Зад: Нека $p(x) \in \Pi_4$ е полиномът, който интерполира $f(x) = \cos x$ в точките $-\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3}$.

Покажете, че ~~Док. че~~

$$\max_{x \in [-\frac{\pi}{3}, \frac{\pi}{3}]} |f(x) - p(x)| \le \frac{1}{120}$$

Имаме, че

$$\max_{x \in [-\frac{\pi}{3}, \frac{\pi}{3}]} |f(x) - p(x)| \le \max_{[-\frac{\pi}{3}; \frac{\pi}{3}]} \frac{|f^{(V)}(x)|}{5!} \cdot \max_{x \in [-\frac{\pi}{3}, \frac{\pi}{3}]} \left|(x+\tfrac{\pi}{3})(x+\tfrac{\pi}{6})(x-0)(x-\tfrac{\pi}{6})(x-\tfrac{\pi}{3})\right|$$

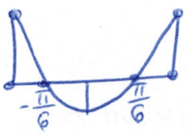
$V \longrightarrow$ пета производна

$$\le \max_{x \in [-\frac{\pi}{3}; \frac{\pi}{3}]} \frac{|-\sin x|}{5!} \max_{x \in [-\frac{\pi}{3}; \frac{\pi}{3}]} \left|(x^2 - \tfrac{\pi^2}{3^2})\right| \max_{x \in [-\frac{\pi}{3}; \frac{\pi}{3}]} \left|x^2 - \tfrac{\pi^2}{6^2}\right| \cdot \max_{x \in [-\frac{\pi}{3}; \frac{\pi}{3}]} |x| =$$

$$= \frac{1}{5!} \cdot \frac{\pi^2}{9} \cdot \frac{\pi^2}{12} \cdot \frac{\pi}{3} \le \frac{1}{120} \cdot \frac{10 \cdot 10 \cdot \pi}{324} \le \frac{1}{120}$$

$-\frac{\pi}{3}$  $\frac{\pi}{3}$

$$\frac{\pi^2}{36} - \frac{\pi^2}{9} = -\frac{\pi^2}{12}$$



Зад: Даден е ~~краен интервал~~ произволен краен интеграл ~~за~~ $[a, b]$. Нека $f(x) = \cos x$ и $L_n(f; x)$ е интерполационен ~~С производните~~ с произволни $n+1$ ~~възли~~ възела

$a \in x_0^{(n)} < x_1^{(n)} < \dots < x_n \le b \qquad n = 0, 1, \dots, n$

Покажете, че $\max_{x \in [a, b]} |f(x) - L_n(f; x)| \xrightarrow[n \to \infty]{} 0$

$$\max_{x \in [a, b]} |f(x) - L_n(f; x)| \le \max_{x \in [a, b]} \left| \frac{\cos^{(n+1)}(x)}{(n+1)!} \right| \cdot \max_{x \in [a, b]} |(x - x_0^{(n)})(x - x_1^{(n)}) \times$$

$$\times (x - x_3^{(n)}) \dots (x - x_m^{(n)})| \le \frac{(b-a)^{n+1}}{(n+1)!} \longrightarrow 0$$

Най-общо сходимостта на интерполационния процес зависи от гладкостта на ф-ята $f(x)$ и от избора на интерполационните на и ~~степе~~ възможности.

Класически пример на ~~Рунге~~ Рунге-функция, която има производни от всеки ред, Зад: Покажете, че $\sum_{k=0}^{n} (x-x_k)^{n+1} l_{k,n}(x) = (-1)^{n}\cos(x)$

Да разгледаме функция $\varphi(t) = (x-t)^{n+1}$ x - фикс. точк

Интерполизационния за $\varphi(t)$ с възм. $x_k = k=0,\dots,n$ по формулата на ~~Л~~ Лагранж и затова така.

$$L_n(\varphi; t) = \sum_{k=0}^{n}$$

Полагаме $t=x$ и използвам

За остатък от интерполиране на $\varphi(t)$ с $L_n(\varphi; t)$

$$(x-t)^{n+1} = \sum_{k=0}^{n} (x-x_k)^{0} l_m(t) = \frac{\varphi^{(m)}(\xi)}{(n+1)!}\omega$$

$(-1)^{n+1}(n+1)!$ и получаваме

$$\sum_{k=0}^{n} (x-x_k)^{n+1} l_{k,n}(x) = (-1)^{n}\omega(x)$$

$$= \frac{(-1)^{(n+1)}(n+1)!}{(n+1)!}\omega(t)$$

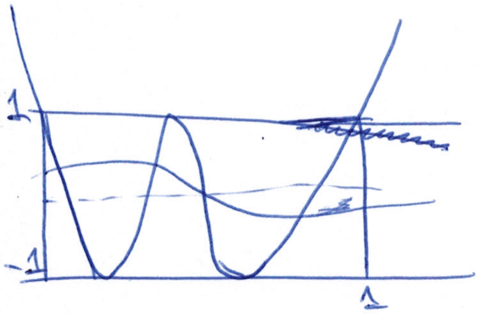
11 възли

$L_{10}(f; x)$

$f(x) = \frac{1}{1+25x^2}$

Пример на Рунге

Полиноми на Чебишов



$$\omega(x) = (x-x_0)(x-x_1)\dots(x-x_n) =$$

$$= 1x^4 + \dots$$

$$\cos\theta = \cos\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$

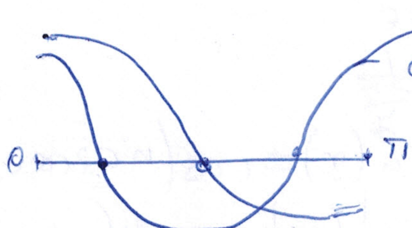
$$\cos(n+1)\theta + \cos(n-1)\theta = 2\cos\theta\cos n\theta$$

$$\cos(n+1)\theta = 2\cos\theta\cos n\theta - \cos(n-1)\theta \qquad n = 1, 2, \dots$$

$$\cos 4\theta = 2\cos\theta(4\cos^3\theta - 3\cos\theta) - (\cos^2 2\theta - 1) = 8\cos^4\theta - 8\cos^2\theta + 1$$

$$\dots$$

$$\cos(n+1)\theta = 2\cos\theta . \cos n\theta - \cos((n-1))\theta =$$



$\cos x - N$

$\cos\theta = x \qquad x = \arccos\theta$

$\theta \in [0, \pi] \Leftrightarrow x \in [1; -1]$

$$T_n(x) \overset{\text{деф. 1}}{=} \cos(n \arccos(x))$$

$$\cos(n+1) = 2\cos\theta\cos(n-1)\theta \qquad n = 1, 2, \dots$$

$T_0 = 1$

$T_1 = x$

$T_2 = 2x^2 - 1$

$T_3 = 4x^3 - 3x$

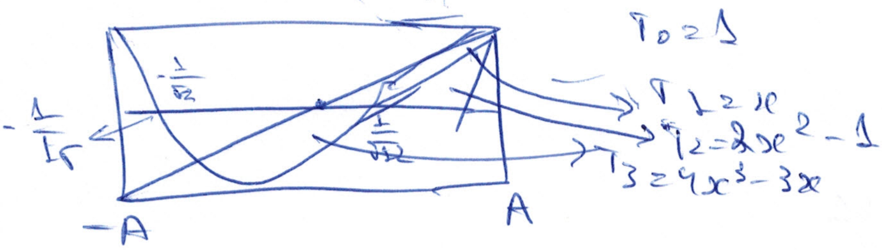
$T_4 = 8x^4 - 8x^2 + 1$

Деф. 2

$T_0 = 1$

$T_1 = x$

$T_{n+1} = 2x T_n - T_{n-1} \quad n = 1, 2, \ldots$

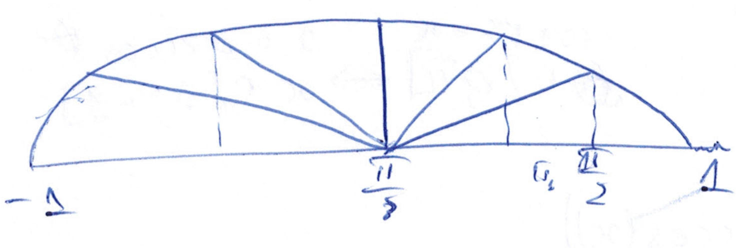


Екстремалните точки (точките на Алтернанс) са

$\eta_k = \cos\frac{k\pi}{n}, \quad k = \overline{0, \ldots, n}$

$T_n(\eta_k) = (-1)^k, \quad k = 0, \ldots, n$

Нулите са $\xi_k = \cos\frac{(2k-1)\pi}{2n} \quad k = 1, \ldots, n$

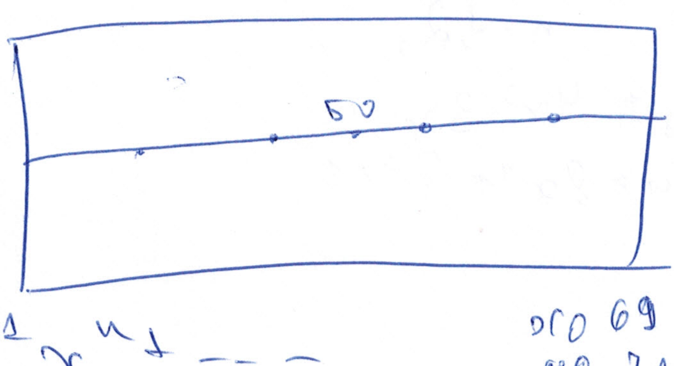


T_6

$T_n(x) = \cos(n \arccos x)$

$T_n'(x) = -\sin($

24 : n.



$T_n = 2^{n-1} x^n + \ldots$

$\omega(x) =$