Домашна работа №2

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Софтуерно инженерство

1. $\lim\limits_{n\to\infty}\frac{\sqrt{n-1}-\sqrt{n_+2}}{\sqrt{n+4}-\sqrt{n+3}} = \lim\limits_{n\to\infty}\frac{(\sqrt{n-1}-\sqrt{n_+2})(\sqrt{n+4}+\sqrt{n+3})}{n+4-n-3} = \lim\limits_{n\to\infty}(\sqrt{n-1}-\sqrt{n+2})(\sqrt{n+4}-\sqrt{n+3}) =$

$$\lim_{n\to\infty}\frac{(n-1-n-2)(\sqrt{n+4}+\sqrt{n+3})}{\sqrt{n-1}+\sqrt{n+2}} = \lim_{n\to\infty}\frac{-3(\sqrt{n+4}+\sqrt{n+3})}{\sqrt{n-1}+\sqrt{n+2}} = -3$$

2. $\lim\limits_{x\to-2}\frac{\sqrt{x^3-x+16}-\sqrt{8-x}}{x^2+8x+12} = \lim\limits_{x\to-2}\frac{x^3-x+16-8+x}{(x^2+8x+12)(\sqrt{x^3-x+16}+\sqrt{8-x})} = \lim\limits_{x\to-2}\frac{(x+2)(x^2-2x+4)}{(x+2)(x+6)(\sqrt{x^3-x+16}+\sqrt{8-x})} =$
$\lim\limits_{x\to-2}\frac{(4+4+4)}{4(\sqrt{26}+\sqrt{10})} = \frac{3}{\sqrt{26}+\sqrt{10}}$

3. $\lim\limits_{n\to\infty}\left(\frac{n^2-3n+2}{n^2+3n+2}\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{-6n}{n^2+3n+2}\right)^n = \lim\limits_{n\to\infty}(\left(1+\frac{-6n}{n^2+3n+2}\right)^{\frac{n^2+3n+2}{-6n}\frac{-6n}{n^2+3n+2}})^n = \lim\limits_{n\to\infty}\left(1+\frac{-6n}{n^2+3n+2}\right)^{\frac{n^2+3n+2}{-6n}\frac{-6n^2}{n^2+3n+2}} = e^{-6}$

4. $\lim\limits_{x\to3}\left(\frac{1}{x-3}-\frac{27}{x^3-27}\right) = \lim\limits_{x\to3}\left(\frac{x^2+3x+9-27}{(x-3)(x^2+3x+9)}\right) = \lim\limits_{x\to3}\left(\frac{x^2+3x-18}{(x-3)(x^2+3x+9)}\right) = \lim\limits_{x\to3}\left(\frac{(x-3)(x+6)}{(x-3)(x^2+3x+9)}\right) =$
$\lim\limits_{x\to3}\left(\frac{(x+6)}{(x^2+3x+9)}\right) = \lim\limits_{x\to3}\left(\frac{9}{9+9+9}\right) = \frac{1}{3}$

5. $\lim\limits_{n\to\infty}\frac{1}{n^3}\lim\limits_{x\to0}\frac{1-\cos x\cos 2x\ldots\cos nx}{x^2} =$
$\lim\limits_{n\to\infty}\frac{1}{n^3}\lim\limits_{x\to0}\frac{1-(\cos x-\cos x)-(\cos x\cos 2x-cosx\cos 2x)-(\cos x\cos 2x\cos 3x-cosx\cos 2x\cos 3x)-\cdots-\cos x\cos 2x\ldots\cos nx}{x^2} =$
$\lim\limits_{n\to\infty}\frac{1}{n^3}\lim\limits_{x\to0}\frac{(1-\cos x)+(\cos x-cosx\cos 2x)+(cosx\cos 2x-cosx\cos 2x\cos 3x)+\cdots+(cosx\cos 2x\ldots\cos(n-1)x-\cos x\cos 2x\ldots\cos nx)}{x^2} =$
$\lim\limits_{n\to\infty}\frac{1}{n^3}\lim\limits_{x\to0}\frac{(1-\cos x)+cosx(1-\cos 2x)+cosx\cos 2x(1-cos3x)+\cdots+cosx\cos 2x\ldots\cos((n-1)x)(1-\cos nx)}{x^2} =$
$\lim\limits_{n\to\infty}\frac{1}{n^3}\Big(\lim\limits_{x\to0}\frac{1-\cos x}{x^2}+\lim\limits_{x\to0}\frac{cosx(1-\cos 2x)}{x^2}+\lim\limits_{x\to0}\frac{cosx\cos 2x(1-cos3x)}{x^2}\ldots+$
$\lim\limits_{x\to0}\frac{(cosx\cos 2x\ldots\cos((n-1)x))\ (1-\text{cosnx})}{x^2}\Big) = \lim\limits_{n\to\infty}\frac{1}{n^3}\Big(\lim\limits_{x\to0}\frac{1-\cos x}{x^2}+\lim\limits_{x\to0}\frac{(1-\cos 2x)}{x^2}+\lim\limits_{x\to0}\frac{(1-cos3x)}{x^2}\ldots+$
$\lim\limits_{x\to0}\frac{(1-\text{cosnx})}{x^2}\Big) = \quad \lim\limits_{n\to\infty}\frac{1}{n^3}\left(\lim\limits_{x\to0}\frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2}+\lim\limits_{x\to0}\frac{\left(1-\left(1-2\sin^2\frac{2x}{2}\right)\right)}{x^2}+\lim\limits_{x\to0}\frac{\left(1-\left(1-2\sin^2\frac{3x}{2}\right)\right)}{x^2}\ldots+\right.$
$\left.\lim\limits_{x\to0}\frac{\left(1-\left(1-2\sin^2\frac{n\,x}{2}\right)\right)}{x^2}\right) = \lim\limits_{n\to\infty}\frac{1}{n^3}\left(\lim\limits_{x\to0}\frac{2\sin^2\frac{x}{2}}{x^2}+\lim\limits_{x\to0}\frac{\left(2\sin^2\frac{2x}{2}\right)}{x^2}+\lim\limits_{x\to0}\frac{\left(2\sin^2\frac{3x}{2}\right)}{x^2}\ldots+\lim\limits_{x\to0}\frac{2\sin^2\frac{n\,x}{2}}{x^2}\right) =$
$\lim\limits_{n\to\infty}\frac{1}{n^3}\left(\lim\limits_{x\to0}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}+\lim\limits_{x\to0}\frac{\left(2\sin^2\frac{2x}{2}\right)}{\frac{4}{4}\left(\frac{2x}{2}\right)^2}+\lim\limits_{x\to0}\frac{\left(2\sin^2\frac{3x}{2}\right)}{\frac{4}{9}\left(\frac{3x}{2}\right)^2}\ldots+\lim\limits_{x\to0}\frac{2\sin^2\frac{n\,x}{2}}{\frac{4}{n^2}\left(\frac{nx}{2}\right)^2}\right) = \lim\limits_{n\to\infty}\frac{1}{n^3}\Big(\lim\limits_{x\to0}\frac{2}{4}+\lim\limits_{x\to0}\frac{2}{1}+$

$\lim_{x\to 0}\frac{2}{\frac{4}{9}}\ldots+\lim_{x\to 0}\frac{2}{\frac{4}{n^2}}\Big)=\lim_{n\to\infty}\frac{1}{n^3}\left(\frac{1}{2}*1^2+\frac{1}{2}*2^2+\frac{1}{2}*3^2\ldots+\frac{1}{2}*n^2\right)=\lim_{n\to\infty}\frac{1}{n^3}\frac{1}{2}(1^2+2^2+3^2\ldots+$
$n^2)=\lim_{n\to\infty}\frac{1}{n^3}\frac{n(n+1)(2n+1)}{12}=\lim_{n\to\infty}\frac{n^3\left(2+\frac{3}{n}+\frac{1}{n^2}\right)}{12n^3}=\frac{1}{6}$

6. Ще докажем $tg(arctg\frac{1}{2}+arctg\frac{1}{8}+\cdots+arctg\frac{1}{2n^2})=\frac{n}{n+1}$
При n=1 $tg\left(arctg\frac{1}{2}\right)=\frac{1}{2}$
Допускаме че е вярно за n: $tg\left(arctg\frac{1}{2}+arctg\frac{1}{8}+\cdots+arctg\frac{1}{2n^2}\right)=\frac{n}{n+1}$

За n+1 : $tg\left(arctg\frac{1}{2}+arctg\frac{1}{8}+\cdots+arctg\frac{1}{2n^2}+arctg\frac{1}{2(n+1)^2}\right)$
Ако $arctg\frac{1}{2}+arctg\frac{1}{8}+\cdots+arctg\frac{1}{2n^2}=\alpha$ и $arctg\frac{1}{2(n+1)^2}=\beta$, прилагаме $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$,
$\tan\alpha=\frac{n}{n+1}$ и $\tan\beta=\frac{1}{2(n+1)^2}$; $\tan(\alpha+\beta)=\frac{\frac{n}{n+1}+\frac{1}{2(n+1)^2}}{1-\left(\frac{1}{2(n+1)^2}\right)\left(\frac{n}{n+1}\right)}=\frac{(n+1)(2n^2+2n+1)}{(n+2)(2n^2+2n+1)}=\frac{n+1}{n+2}$
с индукция доказахме, че твърдението е вярно.
$arctg\frac{1}{2}+arctg\frac{1}{8}+\cdots+arctg\frac{1}{2n^2}=\alpha$ и $\tan\alpha=\frac{n}{n+1}$, т.е. $\alpha=arctg\ \frac{n}{n+1}$

$$\lim_{n\to\infty}\alpha=\lim_{n\to\infty}\left(arctg\ \frac{n}{n+1}\right)=\frac{\pi}{4}$$

7. Нека $x_n=l$ Тогава $\sqrt[3]{260l+1919}=l$

$$l^3-260l-1919=0$$
$$(l^2+19l+360)(l-19)=0$$

Ще докажем с индукция, че $19<x_{n+1}<x_n$
При n=1: $19<\sqrt[3]{260*2012+1919}<2012$ – вярно.
Допускаме, че е вярно за n: $19<x_{n+1}<x_n$
При n+1 може да бъде представено във вида: $19<\sqrt[3]{260*x_{n+1}+1919}<x_{n+1}$, но
$x_{n+1}=\sqrt[3]{260*x_n+1919}$, т.е. $\sqrt[3]{260\sqrt[3]{260*x_n+1919}+1919}<\sqrt[3]{260*x_n+1919}$

$$260\sqrt[3]{260*x_n+1919}<260*x_n$$

$\sqrt[3]{260x_n+1919}<x_n$, което е вярно. Редицата е намаляваща и ограничена отдолу, т.е. е сходима.
$x_n\to l$ и $x_{n+1}\to l$

$$x_{n+1}=\sqrt[3]{260*x_n+1919}$$
$$l=\sqrt[3]{260l+1919}$$

Единственото решение е l = 19