Задачи ЧМА

I тип

Контролно

<u>Задача 1</u>: $B(1,2,4;t)$ в $[1;4]$

По деф. $B(x_0,\dots,x_r;t) = (x-t)_+^{r-1}[x_0\dots x_r]$

B-сп. от $r-1$-ва степен

$$= \sum_{k=0}^{r} \frac{(x_k-t)_+^{r-1}}{\omega'(x_k)}$$

$B(1,2,4;t) = (x-t)_+^1[1,2,4] =$

$$= \frac{(1-t)_+}{\omega'(1)} + \frac{(2-t)_+}{\omega'(2)} + \frac{(4-t)_+}{\omega'(4)} \quad \text{за } t \in [2;4]$$

$\omega(x) = (x-1)(x-2)(x-4)$

$(1-t)_+ = 0$

$(2-t)_+ = 0$

$(4-t)_+ = 4-t$

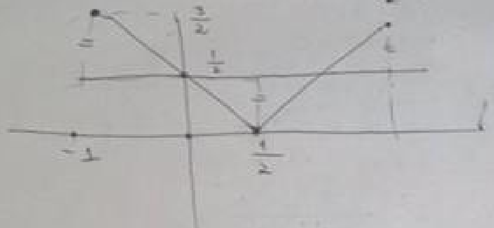
$$B(1,2,4;t) = \frac{(4-t)}{(4-1)(4-2)} = -\frac{1}{6}t + \frac{2}{3}$$

$B(1,2,4;t) = \frac{1}{3}$

$B(1;2;4,t)$ е пол. от π_1 в/у $[1;2]$, който е 0 в 1 и $= \frac{1}{3}$ в 2

$$\Rightarrow B(1,2,4;t) = \begin{cases} \frac{1}{3}t - \frac{1}{3} & t \in [1;2] \\ -\frac{1}{6}t + \frac{2}{3} & t \in [2;4] \end{cases}$$

Задача 2 (1) $f(x) = |x - \frac{1}{2}|$ ПНДРП $E_1(f)$



$p(x) \in \pi_1$

$p(\frac{3}{4}) = \frac{1}{4}$ $E_1 = p(\frac{1}{2})$

$p(-\frac{1}{4}) = \frac{3}{4}$

(2) изпъкнала или вдлъбната

I стъпка: инт полином от I ст. с възли кр. на инт.

II стъпка: x_i : $\max|f(x) - P_I|$

III стъпка: коригиране с $\frac{1}{2}\max|f(x) - P_I|$
+ вдх.
– - изпъкнала

$f(x) = \frac{1}{x+1}$ в $[0,1]$

1) $P_I = -\frac{1}{2}x + 1$

2) $f(x) - P_I \leq 0$ и $x_i = \sqrt{2} - 1$

$f(x) - P(x)$ $\frac{3-2\sqrt{2}}{2}$

$f(\sqrt{2}-1) - P(\sqrt{2}-1) \approx 0,1$ 0,07

3) $P = P_I - \frac{1}{2}\left(\frac{3-2\sqrt{2}}{2}\right)$

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$P(-2) = -8$
$P(0) = 2$
$P(1) = 4$
$P(2) = 12$

	x_i	$P(x_i)$	$P[\cdot,\cdot]$	$P[x_i, \dots]$	
x_0	-2	-8	$5\ P[x_0,x_1]$	-1	$\frac{3}{4}$
x_1	0	2	$2\ P[x_1,x_2]$	2	
x_2	1	4	$4\ P[x_2,x_3]$		
x_3	2	12			

$$\frac{P[x_1]-P[x_0]}{x_1-x_0} = \frac{2+8}{2} = 5$$

$$P(x) = -8 + 5(x-2) + 2(x+2)x + \frac{3}{4}(x+2)x(x-1)$$

$$= -8 + 5x - 10 + 2x^2 + 4x + \frac{3}{4}(x+2)(x^2-x) =$$

$$= -18 + 9x + 2x^2 + \frac{3}{4}(x^3 - x^2 + 2x^2 - 2x) =$$

$$= -18 + 9x + 2x^2 + \frac{3}{4}(x^3 + x^2 - 2x) =$$

$$= \frac{3}{4}x^3 + \frac{11}{4}x^2 + \frac{15}{2}x - 18$$

$$\boxed{P(x) = x^3 + x + 2}$$

	x_i	$P[\cdot]$	$P[\cdot,\cdot]$		
x_0	-2	-8			
			5		
x_1	0	2		-1	
			2		1
x_2	1	4		3	
			8		
x_3	2	12			

(4) $S_k = 1^2 + 2^2 + \dots + k^2$

$S_0 = 0$ $\qquad x = x_0 + kh \qquad x_0 = 0,\ h = 1$

$p(k) = S_k$

x_i	$\Delta P(x_i)$	$\Delta^2 p$	$\Delta^3 p$	$\Delta^4 p$
0	0			
1	1	1	3	
2	5	4		2
3	14	9	5	

$$\Rightarrow p(k) = 0 + \binom{k}{1} + 3\binom{k}{2} + 2\binom{k}{3} = \frac{k(k+1)(2k+1)}{6}$$

(5) $p(0) = -1$

$p'(0) = 1$

$p''(0) = 2$

$p(1) = 0$

$p'(1) = -1$

$$p\{x_0, \dots, x_k\} = \frac{f^{(k)}(x_0)}{k!} \qquad x_0 = x_k$$

$$p\{0,0\} = \frac{p'(0)}{1!}$$

x_i	$p(x_i)$	$p[2\tau]$	$p[3\tau]$	
0	-1	1	1	-3
0	-1	1	-2	
0	-1	-1		
1	0			
1	0			

$3x^3 - 8x^2 + 6x - 4$

$||$

$$H_4(f, x) = -1 + 1(x-0) + 1(x-0)^2 + 3(x-1)^2 = -1 + x + x^2 + 3x^3 - 9x^2 + 9x$$

Числени методи на Анализа

Решения Контролно

1) $p(-1) = 2$
$p(1) = 2$
$p(2) = 5$

$$p(x) = \sum_{k=0}^{2} p(x_k) \prod_{\substack{i=0 \\ i \neq k}}^{2} \frac{x - x_k}{x_k - x_i} = x^2 + 1$$

2) $f(x) = e^x \quad 1, -1, 0$

$$\max_{x \in [-1,1]} = |f(x) - L_2(f,x)| = \frac{f'''(x)}{3!} |x-1||x+1||x|$$

$f'''(x) = e^x$

$$\Rightarrow \max_{x \in [-1,1]} |f(x) - L_2(f,x)| \leq \frac{1}{6} \max e^x \cdot |x||x-1||x+1|$$

$e^x \leq e$

$$\leq \frac{e}{6} \max |x(x-1)(x+1)|$$

$g(x) = x^3 - x$

$g'(x) = 3x^2 - 1 \Rightarrow \max$ в $\pm \frac{1}{\sqrt{3}}$

$$\Rightarrow \max g(x) = g\left(-\frac{1}{\sqrt{3}}\right) = -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}}$$

$$\Rightarrow \leq \frac{e}{6} \cdot \frac{2}{3\sqrt{3}} \leq \frac{1}{5} \qquad \frac{e}{9\sqrt{3}} \leq \frac{1}{5} \qquad 5e \leq 9\sqrt{3}$$

да

x_i		$P[\cdot]$	$P[\cdot,\cdot]$	$P[\cdot,\cdot,\cdot]$		P
x_0	0	-1	1	1	-1	-1
x_1	0	-1	1	0	-2	
x_2	0	-1	1	-2		
x_3	1	0	-1			
x_4	1	0				

$$P\{x_0,x_1,x_2\} = \frac{P''(0)}{2!} = 1$$

$$P\{x_1,x_2,x_3\} = \frac{P\{x_2,x_3\} - P\{x_1,x_2\}}{x_3 - x_1}$$

$$P[x_2,x_3,x_4] =$$

$$P = \cancel{-1 + 1(x-0) + 1(x-0)^2 - 1(x-0)^3 - 1(x-1)^4} = \cancel{x^4 - 4x^3 + 6x^2} - 4x + 1$$

$$(x-1)^3 \qquad -x^4 + 3x^3 - 5x^2 + 5x$$

$$= \cancel{-1 + x + x^2 - x^3} -$$

$$\boxed{P = -x^4 + x^2 + x - 1}$$

$-2-(-1)$

x_i	$P[\cdot]$	$P[\cdot,\cdot]$	$P[\cdot,\cdot,\cdot]$		
0	-1	1	1	-1	
0	-1	1	0	-2	
0	-1	1	-2		
1	0	-1			
1	0				

$$-2 + 7x^3 - x^4 - 5x$$

$$-1 + 1(x-0) + 1(x-0)^2 - 1(x-0)^3 - 1(x-$$

$$= -1 + x + x^2 - x^3 - \cancel{x^4} + 4x^3 - 6x^2 + 4x - 1$$