26.VIII.2013г. Консултация - Табаков

$\boxed{6.9.\ б)}$ 1) e_1, e_2, e_3 – базис на V

$\varphi \in \operatorname{Hom} V$

базис на V, φ е диагонална матрица

$$\varphi(\xi_1 e_1 + \xi_2 e_2 + \xi_3 e_3) = (-\xi_1 + 3\xi_2 - \xi_3)e_1 + (-3\xi_1 + 5\xi_2 - \xi_3)e_2 +$$
$$+(-3\xi_1 + 3\xi_2 + \xi_3)e_3$$

$\xi_1, \xi_2, \xi_3 \in F$

$$A = \begin{pmatrix} \varphi(e_1) & \varphi(e_2) & \varphi(e_3) \\ | & | & | \\ | & | & | \end{pmatrix}$$

1) $\xi_1 = 1 \quad \xi_2 = \xi_3 = 0 \Rightarrow$
$\Rightarrow \varphi(e_1) = -e_1 - 3e_2 - 3e_3$

2) $\xi_2 = 1, \ \xi_1 = \xi_3 = 0 \Rightarrow \varphi(e_2) = 3e_1 + 5e_2 + 3e_3$

3) $\xi_2 = 1, \ \xi_1 = \xi_2 = 0 \Rightarrow \varphi(e_3) = -e_1 - e_2 + e_3$

$$\Rightarrow A = \begin{array}{c} \begin{matrix} \varphi(e_1) & \varphi(e_2) & \varphi(e_3) \end{matrix} \\ \begin{pmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{pmatrix} \end{array}$$

$$f_\lambda(A) = \det(A - \lambda E) = \begin{vmatrix} -1-\lambda & 3 & -1 \\ -3 & 5-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{vmatrix} = (-1-\lambda)(5-\lambda)(1-\lambda) + 9 + 9 -$$

$$-3(5-\lambda) + 3(-1-\lambda) + 9(1-\lambda) = (\lambda - 5)(\cancel{\lambda^2 - 1})^{1-\lambda^2} + \cancel{18} - \cancel{18} + \cancel{3\lambda}$$

$$\cancel{-3} - \cancel{3\lambda} + 9 - 9\lambda = -\lambda^3 + \lambda + 5\lambda^2 - 5 + 9 - 9\lambda^2$$

$$= -\lambda^3 + 5\lambda^2 - \overset{8}{\cancel{10}}\lambda + \cancel{14} + 4 = 0$$

	-1	5	-8	4
1	-1	4	-4	0
2	-1	2	0	0
2	-1	0		

$\pm 1; \pm 2; \pm 4$

$f = (\lambda - 1)(-\lambda^2 + 4\lambda - 4) = (\lambda - 1)(\lambda - 2)^2$

$\lambda_1 = 1 \quad \lambda_{2,3} = 2$

1) $\lambda_1 = 1$

$$A - \lambda E = \begin{pmatrix} -2 & 3 & -1 \\ -3 & 4 & -1 \\ -3 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} -2 & 3 & -1 \\ -1 & 1 & 0 \\ -3 & 3 & 0 \end{pmatrix} \cdot(-3) \sim \begin{pmatrix} -2 & 3 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} -2x_1 + 3x_2 - x_3 = 0 \\ -x_1 + x_2 = 0 \end{cases}$$

u_1	1	1	1

2) $\lambda_2 = \lambda_3 = 2$

$$A - \lambda E = \begin{pmatrix} -3 & 3 & -1 \\ -3 & 3 & -1 \\ -3 & 3 & -1 \end{pmatrix} \sim \begin{pmatrix} -3 & 3 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$-3x_1 + 3x_2 - x_3 = 0$$

	x_1	x_2	x_3
u_2	1	0	-3
u_3	0	1	3

$$D = T^{-1} A T$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -3 & 3 \end{pmatrix} \quad A = \begin{pmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{pmatrix}$$

матрица на прехода

Евклидово пространство

нормирани вектори

$$u_1^* = \frac{1}{\sqrt{u_1}} u_1 = \frac{1}{\sqrt{3}} (1, 1, 1)$$

$$u_2^* = \frac{1}{\sqrt{u_2}} u_2 = \frac{1}{\sqrt{10}} (1, 0, -3)$$

$$u_3^* = \frac{1}{\sqrt{u_3}} u_3 = \frac{1}{\sqrt{10}} (0, 1, 3)$$

$u_2 = (1, 0, -3)$

$u_3 = (0, 1, 3)$

$(u_2, u_3) = 1.0 + 0.1 + (-3)3 = -9$

$u_2 = v_2 \perp v_3$

$u_2 = v_2 = (1, 0, -3)$

$v_3 = u_3 + \lambda u_2 \quad \perp v_2$ (u_2) едно и също е дали е λu_2 или λv_2

$0 = (v_2, v_3) = (u_3, v_2) + \lambda(v_2, v_2)$

$\lambda = -\frac{(u_3, v_2)}{(v_2, v_2)} = \frac{-(-9)}{10} = \frac{9}{10}$

$v_3 = (0, 1, 3) + \frac{9}{10}(1, 0, -3) = \left(\frac{9}{10}, 1, \frac{3}{10}\right)$

$\sqrt{\left(\frac{9}{10}\right)^2 + 1^2 + \left(\frac{3}{10}\right)^2} = \sqrt{\frac{19\cancel{0}}{10\cancel{0}}} =$

$v_3^* = \sqrt{\frac{10}{19}}(0, 1, 3) = \left(0, \sqrt{\frac{10}{19}}, \sqrt{\frac{90}{19}}\right)$

2

$A = \begin{pmatrix} 1 & 3 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -2 \end{pmatrix}$ $\quad V = R^4$

$Ker\,\varphi$ и $Im\,\varphi$ – базис

$Ker\,\varphi = \{ v \in V \mid \varphi(v) = 0 \}$

$Im\,\varphi = \{ v' \in V \mid \exists v \in V : \varphi(v) = v' \}$

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$? $\quad AX = 0$

$$A=\begin{pmatrix}1&3&0&0\\3&1&0&0\\0&0&1&-1\\0&0&2&-2\end{pmatrix}\begin{matrix}1.(-3)\\ \\1.(-2)\\ \end{matrix}\sim\begin{pmatrix}1&3&0&0\\0&-8&0&0\\0&0&1&-1\\0&0&0&0\end{pmatrix}1:(-8)\sim\begin{pmatrix}1&3&0&0\\0&1&0&0\\0&0&1&+1\\0&0&0&0\end{pmatrix}$$

$$\left|\begin{aligned}x_1+3x_2&=0\\x_2&=0\\\underbrace{x_3}_{\text{главен}} \cancel{+}\underbrace{x_4}_{\text{свободен}}&=0\end{aligned}\right.$$

x_1	x_2	x_3	x_4
0	0	1	1

v_1 – базисен за $Ker\varphi$

$v_1(0,0,1,1)$ – базис на $Ker\varphi$

едномерно простр

$Im\varphi$

$$A=\begin{array}{c}\begin{matrix}\varphi(e_1)&\varphi(e_2)&\varphi(e_3)&\varphi(e_4)\end{matrix}\\\begin{pmatrix}1&3&0&0\\3&1&0&0\\0&0&1&-1\\0&0&2&-2\end{pmatrix}\end{array}$$

$$Im\varphi=\ell\Big(\underbrace{\varphi(e_1)}_{a_1},\underbrace{\varphi(e_2)}_{a_2},\underbrace{\varphi(e_3)}_{a_3},\underbrace{\varphi(e_4)}_{a_4}\Big)$$

$$A^T=\begin{pmatrix}1&3&0&0\\3&1&0&0\\0&0&1&2\\0&0&-1&-2\end{pmatrix}\sim\begin{pmatrix}1&3&0&0\\3&1&0&0\\0&0&1&2\\0&0&0&0\end{pmatrix}1.(-3)\sim\begin{pmatrix}1&3&0&0\\0&-8&0&0\\0&0&1&2\\0&0&0&0\end{pmatrix}1:(-8)\sim\begin{pmatrix}1&3&0&0\\0&1&0&0\\0&0&1&2\\0&0&0&0\end{pmatrix}$$

$Im\varphi=\ell(a_1,a_2,a_3)$ – базис на $Im\varphi$ (тримерно простр)

! 5.16 за домашно!

$$\boxed{\begin{aligned}Ker\varphi&=\ell(v_1)\\Im\varphi&=\ell(a_1,a_2,a_3)\end{aligned}}$$

(подходящо за събиране)

~~при~~ – хомогенна система – подходяща за сечение

1) $Ker\varphi+Im\varphi$

$$\begin{array}{c}v_1\\a_1\\a_2\\a_3\end{array}\begin{pmatrix}0&0&1&1\\1&3&0&0\\0&1&0&0\\0&0&1&2\end{pmatrix}\sim\begin{pmatrix}0&0&1&1\\1&3&0&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$$

$$\dim(U+W)=\underset{1}{\dim U}+\underset{3}{\dim W}-\underset{0}{\dim(U\cap W)}$$

4

$$\left| \begin{array}{l} x_1 + 3x_2 = 0 \\ x_2 = 0 \\ \underline{x_3} + 2x_4 = 0 \end{array} \right.$$

ФСР на хомог. с-ма:

$b_1 = \boxed{0 \,|\, 0 \,|\, -2 \,|\, 1}$

$$\mathrm{Im}\,\varphi: \left| \quad -2x_3 + x_4 = 0 \right.$$

$\ker\varphi = \ell\left((0, 0, 1, 1)\right)$

$\left| 0x_1 + 0x_2 + \underline{x_3} + x_4 = 0 \right.$

~~$\boxed{\;|\;|\;-1\;|\;1}$~~

	x_1	x_2	x_3	x_4
c_1	1	0	0	0
c_2	0	1	0	0
c_3	0	0	-1	1

$V = \mathbb{R}^6$

$U: \left| x_3 = 0 \right. \qquad U = \ell(e_1, e_2, e_4, e_5, e_6)$

Когато липсват останалите ~~стойности~~ уравнения, се задават стойности, така че да имате лин. независими базисни вектори.

$$\ker\varphi: \left| \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ -x_3 + x_4 = 0 \end{array} \right.$$

$\ker\varphi \cap \mathrm{Im}\,\varphi$:

$$\left| \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ -x_3 + x_4 = 0 \\ -2x_3 + x_4 = 0 \end{array} \right. \qquad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & +1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \sim \cancel{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

$$\sigma = (0, 0, 0, 0)$$

с-ма с 4 неизвестни — рангът на матрицата = броя базисни вектори

$$\det(A - \lambda E) = \begin{vmatrix} 1-\lambda & 3 & 0 & 0 \\ 3 & 1-\lambda & 0 & 0 \\ 0 & 0 & 1-\lambda & 1 \\ 0 & 0 & 2 & -2-\lambda \end{vmatrix} = (1-\lambda)^3(-2-\lambda) - (1-\lambda)^2(-1).2 -$$

$$-9(1-\lambda)(-2-\lambda) + 9(-1).2 =$$

(3) $V = M_2(F)$ A е фикс. от V

$\varphi: V \to V$ – лин. оператор $\qquad A = \begin{pmatrix} 2 & 5 \\ 6 & 8 \end{pmatrix}$

$\varphi(X) = AX, \ \forall x \in V$

Матрицата на φ в базиса $E_{11}, E_{12}, E_{21}, E_{22}$

1) $\varphi(X+Y) = \varphi(X) + \varphi(Y)$

$\varphi(X+Y) = A(X+Y) = AX + AY = \varphi(X) + \varphi(Y)$

φ е линеен оператор

2) $\varphi(\lambda X) = \lambda \varphi(X), \ \lambda \in F$

$\varphi(\lambda X) = A(\lambda X) = \lambda (AX) = \lambda \varphi(X)$

$\cancel{\varphi(E_{11}) \quad \varphi(E_{12}) \quad \varphi(E_{21}) \quad \varphi(E_{22})}$

$$A = \begin{array}{cccc} \varphi(E_{11}) & \varphi(E_{12}) & \varphi(E_{21}) & \varphi(E_{22}) \end{array}$$
$$A = \begin{pmatrix} 2 & 0 & 5 & 0 \\ 0 & 2 & 0 & 5 \\ 6 & 8 & 8 & 0 \\ 0 & 0 & 0 & 8 \end{pmatrix}$$

$\varphi(E_{11}) = AE_{11} = \begin{pmatrix} 2 & 5 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 6 & 0 \end{pmatrix} = 2E_{11} + 6E_{21}$

$\varphi(E_{12}) = AE_{12} = \begin{pmatrix} 2 & 5 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 6 \end{pmatrix} = 2E_{12} + 6E_{22}$

$\varphi(E_{21}) = AE_{21} = \begin{pmatrix} 2 & 5 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 8 & 0 \end{pmatrix} = 5E_{11} + 8E_{21}$

$\varphi(E_{22}) = AE_{22} = \begin{pmatrix} 2 & 5 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 0 & 8 \end{pmatrix} = 5E_{12} + 8E_{22}$

(6) $\varphi(X) = X + E \qquad \varphi(X+Y) = (X+Y) + E$

$\varphi(X) + \varphi(Y) = X + E + Y + E$

4) $a_1=(3,1,5,4)$
$a_2=(12,-7,4,11)$
$a_3=(2,-3,-2,1)$
$v=(8,-1,\lambda+6,\lambda+7)$

$\lambda=?$
v се представя по повече от 1 начин като лин. комбинация
2 различни представяния

$$v=\mu_1 a_1+\mu_2 a_2+\mu_3 a_3$$

$$8e_1+-e_2+(\lambda+6)e_3+(\lambda+7)e_4=\mu_1(3e_1+e_2+5e_3+4e_3)+$$
$$+\mu_2(12e_1+-7e_2+4e_3+11e_4)+\mu_3(2e_1+-3e_2-2e_3+e_4)$$

$$\left|\begin{aligned} 8&=3\mu_1+12\mu_2+2\mu_3\\ -1&=\mu_1-7\mu_2-3\mu_3\\ \lambda+6&=5\mu_1+4\mu_2-2\mu_3\\ \lambda+7&=4\mu_1+11\mu_2+\mu_3\end{aligned}\right.$$

$$\begin{array}{cccc} a_1 & a_2 & a_3 & v \\ \end{array}$$
$$\left(\begin{array}{ccc|c} 3 & 12 & 2 & 8\\ 1 & -7 & -3 & -1\\ 5 & 4 & -2 & \lambda+6\\ 4 & 11 & 1 & \lambda+7\end{array}\right) |(3) \quad |.(-5) \quad |.(-4) \sim$$

$$\sim\left(\begin{array}{ccc|c} 1 & -7 & -3 & -1\\ 0 & 39 & 11 & 11\\ 0 & 39 & 13 & \lambda+11\\ 0 & 39 & 13 & \lambda+11\end{array}\right) |:11 \sim \left(\begin{array}{ccc|c} 1 & -7 & -3 & -1\\ 0 & 3 & 1 & 1\\ 0 & 39 & 13 & \lambda+11\\ 0 & 0 & 0 & 0\end{array}\right) |.3 \quad |.(-13) \sim$$

$$\sim\left(\begin{array}{ccc|c} 0 & 3 & 1 & 1\\ 1 & 2 & 0 & 2\\ 0 & 0 & 0 & \lambda-2\\ 0 & 0 & 0 & 0\end{array}\right)$$

1) $\lambda\neq 2$ с-мата е несъвместима
2) $\boxed{\lambda=2} \Rightarrow \left|\begin{aligned} 3\mu_2+\mu_3&=1\\ \mu_1+2\mu_2&=2\end{aligned}\right.$ с-мата е неопределена

$\mu_3=p \quad \mu_2=\frac{1-p}{3} \quad \mu_1=\frac{6-2+2p}{3}=\frac{4+2p}{3}$

$(\mu_1,\mu_2,\mu_3)=\left(\frac{4+2p}{3},\frac{1-p}{3},p\right)$

$v=\frac{4+2p}{3}a_1+\frac{1-p}{3}a_2+pa_3 \quad \forall p\in F=\mathbb{Q}$

1) $p=0 \quad v=\frac{4}{3}a_1+\frac{1}{3}a_2$
2) $p=1 \quad v=\frac{2}{3}a_1+a_3$

$AX=B$

$(A|B)=(E|X)$

$XA=B \quad |.^t$
$A^tX^t=B^t$

$(A^t|B^t)\sim(E|X^t)$