k!(n-k)!} p^k q^{n-k} =$$

$$np \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!(n-k)!} p^{k-1} q^{n-k} = np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} q^{n-k}$$

Нека $k-1 = l \Rightarrow l+1 = k \Rightarrow np \sum_{l=0}^{n-1} \binom{n}{l} p^l q^{n-l-1}$

$np(\underbrace{p+q}_{1})^{n-1} \Rightarrow EX = np$

$$EX^2 = \sum_{k=0}^{n} k^2 \binom{n}{k} p^k q^{n-k} = np \sum_{k=1}^{n} k \frac{(k-1)(n-1)!}{(k-1)!(n-k)!} p^{k-1} q^{n-k}$$

$$= np^2(n-1) \sum_{k=1}^{n} \frac{(n-2)!}{(k-2)!(n-k)!} p^{k-2} q^{n-k} + np =$$

$$np^2(n-1) + np.$$

$$DX = n(n-1)p^2 + np - (np)^2 = np - np^2 = np(1-p) = \underline{npq}$$

Геометрично разпределение

$$P(X=k) = q^{k-1}p, \quad k=\overline{1,\infty}$$

$$EX = \sum_{k=1}^{\infty} kpq^{k-1} = p\sum_{k=1}^{\infty} kq^{k-1} = p\frac{\partial}{\partial q}\left(\sum_{k=1}^{\infty} q^k\right) =$$

$$p\frac{\partial}{\partial q}\left(\frac{q}{1-q}\right) = p\frac{1-q+q}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

$$EX^2 = p\sum_{k=1}^{\infty} k^2 q^{k-1} = p\frac{\partial}{\partial q}\left(\sum_{k=1}^{\infty} kq^k\right) = p\frac{\partial}{\partial q}\left(q\sum_{k=1}^{\infty} kq^{k-1}\right)$$

$$= p\frac{\partial}{\partial q}\left[q\frac{\partial}{\partial q}\left(\sum_{k=1}^{\infty} q^k\right)\right] = p\frac{\partial}{\partial q}\left[q\frac{1}{(1-q)^2}\right] = p\frac{(1-q)^2+q2(1-q)}{(1-q)^4} =$$

$$= \frac{1-q+2q}{p^2} = \frac{1+q}{p^2}$$

$$DX = \frac{1+q}{p^2} - \frac{1}{p^2} = \frac{q}{p^2}$$

Поасоново разпределение

$$P(X=k) = \frac{\lambda^k}{k!}e^{-\lambda}, \quad k=\overline{0,\infty}$$

$$EX = \sum_{k=0}^{\infty} k\frac{\lambda^k}{k!}e^{-\lambda} = \sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!} = \lambda e^{-\lambda}\overbrace{\sum_{k=1}^{\infty}\frac{\lambda^{k-1}}{(k-1)!}}^{e^{\lambda}} = \lambda$$

$$EX^2 = \sum_{k=1}^{\infty} k^2\frac{\lambda^k}{k!}e^{-\lambda} = e^{-\lambda}\lambda\sum_{k=1}^{\infty} k^{(+1-1)}\frac{\lambda^{k-1}}{(k-1)!} =$$

$$= \lambda^2 e^{-\lambda}\sum_{k=2}^{\infty}\frac{\lambda^{k-2}}{(k-2)!} + \lambda = \lambda^2+\lambda$$

$$DX = \lambda^2+\lambda-\lambda^2 = \lambda$$

$$FI = \frac{DX}{EX} = 1 \;!!!$$ – Фишер – само при Поасоновото е вярно

Ф-я от пораждаща величина – Пораждаща ф-я

$$\psi(s) = E s^X = \sum_{k=0}^{\infty} s^k p_k .$$ Предполагаме че редът е сходящ.

$$\psi'(s) = \sum_{k=1}^{\infty} k s^{k-1} p_k, \quad \psi'(1) = \sum_{k=1}^{\infty} k p_k = EX, \quad p_k = \frac{\psi^{(k)}(0)}{k!}$$

$$\psi''(s) = \sum_{k=2}^{\infty} k(k-1) s^{k-2} p_k, \quad \psi''(1) = \sum_{k=2}^{\infty} k(k-1) p_k = E[X(X-1)] = E(X^2 - X)$$

$$EX^2 = \psi''(1) - \psi'(1); \quad DX = \psi''(1) + \psi'(1) - [\psi'(1)]^2$$

За биномната ф-я
$$\psi(s) = \sum_{k=0}^{\infty} s^k \binom{n}{k} p^k q^{n-k} = (ps+q)^n$$

За геометричната ф-я
$$\psi(s) = p \sum_{k=1}^{\infty} s^k q^{k-1} = \frac{ps}{1-qs} = \frac{ps}{1-(1-p)s}$$

За поасоновата ф-я
$$\psi(s) = \sum_{k=0}^{\infty} s^k \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\lambda} e^{\lambda s} = e^{-\lambda(1-s)}$$