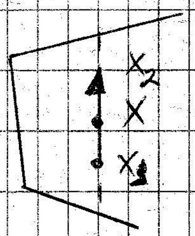
МО лекции 31.03.15

I. $P = \{x \in R^n : Ax = b,\ x \geq 0\}$, $x = \sum_{i \in I} \lambda_i V_i + d_i$

$\lambda_i \geq 0,\ i \in I$, d - посока, $\sum_{i \in I} \lambda_i = 1$



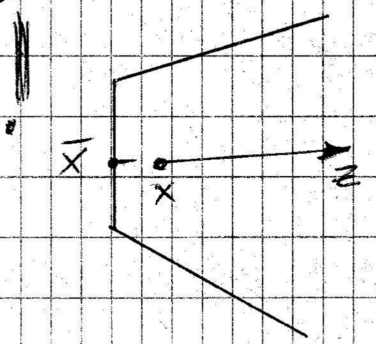
$x = \gamma x_1 + (1-\gamma) x_2,\ \gamma \in (0,1)$

$x_1 \in P \ni x_2$, $z = x_2 - x_1 \neq 0$

$x = \theta \bar{x}_1 + (1-\theta)\bar{x}_2,\ \theta \in (0,1)$

Ако $0 = x_i = \gamma x_{1i} + (1-\gamma) x_{2i} \Rightarrow x_{1i} = x_{2i} = 0$

II.



$P \ni x + \lambda z,\ \lambda \geq 0$, $P \ni x \Rightarrow \begin{cases} x \geq 0 \\ Ax = b \end{cases}$

$A(x + \lambda z) = \underset{b}{Ax} + \lambda \overset{0}{Az} = b$ $\square$

$Az = A(x_2 - x_1) = Ax_2 - Ax_1 = b - b = 0$

$x - \mu z \geq 0,\ \mu \geq 0,\ x_i - \mu z_{i_0} \geq 0 \Rightarrow$

$\mu \leq \frac{x_i}{z_{i_0}}$

$\bar{x} = x - \mu z \Rightarrow x = \bar{x} + \mu z \Rightarrow \bar{x} = \sum_{i \in I} \lambda_i V_i + d_i \Rightarrow$

$x = \sum \lambda_i V_i + d_i + \mu z \Rightarrow$ Th е вярна

посока

Число умножено по посока е посока, ако за произволна точка е изпълнено

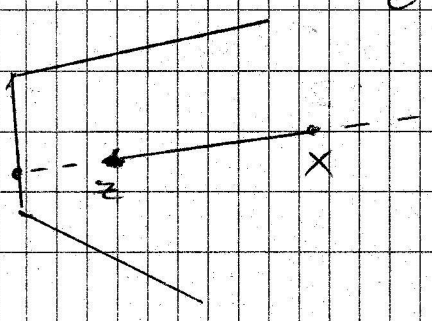
$x + \lambda z \in P$, $d \neq 0 \longleftrightarrow Ad = 0$

$d_i \geq 0$

III.

$0 \neq z \leq 0 \longleftrightarrow z_i \leq 0,\ i = 1..n$

$x + \mu z,\ \mu \geq 0$



$x_i + \mu z_i \geq 0 \Rightarrow 0 \leq \mu \leq -\frac{x_i}{z_i} > 0$

$\bar{\mu} = \min\left\{-\frac{x_i}{z_i},\ z_i < 0\right\}$

Нека $x \in P = \{x \in R^n, Ax = b, x \geq 0\} \neq \emptyset$

$x = \sum_{i \in I} \alpha_i v_i + d_i$, $\alpha_i \geq 0$, $i \in I$, $\sum_{i \in I} \alpha_i = 1$

Едно мн-во P е неограничено $\Leftrightarrow$ P има посока

$$\left.\begin{array}{l} d \neq 0 \\ x_0 \in P \\ \lambda \geq 0 \end{array}\right\} \begin{array}{l} x_0 + \lambda d \in P \\ \|x_0 + \lambda d\| \geq \lambda \|d\| - \|x_0\| \xrightarrow[\lambda \to \infty]{} \infty \end{array}$$

$x_i = \sum_{i \in I} \alpha_i v_i$, $\|x\| \leq \sum \alpha_i \|v_i\| \leq \max_{i \in I} \|v_i\| \underbrace{\sum_{i \in I} \alpha_i}_{1} \leq$

$\leq \max_{i \in I} \|v_i\|$

Допускаме се P няма връх $\Rightarrow x = d$, d-посока

$\Rightarrow Ad = 0$, $d \geq 0 \Rightarrow 0 \in P$ ↯ понеже $x \in P \neq \emptyset$

Нека $c \neq 0$, $\langle c, x \rangle \to \min_{x \in P}$, $x \in P$

I или $\min_{x \in P} \langle c, x \rangle = \langle c, v \rangle$, v - връх

II редицата $x_n \in P$, $\langle c, x_n \rangle \to -\infty$

Док

I $x \in P \Rightarrow x = \sum_{i \in I} \alpha_i v_i + d_i \Rightarrow \langle c, x \rangle =$

$\sum_{i \in I} \alpha_i \langle c, v_i \rangle + \langle c, d_i \rangle \geq \min_{i \in I} \langle c, v_i \rangle$

II $\exists$ посока $\langle c, d\rangle < 0$

$x_0 + \lambda d \in P, \lambda \geq 0$

$\langle c, x_0\rangle + \lambda \underset{\vee 0}{\langle c, d\rangle} \underset{\lambda\to\infty}{\Rightarrow} -\infty$

Базисна точка е $\bar{x} = (\bar{x}_B, \overset{=0}{\bar{x}_N})$

$Ax = b \Leftrightarrow Bx_B + Nx_N = b, \quad x_B + B^{-1}Nx_N = B^{-1}b$

$x = \sum_{j=1}^{n} a_j x_j$, където a_j са стълбовете на $A \Rightarrow$

$Bx_B + \sum_{j\in N} a_j x_j = b$ т.е.

$$\left.\begin{array}{l} x_B + \sum_{j\in N} B^{-1} a_j x_j = B^{-1} b \\ x_q = \theta, \quad \theta \geq 0 \\ x_j = 0, \quad j \in N \setminus \{q\} \end{array}\right\} \Rightarrow x_B = B^{-1}b - B^{-1}a_q\theta$$

$x = \bar{x} + \theta e$, ако e е ограничена - отсечка

x - начална точка, ако e е неограничена - лъч

$$\begin{pmatrix} x_B \\ x_q \\ x_j \end{pmatrix} = \overset{=\bar{x}}{\begin{pmatrix} x_B \\ 0 \\ 0 \end{pmatrix}} + \theta \begin{pmatrix} -B^{-1}a_q \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$\langle c, \theta\rangle = \langle c_B, x_\theta^B\rangle + \sum_{j\in N} c_j x_{j\theta} =$

$= c_B^T (x_B - B^{-1}a_q) + c_q\theta = \langle c, \bar{x}\rangle + \theta(c_q - c_B^T B^{-1} a_q)$