ЧДУ 07.10.14

Характеристика - повърхнина с определени св-ва

$\sum_{i,j=1}^{n} a_{ij}(x) u_{x_i x_j} + \dots$

$y = \varphi(x)$. $A(y) = \sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial \varphi_k}{\partial x_i} \frac{\partial \varphi_e}{\partial x_j}$

Такава повърхнина $S: \Phi(x_1 \dots x_n) = 0$ $(\operatorname{grad} \Phi \neq 0)$

т.е.

$x \in S: \sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial \Phi}{x_i} \frac{\partial \Phi}{x_j} = 0$

Зад

$y^2 u_{xx} + 2xy u_{xy} + 2x^2 u_{yy} + y u_y = 0$, $xy \neq 0$

$\Delta = (xy)^2 - y^2 2x^2 = -x^2 y^2 < 0 \Rightarrow$ елиптично

У-е на характеристиките

$y^2 dy^2 - 2xy\, dx\, dy + 2x^2 dx^2 = 0 \quad / dx^2$

$y' = \frac{xy \pm \sqrt{-x^2 y^2}}{y^2} = \frac{xy \pm ixy}{y^2}$

$y' = \frac{x(1+i)}{y}$, $yy' = x(1+i) \Rightarrow$ с разд. променливи

$\frac{1}{2} y^2 = \frac{1}{2} x^2 (1+i) + C$

$y^2 - x^2 - ix^2 = C_1$

Полагаме

$\left| \begin{array}{l} \xi = y^2 - x^2 \\ \eta = x^2 \end{array} \right\} \Rightarrow$

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от гл уравн

$0\cdot$ | $u_x = u_\xi(-2x) + u_\eta(2x)$

$y\cdot$ | $u_y = u_\xi(2y) + u_\eta\cdot 0$

$2xy$ | $u_{xy} = u_{\xi\xi}(2y)(-2x) + u_{\xi\eta}(-2x)\cdot(0) +$
$+ u_{\eta\xi}(2x)(2y) + u_{\eta\eta}(2x)\cdot(0)$

$y^2\cdot$ | $u_{xx} = u_{\xi\xi}4x^2 + u_{\xi\eta}(-4x^2) + u_\xi(-2) +$
$+ u_{\eta\xi}(-2x)(2x) + u_{\eta\eta}4x^2 + u_\eta\cdot 2$

$2x^2$ | $u_{yy} = u_{\xi\xi}4y^2 + u_{\xi\eta}\cdot 0\cdot 2y + 2u_\xi$

$$0 = u_{\xi\xi}(-8x^2y^2 + 4x^2y^2 + 8x^2y^2) +$$
$$u_{\xi\eta}(8x^2y^2 - 4x^2y^2 - 4x^2y^2) +$$
$$u_{\eta\eta}(4x^2y^2) + u_\xi(\cancel{2y^2} + 4x^2 - \cancel{2y^2}) +$$
$$u_\eta(2y^2) =$$
$$4x^2y^2(u_{\xi\xi} + u_{\eta\eta}) + u_\xi\cdot 4x^2 + u_\eta 2y^2 = 0$$

$$u_{\xi\xi} + u_{\eta\eta} + \frac{1}{y^2}u_\xi + \frac{1}{2x^2}u_\eta = 0$$

$$u_{\xi\xi} + u_{\eta\eta} + \frac{1}{\xi+\eta}u_\xi + \frac{1}{2\eta}u_\eta = 0$$ за $y^2 = \xi+\eta$

$$x^2 u_{xx} - 2x u_{xy} + u_{yy} + 2u_y = 0$$

$\Delta = 4x^2 - 4x^2 = 0 \Rightarrow$ параболично

у-е на характеристиките

$$x^2 dy^2 + 2x\,dx\,dy + dx^2 = 0$$

$$x^2 y'^2 + 2x y' + 1 = 0$$

$$y' = -\frac{x}{x^2} = -\frac{1}{x} \quad (\text{от } (xy' = -1))$$

$$\Rightarrow y = -\ln x + C, \quad x, y > 0$$

$\begin{cases} \xi = y + \ln x \\ \eta = x \end{cases}$ (от якобиана $\begin{vmatrix} \frac{1}{x} & 1 \\ 1 & 0 \end{vmatrix}$) (каквото число искаме в η стига якобиана $\neq 0$)

$0.\ \big|\ u_x = u_\xi \frac{1}{x} + u_\eta \cdot 1$

$2\ \big|\ u_y = u_\xi \cdot 1 + u_\eta \cdot 0$

$x^2\ \big|\ u_{xx} = u_{\xi\xi}\left(\frac{1}{x^2}\right) + u_{\xi\eta}\frac{1}{x} + u_\xi\left(-\frac{1}{x^2}\right) + u_{\eta\xi}\left(\frac{1}{x}\right) + u_{\eta\eta}$

$1\ \big|\ u_{yy} = u_{\xi\xi}$

$-2x\ \big|\ u_{yx} = u_{\xi\xi}\frac{1}{x} + u_{\xi\eta} \cdot 1$

$$0 = u_{\xi\xi}(1+1-2) + u_{\xi\eta}(x+x-2x) + u_{\eta\eta}x^2 + u_\xi$$

$$\Rightarrow x^2 u_{\eta\eta} + u_\xi = 0 \quad /: x^2$$

$u_{\eta\eta} + \frac{1}{x^2}u_\xi = 0$ (от $\eta = x$) $\Rightarrow u_{\eta\eta} + \frac{1}{\eta^2}u_\xi = 0$

У-е на струната

$u_{tt} - a^2 u_{xx} = 0 \quad , a > 0$

$dx^2 - a^2 dt^2 = 0$ т.е. $x'^2 - a^2 = 0 \Rightarrow x' = \pm a$

$\Delta = 0 - 1(-a^2) = a^2 > 0$ - хиперболично

$x = \pm at + c$ смяна на променливите

$$\begin{cases} \xi = x - at \\ \eta = x + at \end{cases}$$

$u_t = u_\xi(-a) + u_\eta(a)$

$u_x = u_\xi + u_\eta$

$1 \quad u_{tt} = u_{\xi\xi} a^2 + u_{\xi\eta}(-a^2) + u_{\eta\xi}(-a^2) + u_{\eta\eta} a^2$

$-a^2 \quad u_{xx} = u_{\xi\xi} + u_{\xi\eta} + u_{\eta\xi} + u_{\eta\eta}$

$0 = u_{\xi\xi}(a^2 - a^2) + u_{\xi\eta}(-4a^2) + u_{\eta\eta}(a^2 - a^2)$

$\Rightarrow u_{\xi\eta} = 0$

$u(\xi, \eta) = f(\xi) + g(\eta)$

$u(x, y) = f(x - at) + g(x + at)$