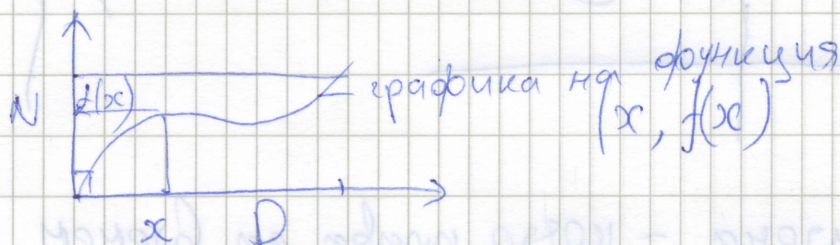


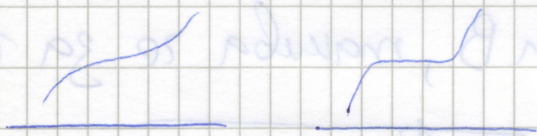
1.2 QUC 1 - Анализ - доц. Недевски
 учебник на Анализ 1 и 2 кл. - ползва се упражнение

$f(x)$
 $D \rightarrow N$
 закон
 диоф. множество

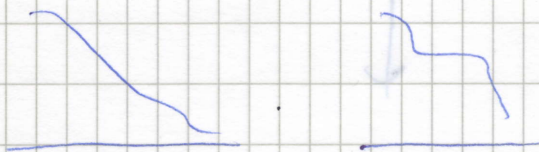
$$x \in D \Rightarrow f(x) \in N$$



Монотонни функции: $x_1 < x_2$, то $f(x_1) \leq f(x_2)$ - растяща
 $x_1 < x_2$, $f(x_1) < f(x_2)$ - строго растяща



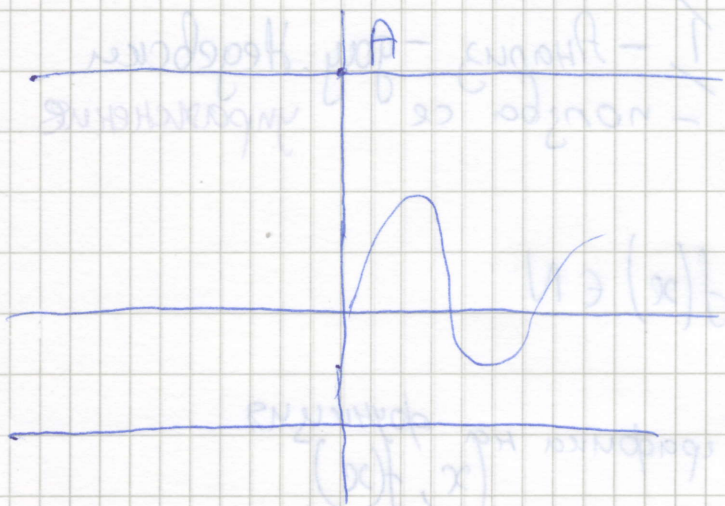
$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$ - намаляваща



Ограничени функции:

Частично монотонна - диоф. област се разлага на ^{краен брой} интерв. във всеки, който е монотонна

Ограничена отгоре: същ. Ано същ. число А, такова е $f(x) \leq A$ за $\forall x$



A - горна граница
различава се от граница
тогва горна граница



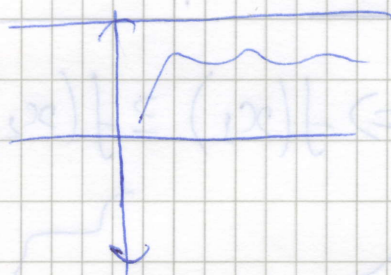
Неограничена - която права да вземем да има точки
над нея

$$\forall A \exists x_A : f(x_A) > A$$

Съществуват две числа A и B, такива че за $\forall x$
 $A \leq f(x) \leq B$

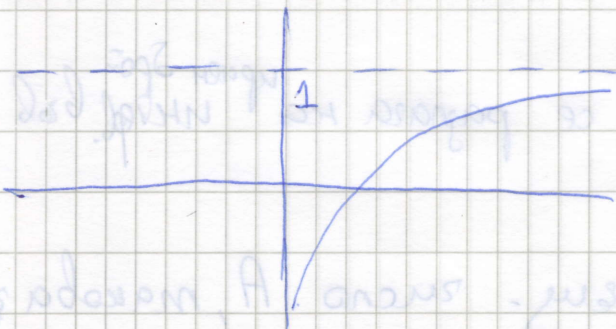
$$|f(x)| \leq A$$

$$-A \leq f(x) \leq A$$



A и B

тогва горна граница - най-малката горна граница



най-голяма стойност на $f(x)$
~~локален максимум~~
 x_0 $f(x_0) \leq f(x)$ за всяко x

локален максимум - x_0 , ако същ. отворен интервал
с центр. т. x_0 , за който са изп. 2 неща:
- за вс. точки в инт. $f(x)$ е дефинирана

1.2.2

$f(x_0)$ най-голямата стойност на $f(x)$ в този интервал
вътрешна за дефо. област

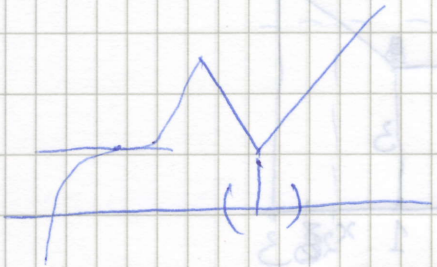
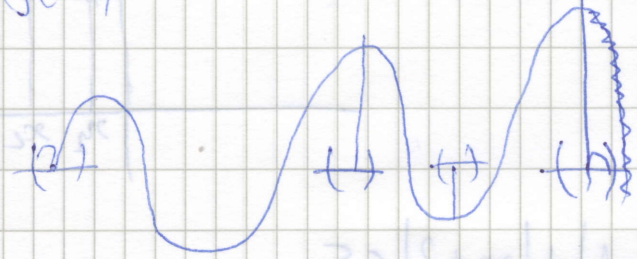
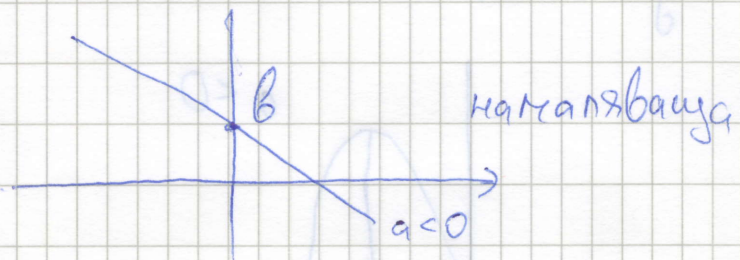
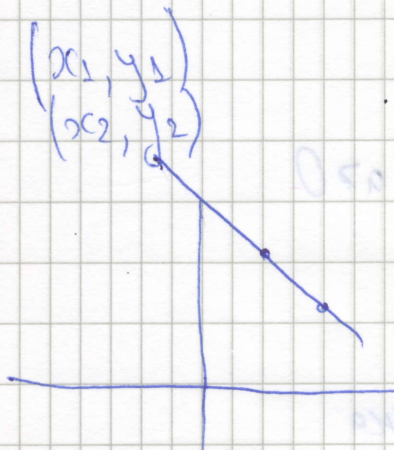
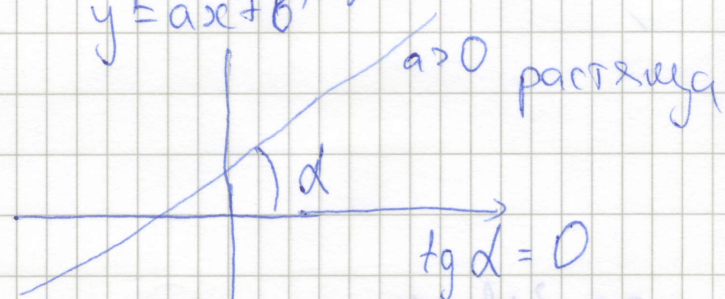
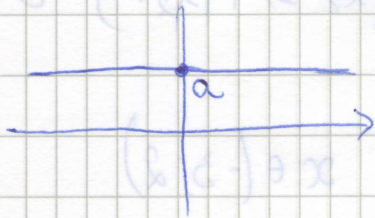


График на елементарни функции
 $y = a$ (растяща и намаляваща) $y = ax + b$



$$y = ax + b$$

$$y_1 = ax_1 + b$$

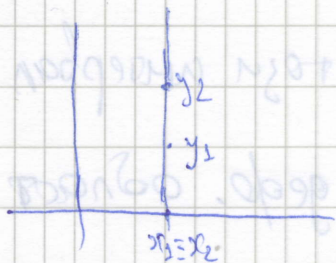
$$y_2 = ax_2 + b$$

b -отрез

$\alpha(x_0, y_0)$

$$y - y_0 = (x - x_0) \cdot \text{tg } \alpha$$

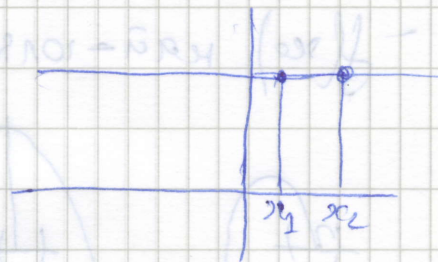
$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$



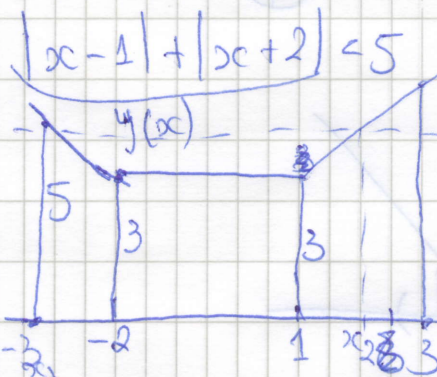
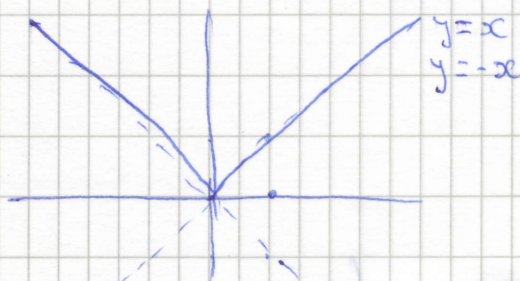
$$x_2 \geq x_1 \geq a$$

$$x \geq a$$

$$y_2 = y_1$$



$$y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



$$y(x) < 5$$

$$y(x) = 5$$

$$x_2 > 1$$

$$x_2 > -2$$

$$|x_2 - 1| + |x_2 + 2| = (x_2 - 1) + (x_2 + 2) = 5$$

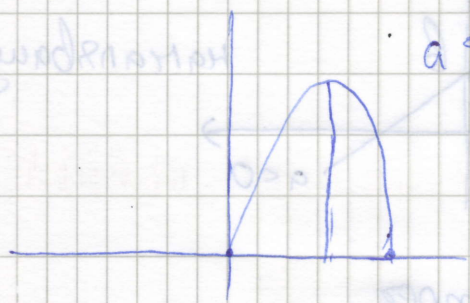
$$2x_2 + 1 = 5$$

$$2x_2 = 4$$

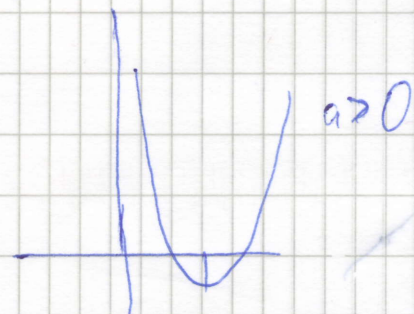
$$x_2 = 2$$

$$x \in (-3; 2)$$

$$y = ax^2 + bx + c$$



$$\frac{x_1 + x_2}{2} = -\frac{b}{2a}$$



$$D > 0$$

$$b^2 - 4ac$$

$$D = 0$$

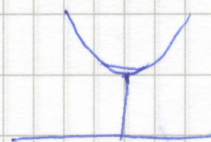
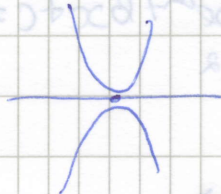
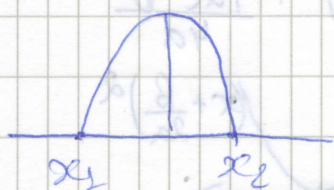
$$D < 0$$

2 корня

1 двойной корень

2.2.4

$$\frac{x - x_0}{x - x_1} = \frac{y - y_0}{y - y_1}$$



$$y = ax^2 + bx + c = a \left(x^2 + \frac{2b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} \right) = a \left(x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a}$$

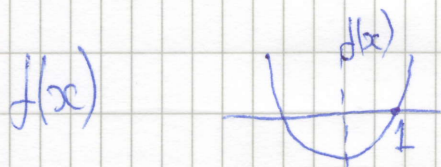
$$x_1 + x_2 = -\frac{b}{a} \quad x_1 x_2 = \frac{c}{a}$$

$$ax^3 + bx^2 + cx + d = 0 \quad x_1, x_2, x_3$$

$$x_1 + x_2 + x_3 = -\frac{b}{a}$$

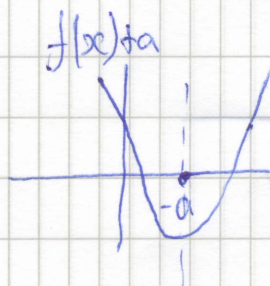
$$x_1 \cdot x_2 + x_1 x_3 + x_2 x_3 = \frac{c}{a}$$

$$x_1 x_2 x_3 = -\frac{d}{a}$$



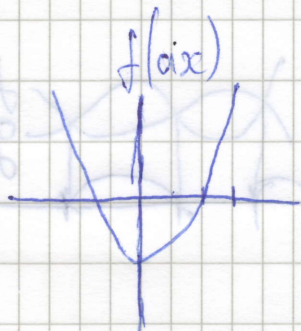
$$0 = x + a$$

$$x = -a$$



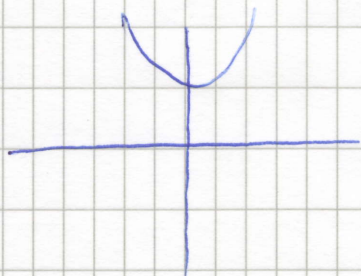
$$ax^2 = 1$$

$$x = \pm \frac{1}{a}$$

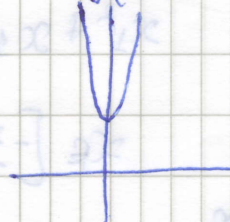


$f(-x)$
симметрия относительно O_y

$$f(x) + a$$



$$af(x)$$

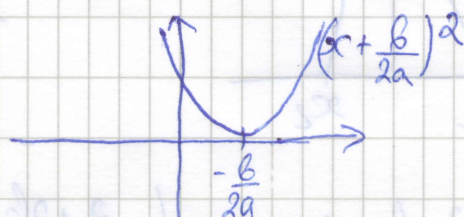
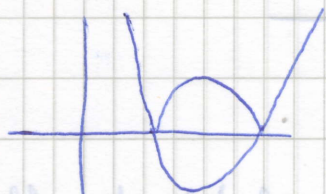


$$-f(x)$$

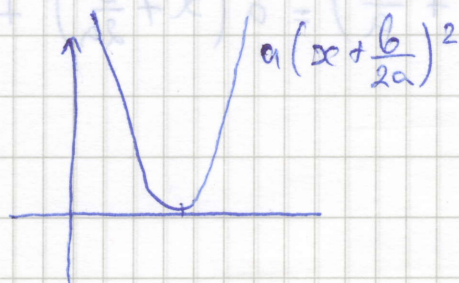


$|f(x)|$

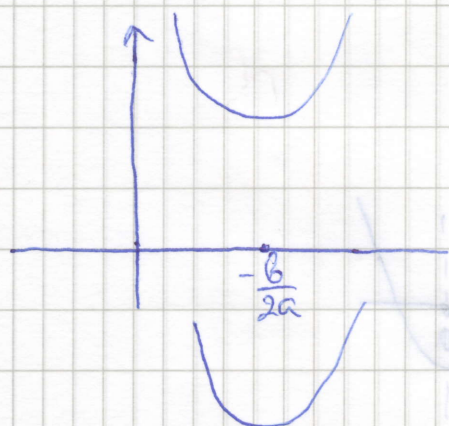
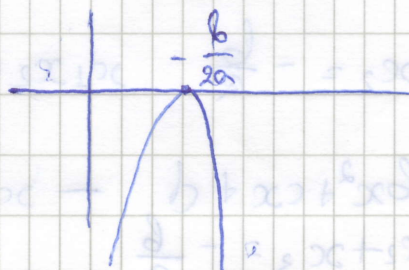
$$f(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$



1) cn.
 $a > 0$

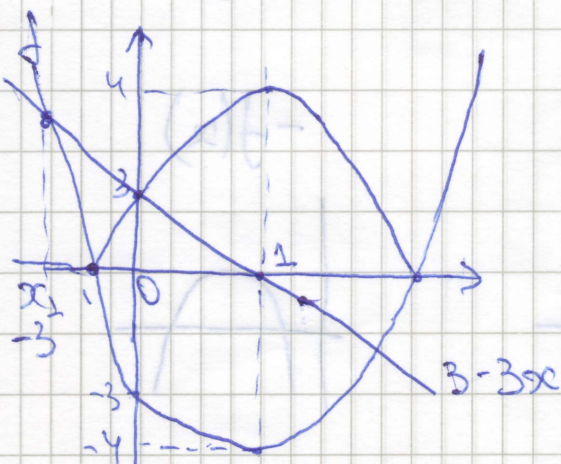
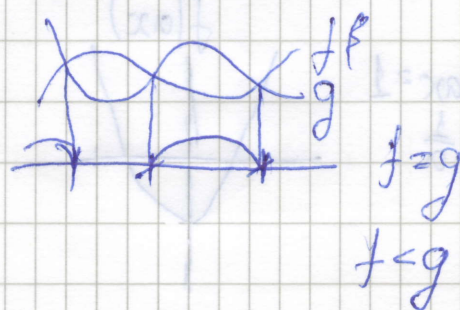


1) cn.
 $a < 0$



Да се реши неравенството:

$$\underbrace{|x^2 - 2x - 3|}_f \leq \underbrace{3 - 3x}_g$$



$$\begin{aligned} x_1^2 - 2x_1 - 3 &= 3 - 3x_1 \\ x_1^2 + x_1 - 6 &\geq 0 \end{aligned}$$

$$x \in [-3; 0]$$

1.2.6

1. Уравнение, от което и да е степен, с цели коефициенти, търсим рационални корени. Пред най-високата степен, ако е единица, то те непременно се дели своият член и те са цели числа.

$$x^3 - 3x^2 + 2 = 0 \quad \pm 1, \pm 2$$

$$x^3 - 3x^2 + 2$$

$$1^3 - 3 \cdot 1^2 + 2$$

$$\begin{aligned} x^3 - 1^3 - 3(x^2 - 1^2) &= (x-1)(x^2 + x + 1) - 3(x-1)(x+1) = (x-1)(x^2 + x + 1 - 3x - 3) \\ &= (x-1)(x^2 - 2x - 2) \end{aligned}$$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 0 & 2 \\ & 1 & -2 & -2 & 0 \end{array} \quad \text{корен}$$

$$f(x) = x^a$$

$$x \geq 0$$

$$a > 1$$

$$(a=2)$$

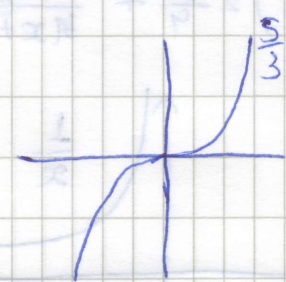
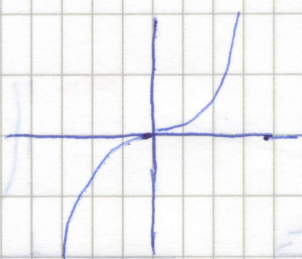
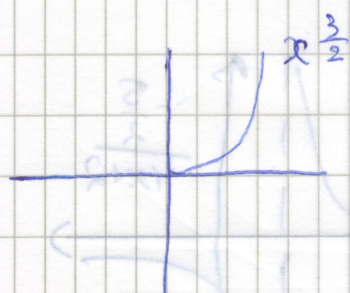
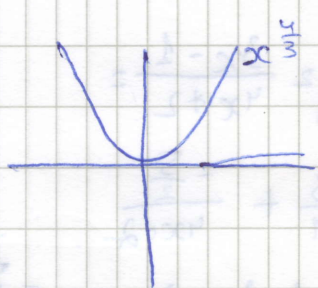
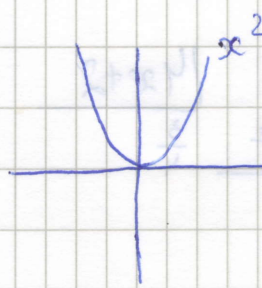
$$> 0 < a < 1$$

$$(a = \frac{1}{2} \sqrt{x})$$

$$a < 0$$



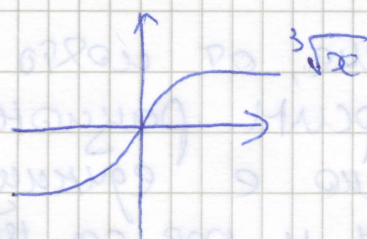
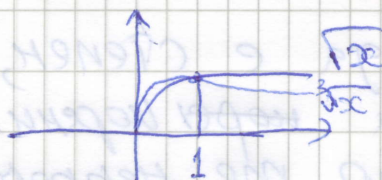
$$x^{\frac{1}{3}}$$



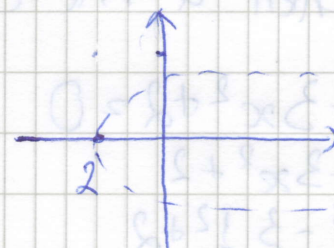
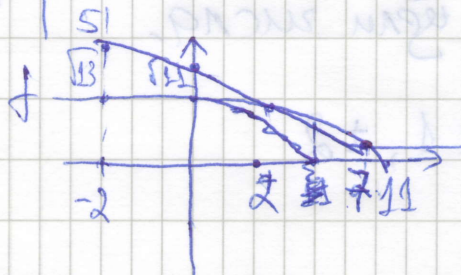
$$(x)^{\frac{4}{3}} = x^{\frac{4}{3}}$$

$$(-x)^{\frac{5}{3}} = \sqrt[3]{(-x)^5} = -x^{\frac{5}{3}}$$

$$\begin{matrix} x^2 \\ x^3 \\ x^{\frac{4}{5}} \end{matrix} \quad \begin{matrix} \sqrt{x} \\ \sqrt[3]{x} \\ x^{\frac{5}{4}} \end{matrix}$$



$$\underbrace{\sqrt{11-x}}_f < \underbrace{5-\sqrt{2+x}}_g$$



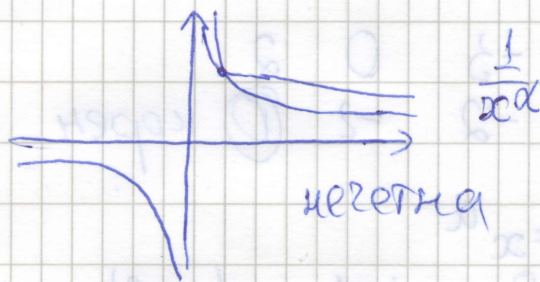
$$\begin{aligned} \sqrt{11} &\neq 5 - \sqrt{2} \\ 11 &\neq 25 - 10\sqrt{2} + 2 \end{aligned}$$

$$[-2; 2] \cup [7; 11]$$

$$10\sqrt{2} \neq 16$$

$$200 \neq 256$$

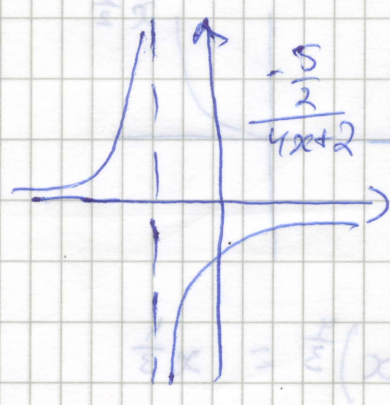
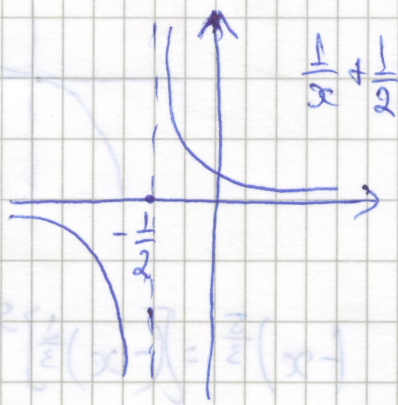
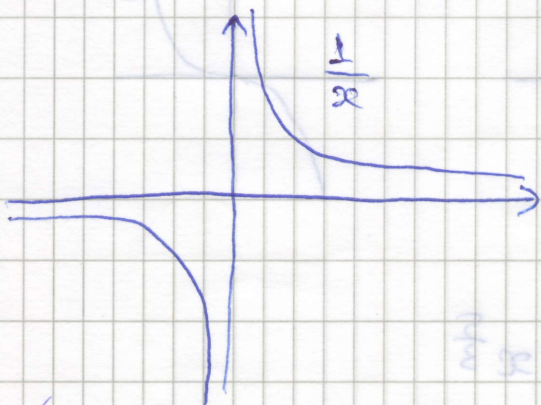
$$y = \frac{1}{x}$$



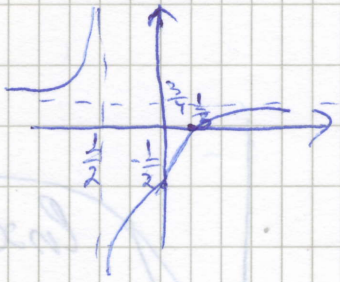
$$\begin{aligned} y^2 \frac{3x-1}{4x+2} &= \\ &= \frac{3}{4} + \frac{-\frac{5}{2}}{4x+2} \end{aligned}$$

$$\begin{aligned} &\frac{3x-1}{3x+\frac{3}{2}} \cdot \frac{4x+2}{\frac{3}{4}} \\ &= \frac{-\frac{5}{2}}{-\frac{5}{2}} \end{aligned}$$

$$y(x) = \frac{3}{4} + \frac{-\frac{5}{2}}{4x+\frac{1}{2}}$$



1.2.8



$$x^x$$

$$x^2$$

$$\frac{1}{x}$$

$$x - 3 - \sqrt{3x - 5} \leq 0$$

$$x + \sqrt{x+1} = 7$$

$$f = x$$

$$g = \sqrt{x+1}$$

$$x_1 < x_2$$

- домашно

- домашно

$$f(x_1) \leq f(x_2)$$

$$g(x_1) \leq g(x_2)$$

$$(f+g)(x_1) \leq (f+g)(x_2)$$

$$a^x$$

$$1 \neq a > 0$$

$$e^x$$

$$e \sim 2,7$$

$$N - 1, 2, 3, \dots$$

$$Z - -3, -2, -1, 0, 1, 2, 3, \dots$$

$$\text{рационални } Q = \frac{p}{q} \quad p, q - \text{цели}$$

$$\log_a x \quad \ln x - \text{натурален логаритъм}$$

$$\text{реални } R - \text{реал}$$

$$a \log_a x = x$$

$$e^{\log_e x} = x$$

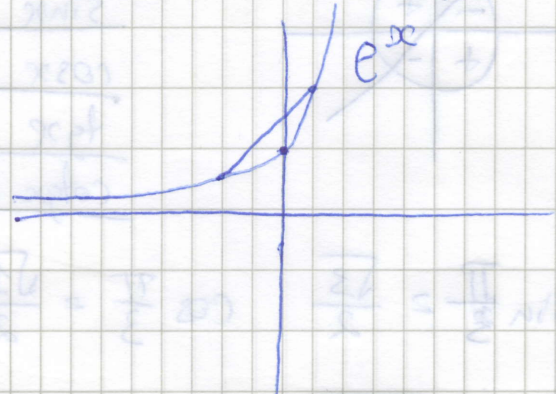
$$e^{\ln x} = \ln e^x = x$$

$$a^x \stackrel{d.}{=} e^{x \cdot \ln a}$$

рационални числа

алгебрични

трансцендентни



$$\log_a x = \frac{\ln x}{\ln a}$$

$$a^x \quad 0 < a \neq 1 \quad 0 \geq e^{\frac{x+y}{2}} < \frac{e^x + e^y}{2}$$

$$a^0 = 1$$

$$a^x \cdot a^y = a^{x+y}$$

$$a^{-x} = \frac{1}{a^x}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^x > 0$$

$$e^{\frac{x+y}{2}} < \frac{e^x + e^y}{2}$$

до-ста е ~~изисквана~~

$$D: x > 0$$

$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln xy = \ln x + \ln y$$

$$x < 0 \quad y < 0 \quad \ln xy = \ln(-x) + \ln(-y)$$

$$xy > 0 \quad \ln|x| + \ln|y|$$

$$\ln \frac{1}{x} = -\ln x$$

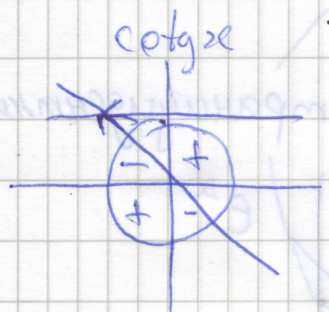
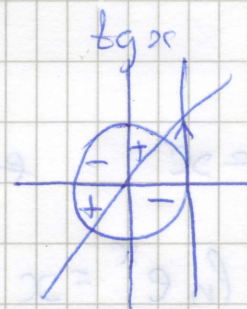
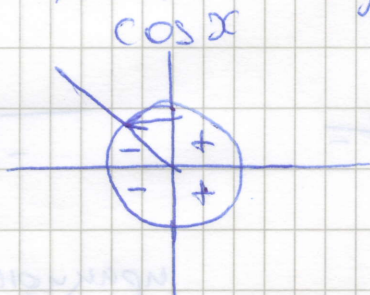
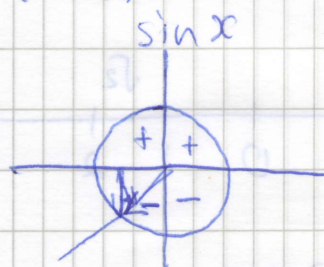
$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\ln x^a = a \cdot \ln x$$

$$\ln x^2 = 2 \ln|x|$$

$$\ln\left(\frac{x+y}{2}\right) > \frac{\ln x + \ln y}{2}$$

- до-ста е ~~изисквана~~



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin x	0	1	2	3	4
cos x	4	3	2	1	0
tg x	0	3	9	27	∞
cotg x	∞	27	9	3	0

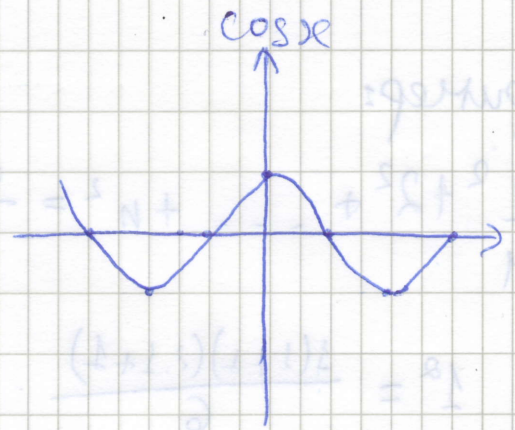
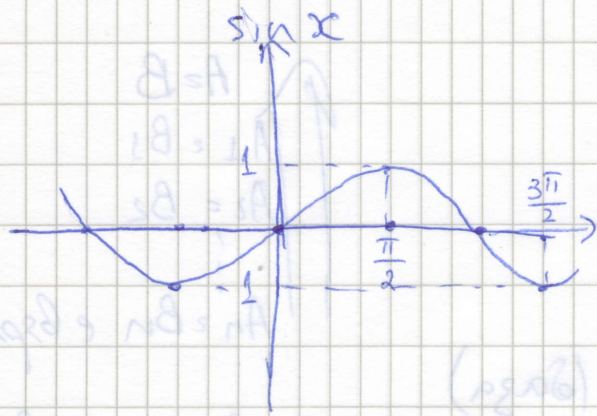
$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\frac{\text{tg}}{\text{cotg}} = \frac{\sqrt{3}}{3}$$

$$\frac{\sin}{\cos} = \frac{\sqrt{2}}{2}$$

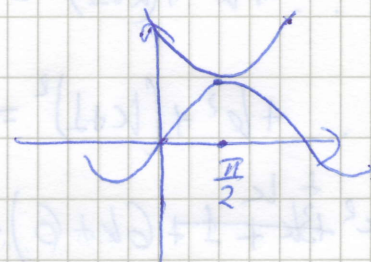


$$x^2 - \pi x + 1 = \sin x - \frac{\pi^2}{4}$$

$$x^2 - \pi x + 1 = \cos x - \frac{\pi^2}{4}$$

$$x^2 - \pi x + 1 + \frac{\pi^2}{4} = \sin x \cos x$$

$$\frac{\pi^2}{4} - \frac{\pi^2}{2} + 1 + \frac{\pi^2}{4} = 1$$



Принцип за математическа индукция.
Аксиома на рачуни

$$M \subset N$$

$$1. 1 \in M$$

$$2. \text{ Ако } k \in M \Rightarrow k+1 \in M$$

$$M = N$$

Поврдување: $T(n)$

$$1. T(1) \text{ е вярно}$$

$$2. \text{ Ако } T(k) \text{ е вярно} \Rightarrow T(k+1) \text{ е вярно}$$

$$T(n) \text{ е вярно за воеко } n \in N$$

Пример:

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$A=B$
 $A_1=B_1$
 $A_2=B_2$
 $\vdots$
 $A_n=B_n$ е вярно

1) $1^2 = \frac{1(1+1)(2 \cdot 1+1)}{6} \Rightarrow 1 \in M$ (базис)

$A=A_1=A_2=\dots=B$

$A-B=\dots=0$

2) $k \in M \quad 1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$

$\Rightarrow k+1 \in M$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$$

$$= \frac{k+1}{6} (2k^2 + \cancel{3k} + 1 + 6k + 6) = \frac{k+1}{6} (2k^2 + 9k + 7) = \frac{k+1}{6} (k+2)(k+\frac{7}{2}) =$$

$$k_1 = -2 \quad k_2 = -\frac{7}{2} \quad = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} + (n-1)(n-2)(n-3)(n-4) \dots (n-100)$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2} \right)^2$$

1) $1^3 = \left(\frac{1 \cdot 2}{2} \right)^2 \quad 1=1 \Rightarrow$ това е вярно

2) $k \in M \quad 1^3 + 2^3 + 3^3 + \dots + k^3 = \left(\frac{k(k+1)}{2} \right)^2$ допускаме, че това е вярно

$$1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2} \right)^2$$

$$\left(\frac{k(k+1)}{2} \right)^2 + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2} \right)^2$$

1.2.12

$$\frac{k^2(k^2+2k+1)}{2} \quad \frac{(k+1)^2}{4} (k^2+k+4) = \frac{(k+1)^2}{4} (k^2+2k+4)$$

$$\frac{(k+1)^2}{4} (k^2+2k+4) = \frac{(k+1)^2}{4} (k^2+2k+4+2k+4)$$

$$\frac{(k+1)^2}{4} (k^2+2k+4) = \frac{(k+1)^2}{4} (k^2+4k+8)$$

$$\frac{(k+1)^2(k+2)^2}{4} = \frac{(k+1)^3}{4} \Rightarrow T(n) \text{ верно за } \forall k$$

$$1^4 + 2^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

$$1 + 2 + \dots + n = \frac{n^2}{2} + \frac{n}{2}$$

$$1^2 + 2^2 + \dots + n^2 = \frac{n^3}{3} + a \cdot n^2 + b \cdot n$$

$$1^3 + 2^3 + \dots + n^3 = \frac{n^4}{4} + \dots n^3 + \dots n^2$$

$$1^4 + 2^4 + \dots + n^4 = \frac{n^5}{5} + \dots n^4$$

$$1^k + 2^k + \dots + n^k = \frac{n^{k+1}}{k+1} + \dots n^k$$

$P_k(n)$

$T(n)$

$$2^n > n^2$$

$$1) n=1 \quad 2 > 1$$

$$2) 2^k > k^2 \quad / \cdot 2$$

$$2^{k+1} > (k+1)^2$$

$$2^{k+1} > 2k^2$$

$$2k^2 > (k+1)^2$$

$$k \geq 2$$

$$k+1 \geq 3$$

$$2k^2 - k^2 - 2k - 1 > 0$$

$$k^2 - 2k - 1 > 0$$

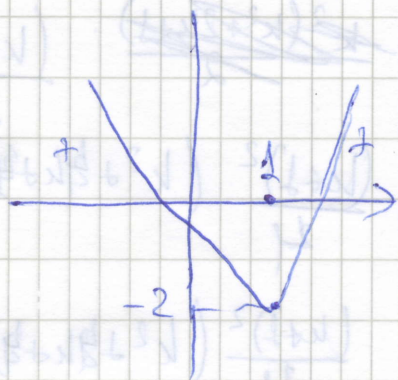
$$k_1 = 1 + \sqrt{2}$$

$$k > 1 + \sqrt{2}$$

$$k \in \mathbb{N} \quad k \geq 3$$

$$\| T(k) \Rightarrow T(k+1)$$

$$k \geq 3$$



$$T(n) \quad n_0 \in \mathbb{Z} \quad 1) T(n_0) \text{ е вярно}$$

$$2) T(n) \text{ е вярно} \Rightarrow T(n+1) \text{ е вярно.} \quad n \in \mathbb{Z} \quad n \geq n_0$$

$$2^{10} = 1024 > 10$$

$$T(5) \text{ е вярно}$$

$$n \geq 3$$

$$n \geq 5$$

$$2^n > n^2 \text{ е вярно}$$

$$2^5 = 32 > 5^2$$

$$\text{за } \forall n \geq 5$$

$$2^n > n^3 \quad - \text{домашно}$$

$$2^n > n^3$$

$$1) n=1 \quad 2 > 1 \quad \checkmark$$

$$2) 2^k > k^3 \quad / \cdot 2 \quad \checkmark$$

$$2^{k+1} > (k+1)^3$$

$$2^{k+1} > 2k^3$$

$$2k^3 > (k+1)^3$$

$$2k^3 - k^3 - 3k^2 - 3k - 1 > 0$$

$$k^3 - 3k^2 - 3k - 1 > 0 = \frac{k^3}{3} + \frac{k^3}{3} + \frac{k^3}{3} - 3k^2 - 3k - 1 =$$

$$= \frac{k^3}{3} (k-9) + \frac{k^3}{3} (k^2-9) + \frac{k^3}{3} - 1 \geq 0 \quad \text{при } k \geq 9$$

$$2^n > n^4 \quad - \text{домашно}$$

$$T(k) \Rightarrow T(k+1)$$

$$k \geq 9$$

$$T(10)$$

$$512 < 729$$

$$1024 > 1000$$

Да се докаже, че $(1+x)^n \geq 1+nx$ за $\forall n \geq 1, n \in \mathbb{N}$
 $x \geq -1$

1) ~~Q~~ при $n=1$ $1+x \geq 1+x$ $\checkmark$

2) Допускаме, че $\{ n=2 \}$ $(1+x)^k \geq 1+kx$ $\forall (1+x)^{x \neq -1}$

~~$1+x+x^2 \geq 1+x$~~ $(1+x)^{k+1} \geq 1+(k+1)x$

$(1+x)^{k+1} \geq (1+kx)(1+x)$
 $(1+x)^{k+1} \geq 1+x+kx+kx^2 \geq 1+x+kx$

Нека $a, b \geq 0$ да се докаже: $\frac{a+b}{2} \geq \sqrt{ab}$

$a \leq b \Leftrightarrow a^2 \leq b^2$
 $\begin{matrix} a \geq 0 \\ b \geq 0 \end{matrix}$

~~$a+b \geq 2\sqrt{ab}$~~

$(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{a}\sqrt{b} \geq 0$

$(\sqrt{a} - \sqrt{b})^2 \geq 0$
 $a \neq b$

$a_1, a_2, \dots, a_n \geq 0$

$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}$

Ако $\forall$ изела са равнина имаме
 равенство, ако ли не е строго неравенство
 неравенство на Куши (ср. аритм. и
 ср. геометрично)

$x_1, x_2, \dots, x_n > 0 \Rightarrow x_1 \cdot x_2 \cdot x_3 \dots x_n = 1 \Rightarrow$

$\Rightarrow x_1 + x_2 + \dots + x_n \geq n$

T(2) $x_1 \cdot x_2 = 1 \Rightarrow x_1 + x_2 \geq 2$ $k \Rightarrow k+1$ - за домашно

$$2 \Rightarrow 3 \quad x_1 x_2 x_3 = 1 \Rightarrow x_1 + x_2 + x_3 \geq 3 \quad x_1 = x_2 = x_3 = 1 \Rightarrow x_1 + x_2 + x_3 = 3$$

$$x_1 > 1 \quad (x_2 \geq 1 \quad x_3 \geq 1) \quad x_2 < 1 \quad (1-x_1)(1-x_2) \leq 0$$

$$x_1 + x_2 > 1 + x_1 x_2$$

$$x_1 + x_2 + x_3 > 1 + \overset{a}{(x_1 x_2)} + \overset{b}{x_3} \geq 1 \quad | + 2 = 3$$

$$ab = 1 \Rightarrow a + b \geq 2 \quad \Sigma \cancel{a+b} \Rightarrow x_1 + x_2 > 1 + x_1 x_2$$

Ретроградна математическа
индукция

$$1) \quad M \xrightarrow{n < n_1} M \quad T(n) \text{ е вярно. } n \in M$$

$$2) \quad T(n) \text{ е вярно } \Rightarrow T(n-1) \text{ е вярно}$$

Метод на ретроградната индукция
Метод на ретроградната индукция е верен.

$$k = 2^n \quad k = 1$$

$$n \geq 1 \quad \frac{a_1 + b_1}{2} \geq \sqrt{a_1 b_1}$$

$$M: 2, 4, 8, 16, \dots$$

Допускаме за намиране
на точное n.

$$\frac{a_1 + a_2 + \dots + a_{2^n}}{2^n} \geq \sqrt[2^n]{a_1 a_2 \dots a_{2^n}}$$

$$\underbrace{a_1, a_2, \dots, a_{2^n}}_{2^n}, \underbrace{a_{2^n+1}, a_{2^n+2}, a_{2^n+3}, \dots, a_{2^{n+1}}}_{2^n}$$

$$a_1, a_2, \dots, a_4, a_5, a_6, a_7, a_8$$

$$2^n$$

$$\cancel{a_1 + a_2 + \dots + a_{2^n}}$$

$$1. 2. 16$$

$$\frac{a_1 + a_2 + \dots + a_{2^n} + a_{2^{n+1}} + \dots + a_{2^{n+1}}}{2^{n+1}} \geq \frac{1}{2^{n+1}} \left\{ \geq \right.$$

$$= \frac{1}{2^{n+1}} \left(2^n \sqrt[n]{a_1 \cdot a_2} + 2^n \sqrt[n]{a_{2^{n+1}} \cdot \dots \cdot a_{2^{n+1}}} \right)$$

$$\Sigma = \frac{1}{2} \sqrt[n]{a_1 \cdot a_2} \cdot \sqrt[n]{a_{2^{n+1}} \cdot \dots \cdot a_{2^{n+1}}} = \sqrt[n]{a_1 \cdot \dots \cdot a_{2^{n+1}}}$$

$$\frac{1}{2^n} - \frac{1}{2} = \frac{1}{2^{n-1}}$$

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n} \quad n=2^k$$

$$n < 2^n \quad 3 < 2^3 \quad 100 < 2^{100}$$

$$A = \frac{a_1 + \dots + a_{n-1}}{n-1} \geq \sqrt[n-1]{a_1 a_2 \dots a_{n-1}}$$

$$A = \frac{a_1 + a_2 + \dots + a_{n-1} + A}{n} \geq \sqrt[n]{a_1 a_2 \dots a_{n-1} \cdot A}$$

$$A^{n-1} \geq a_1 a_2 \dots a_{n-1}$$

$$\frac{a_1 + \dots + a_{n-1}}{n-1} \geq \sqrt[n-1]{a_1 \dots a_{n-1}} \Rightarrow \forall n$$

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

Бином на Нютон

Сингмент коефициенти

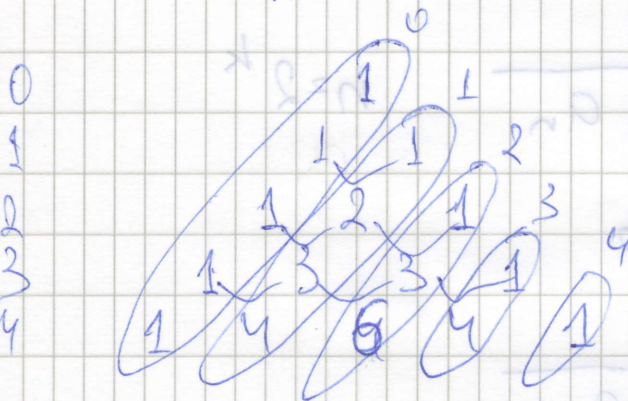
$$\binom{\alpha}{k} = \begin{cases} \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-k+1)}{k(k-1)\dots 3 \cdot 2 \cdot 1} & k \geq 1 \\ 1 & k=0 \end{cases} = C_n^k$$

$$\binom{5}{3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 35$$

$$C_n^k$$

$$C_n^{\frac{1}{2}}$$

$$\binom{\frac{1}{2}}{2} = \frac{\frac{1}{2}(\frac{1}{2}-1)}{2 \cdot 1} = -\frac{1}{8}$$



$$\textcircled{1} \binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\binom{n}{0} + \binom{n}{1} = \binom{n+1}{1}$$

$$\textcircled{2} \binom{n}{1} = n \quad \binom{n}{n} = 1$$

$$\binom{n}{k}$$

$$n, k \geq 0$$

$$0 \leq k \leq n$$

$$k \neq 0$$

$$\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)(n-k)(n-k-1)\dots 3 \cdot 2 \cdot 1}{k(k-1)\dots 1} = \frac{n!}{(k!)(n-k)!}$$

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!} = \frac{n!}{(n-k)! \cdot k!}$$

Да следователно, че $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$ за $\forall x$ и $\forall n \geq 0$ е изпълнено.

$$(1+x_1)(1+x_2)(1+x_3) = (1+x_1+x_2+x_1x_2)(1+x_3) =$$

$$= 1 + x_1 + x_2 + x_1x_2 + x_3 + x_1x_3 + x_2x_3 + x_1x_2x_3$$

$$(1+x_1)(1+x_2)(1+x_3) = 1 + (x_1 + x_2 + x_3) + (x_1x_2 + x_1x_3 + x_2x_3) + x_1x_2x_3$$

$$x_1 = x_2 = x_3 \quad (1+x)^3 = 1 + \binom{3}{1}x + \binom{3}{2}x^2 + \binom{3}{3}x^3$$

1) Если $n=1$

$$(1+x)^1 = \binom{1}{0} + \binom{1}{1}x = 1 + x$$

$$1+x = 1 + 1 \cdot x = 1+x$$

$$1+x \equiv 1+x$$

2) допустим, что $T(n)$ верно для некоего n

$$(1+x)^{n+1} = \binom{n+1}{0} + \binom{n+1}{1}x + \binom{n+1}{2}x^2 + \dots + \binom{n+1}{n}x^n + \binom{n+1}{n+1}x^{n+1}$$

$$(1+x)^n(1+x) = \left(\binom{n}{0} + \binom{n}{1}x + \dots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n \right) (1+x)$$

$$= \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n + \binom{n}{n+1}x^{n+1}$$

$$x = \frac{b}{a}$$

$$1 - q^n = (1 - q)(1 + q + q^2 + \dots + q^{n-1})$$

$$q = \frac{b}{a} \quad a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \dots + b^{n-1})$$

$$(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3}) = 2 - 3$$

$$(\sqrt[3]{2} - \sqrt[3]{3})((\sqrt[3]{2})^2 + \sqrt[3]{2}\sqrt[3]{3} + (\sqrt[3]{3})^2) = 2 - 3$$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$$

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^n \binom{n}{n} = 0$$

$$1^4 + 2^4 + \dots + n^4 = S_4$$

$$\left. \begin{aligned} (1+1)^5 &= 1^5 + 5 \cdot 1^4 + 10 \cdot 1^3 + 10 \cdot 1^2 + 5 \cdot 1 + 1 \\ (1+2)^5 &= 1 + 5 \cdot 2 + 10 \cdot 2^2 + 10 \cdot 2^3 + 5 \cdot 2^4 + 1 \cdot 2^5 \\ (1+3)^5 &= 1 + 5 \cdot 3 + 10 \cdot 3^2 + 10 \cdot 3^3 + 5 \cdot 3^4 + 1 \cdot 3^5 \\ \vdots \\ (1+n)^5 &= 1 + 5 \cdot n + 10 \cdot n^2 + 10 \cdot n^3 + 5 \cdot n^4 + n^5 \end{aligned} \right\} \star$$

$$(1+n)^5 = 1 + n + 5(1+2+3+\dots+n) + 10(1^2+2^2+3^2+\dots+n^2) + 10(1^3+2^3+\dots+n^3) + 5(1^4+2^4+\dots+n^4) \\ \stackrel{S_4}{\quad}$$

$$n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1 - 1 = n - 5 \left\{ \frac{n(n+1)}{2} \right\} - 10 \frac{n(n+1)(2n+1)}{6} - 10 \frac{n^2(n+1)^2}{4} - 5S_4$$

$$\boxed{S_4 = \frac{n^5}{5} + P_4(n)} \quad S_4 = \frac{n^5}{5} + \frac{n^4}{5} \left(5 - \frac{10}{4} \right)$$

$$1^k + 2^k + \dots + n^k = \frac{n^{k+1}}{k+1} + P_k(n)$$

$T(n)$ ① $T(1)$ е вярно

② Допускаме, че $T(1), T(2), \dots, T(n) \Rightarrow T(n+1)$

$\Rightarrow T(n)$ е вярно за $\forall n$

$$T(1) = 1 + 2 + \dots + n = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} = \frac{n^2}{2} + P_1(n)$$

$$T(2) = 1^2 + 2^2 + \dots + n^2 = \frac{n^3}{3} + P_2(n)$$

$$T(3) = 1^3 + 2^3 + \dots + n^3 = \frac{n^4}{4} + P_3(n)$$

$$\tilde{T}(k) = 1^k + 2^k + \dots + n^k = \frac{n^{k+1}}{k+1} + P_k(n)$$

1.2.20

$$T(k+1) = \frac{n^{k+2}}{k+2} + P_{k+1}(n)$$

$$(1+1)^{k+2} = \binom{k+2}{0} + \binom{k+2}{1} 1^1 + \binom{k+2}{2} 1^2 + \dots + \binom{k+2}{k+1} 1^{k+1} + \binom{k+2}{k+2} 1^{k+2}$$

$$(1+2)^{k+2} = \binom{k+2}{0} + \binom{k+2}{1} 2^1 + \binom{k+2}{2} 2^2 + \dots + \binom{k+2}{k+1} 2^{k+1} + \binom{k+2}{k+2} 2^{k+2}$$

$$(1+n)^{k+2} = \binom{k+2}{0} + \binom{k+2}{1} n + \dots + \binom{k+2}{k+1} n^{k+1} + \binom{k+2}{k+2} n^{k+2}$$

$$S_{k+1} = 1^{k+1} + \dots + n^{k+1} = P_{k+1}(n)$$

$$(1+n)^{k+2} - 1 = n + \binom{k+2}{2} (1^2 + 2^2 + \dots + n^2) + \dots + \binom{k+2}{k+1} (1^{k+1} + 2^{k+1} + \dots + n^{k+1})$$

$$n^{k+2} + \frac{S_{k+1}}{k+1} = (1+n)^{k+2} - 1 \quad \binom{k+2}{k+1} = \binom{k+2}{1} = k+2$$

$$n^{k+2} + P_{k+1}(n) = (k+2) S_{k+1}$$

Редици

$\{a_n\}$

a_n - редицата е общ член a_n

Функция с деф. множество от ест. числа - редица.

a_n не е ограничена отгоре - съществува число A , за $\forall n$ е изпълнено $a_n \leq A$

a_n не е ограничена отгоре - за $\forall A$, същ. число n_A , такова че $a_{n_A} > A$.

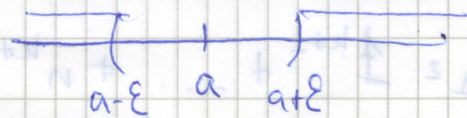
$a_n \rightarrow +\infty$ за $\forall A$, $\exists n_A$, такова че за $\forall n > n_A$ е изпълнено $a_n > A$

Примери:

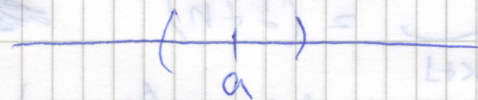
$1, 1, 1, 1, \dots$
 $1, 0, 1, 0, \dots$
 $1, \frac{1}{2}, \frac{1}{3}, \dots$
 $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \dots$
 $1, 2, 3, 4, \dots \rightarrow +\infty$
 $1, 2^2, 3^2, 4^2, \dots \rightarrow +\infty$
 $1, -2, 3, -4, 5, \dots \times$
 $1, 2, 3, 1, 4, 1, 5, 1, \dots \times$

$a \leftarrow a_n \rightarrow a$

ограничен
граница
горна граница



$a \leftarrow a_n \rightarrow a$ за $\forall \varepsilon > 0$, $\exists N(\varepsilon)$, такова че за $\forall n > N$ е
 $\Rightarrow |a_n - a| < \varepsilon$
 $\Rightarrow a$ е точката съвпадане за $\forall \varepsilon > 0$, и $\forall n$, $\exists n_0$, такова че $n_0 > n$ и $|a_{n_0} - a| < \varepsilon$



		b_n		
		B	$+\infty$	$-\infty$
a_n	A	$A+B$	$+\infty$	$-\infty$
	$+\infty$	$+\infty$	$+\infty$	?
	$-\infty$	$-\infty$	?	$-\infty$

$\infty - \infty$
неопределеност

$n+2 \rightarrow +\infty$
 $n \rightarrow +\infty$
 $(n+2) - n = 2 \rightarrow 2$

	$B < 0$	$B = 0$	$B > 0$	$+\infty$	$-\infty$
$A < 0$	$A \cdot B$	$A \cdot B$	$A \cdot B$	$-\infty$	$+\infty$
$A = 0$	$A \cdot B$	$A \cdot B$	$A \cdot B$		
$A > 0$	$A \cdot B$	$A \cdot B$	$A \cdot B$	$+\infty$	$-\infty$
$+\infty$					$\frac{1}{+\infty} \rightarrow 0$
$-\infty$					$\frac{1}{-\infty} \rightarrow 0$

$\infty - \infty, 0 \cdot \infty, \frac{\infty}{\infty}, \frac{0}{0}$
 $a_n^x \rightarrow a^x$
 $a_n^x \rightarrow a^x$
 $\sqrt[n]{a_n} \rightarrow \sqrt[n]{a}$
 $a_n^2 \rightarrow a^2$

$$(a_n)^{b_n} \Leftrightarrow 1^{b_n} = 1 \rightarrow 1 \quad 2 \leq \left(1 + \frac{1}{n}\right)^n \leq 3 \quad e$$

$$a_n \rightarrow 1 \quad b_n \rightarrow \infty \quad \frac{a_n}{b_n} \rightarrow \frac{a}{b} \quad b_n \neq 0 \quad b \neq 0$$

Задача: 1) $\frac{\infty}{\infty}$

$$\frac{n^2 - n + 3}{3n^2 - 3n + 2} = \frac{n^2(1 - \frac{1}{n} + \frac{3}{n^2})}{n^2(3 - \frac{3}{n} + \frac{2}{n^2})} \Rightarrow \frac{1}{3}$$

$$a_n \xrightarrow{n \rightarrow \infty} a$$

$$a_n \rightarrow a$$

$$n \rightarrow \infty$$

$$n^k \rightarrow \infty$$

$$k > 0$$

$$\frac{1}{n^k} \rightarrow 0$$

$$k > 0$$

$$\frac{n^2 - 5n^3 - 10n^4}{n^3 - 3n^2 + 1} = \frac{n^4(\frac{1}{n^2} - \frac{5}{n} - 10)}{n^3(\frac{1}{n} - \frac{3}{n^2} + \frac{1}{n^3})} \Rightarrow \frac{n \cdot (-10)}{1} \Rightarrow -\infty$$

$$\frac{a_0 n^k + a_1 n^{k+1} + \dots + a_k}{b_0 n^l + b_1 n^{l-1} + \dots + b_l} = \frac{n^k}{n^l} \frac{a_0 + \frac{a_1}{n} + \frac{a_2}{n^2} + \dots}{b_0 + \frac{b_1}{n} + \frac{b_2}{n^2} + \dots} = n^{k-l} \left(\frac{a_0}{b_0} \right)$$

Если $k > l$ $\rightarrow \infty \cdot \left(\frac{a_0}{b_0} \right)$
 $k = l$ $\left(\frac{a_0}{b_0} \right)$
 $k < l$ 0

$$\frac{1^2 + 2^2 + \dots + n^2}{n^3} = \frac{n(n+1)(2n+1)}{6n^3} \Rightarrow \frac{2}{6} = \frac{1}{3}$$

$$\frac{1^3 + 2^3 + \dots + n^3}{n^4} = \frac{n^4}{4} + P_3(n) \rightarrow \frac{1}{4}$$

$$\frac{1}{na} \rightarrow 0$$

$a > 0$

$$\begin{array}{l} \uparrow a_n \rightarrow a \\ a_n^x \rightarrow a^x \\ \downarrow |a_n| \rightarrow |a| \end{array}$$

$$\begin{array}{l} a_n \rightarrow a \Rightarrow |a_n| \rightarrow |a| \\ a_n \rightarrow 0 \Rightarrow |a_n| \Rightarrow 0 \end{array}$$

$$a_n \rightarrow a$$

$$a_n - a \rightarrow 0$$

$$|a_n - a| \Rightarrow 0$$

$$a_n \leq b_n \leq c_n$$

$\searrow \quad \swarrow$
 a

$$\begin{array}{cc} a_n \leq b_n & \\ \downarrow & \downarrow \\ a & b \\ \hline a \leq b \end{array}$$

за безбројности

$$a_n \leq b_n \text{ при } n \geq n_0$$

$$\downarrow$$

$$b_n \rightarrow \infty$$

Теорема за 2-та ниво

$$\sqrt[n]{a} = \cancel{a} = a^{\frac{1}{n}} \rightarrow a^0 = 1$$

$$a > 0$$

$$a > 1 \quad \sqrt[n]{a} - 1 = d_n > 0 \quad (d_n + 1)^n = a \geq n d_n \quad 0 < d_n \leq \frac{a}{n} \text{ за } n\text{-ест. зано}$$

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n \geq 1 + nx \quad x \geq 0$$

$$\frac{n(n-1)(n-2) \dots (n-k+1)}{k(k-1) \dots 1} \geq 1 \quad 0 \leq k \leq n$$

$$(1+x)^n \geq 1 + nx \quad x \geq 0 \quad x \geq -1$$

$$\sqrt[n]{n} \rightarrow 1$$

$$\sqrt[n]{\frac{1}{a}} \rightarrow 1$$

$$\sqrt[n]{a} \rightarrow 1$$

$$\sqrt[n]{n} - 1 = d_n$$

$$(d_n + 1)^n = n \geq n d_n$$

$$d_n \leq 1$$

$$0 \leq d_n \leq 1$$

$$(1+x)^n \geq \frac{n(n-1)}{2} x^2 \geq 2$$

$$n = (d_n + 1)^n \geq \frac{n(n-1)}{2} d_n^2$$

$$0 \leq d_n^2 \leq \frac{2}{n-1}$$

$$\sqrt{d_n^2} = |d_n|$$

$$0 < d_n \leq \frac{\sqrt{2}}{\sqrt{n-1}}$$

$$q > 1$$

$$q^n \rightarrow +\infty$$

$$q^n = (1+x)^n \geq nx$$

$$q^n \geq nx \rightarrow +\infty$$

$$q = 1+x \text{ with } x > 0 \text{ for } x \neq 0$$

$$q > 1 \Rightarrow q^n \rightarrow +\infty$$

$$a_n \rightarrow \infty \quad |a_n| \rightarrow +\infty$$

$$-1 < q < 1 \quad q^n \rightarrow 0$$

$$0 < |q| < 1$$

$$1 < \frac{1}{|q|}$$

$$\frac{1}{|q|^n} \rightarrow \infty$$

$$|q|^n \rightarrow 0$$

$$q^n \rightarrow 0 \quad n^k q^n \rightarrow 0 \quad k \geq 0 \quad -1 < q < 1$$

$$n^3 q^n \rightarrow 0$$

$$|q| < 1$$

$$0 < q < 1$$

$$\frac{1}{n^3 q^n} \rightarrow \infty$$

$$\frac{1}{q} = 1+x \quad x > 0$$

$$\left(\frac{1}{q}\right)^n = (1+x)^n \geq n \cdot x$$

$$(1+x)^n \geq \binom{n}{4} x^4 \quad n \geq 4$$

$$\frac{1}{n^3 q^3} \geq \frac{x^4}{n^4} \rightarrow 0$$

$$\frac{1}{q^n} = (1+x)^n \geq x^4 \frac{n(n-1)(n-2)(n-3)}{4!}$$

$$\frac{1}{q^n \cdot n^3} \geq x^4 \frac{n(n-1)(n-2)(n-3)}{4! \cdot n^3} \rightarrow +\infty$$

$$\frac{x^4}{4!} > 0 \quad n \geq 4$$

$$n^{10} q^n$$

$$\frac{n^{1000}}{2^n} \rightarrow 0$$

$$\lim S_n = 1 + \frac{1}{3} + \frac{1}{9} + \dots + \frac{1}{3^{n-1}} = \frac{4}{3}$$

$$S_n = \frac{1 - \left(\frac{1}{3}\right)^n}{1 - \frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$S_n = \frac{1}{6} - \frac{1}{18} + \frac{1}{54} - \dots - (-1)^{n+1} \cdot \frac{1}{2 \cdot 3^n} = \frac{1}{6} \left(1 - \left(\frac{1}{3}\right)^n \right)$$

$$= \frac{1}{6} \frac{1 - \left(-\frac{1}{3}\right)^n}{1 + \frac{1}{3}} \rightarrow \frac{1}{2 \cdot 4} = \frac{1}{8}$$

$$1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \dots + \frac{n+2}{2^{n+1}} = \frac{A}{1 - \frac{1}{2}}$$

$$1 + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{n+1}} + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{n+1}} + \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{n+1}} + \frac{1}{2^{n+1}}$$

$$\frac{1}{n^a} \xrightarrow{a > 0} 0$$

$$n^a \xrightarrow{a > 0} \infty$$

$$q^n \rightarrow 0 \quad |q| < 1$$

$$q^n \rightarrow \infty \quad |q| > 1$$

$$q = 1$$

$$\sqrt[n]{a} \xrightarrow{a > 0} 1$$

$$\sqrt[n]{n} \rightarrow 1$$

$$a_n \rightarrow a > 0$$

$$\frac{1}{n} \rightarrow 0$$

$$n \rightarrow \infty$$

$$a_n \rightarrow a$$

$$\left(\frac{1}{2}, \frac{a}{2}, a, \frac{3a}{2} \right)$$

$$\exists n_1, n > n_1$$

$$\frac{1}{n} < \frac{a}{2}$$

$$n > n_0$$

$$\exists n_2, n > n_2$$

$$n > \frac{3a}{2}$$

$$\frac{1}{n} < a_n < n$$

$$\exists n_3, n > n_3$$

$$\frac{a}{2} < a_n < \frac{3a}{2}$$

$$n \geq n_0$$

$$n_0 = \max(n_1, n_2, n_3)$$

$$\frac{1}{\sqrt{n}} < \sqrt{a_n} < \sqrt{n}$$

$$\rightarrow 1$$

$$a_n \leq b_n \text{ безброй много}$$

$$\downarrow \quad \downarrow$$

$$a \leq b$$

$$a > b$$

$$\left(b, \frac{a+b}{2}, a \right)$$

$$a_n \rightarrow a$$

$$b_n \rightarrow b$$

$$\exists n_1: n > n_1$$

$$\frac{a+b}{2} < a_n$$

$$\frac{a+b}{2} < a_n$$

$$n_0 = \max(n_1, n_2)$$

$$n > n_0 \Rightarrow$$

$$b_n < \frac{a+b}{2} < a_n$$

$$b_n < a_n$$

$$a_n \rightarrow a > 0$$

$$\sqrt[n]{a_n} \rightarrow 1$$

$$\lim \sqrt[n]{1+2^n} = \lim 2^{\frac{1}{n} \log(1+2^n)} = \lim 2 = 2$$

$$n^n \rightarrow \infty$$

$$\sqrt[n]{n^n} = n \rightarrow +\infty$$

$$\sqrt[n]{1+2^n} = 2 \sqrt[n]{(\frac{1}{2})^n + 1} \rightarrow 2$$

$$\sqrt[n]{a^n + b^n} = \max(a, b) \quad a, b > 0$$

I $a > b$

$$a \sqrt[n]{1 + (\frac{b}{a})^n} \rightarrow 1$$

$$0 < \frac{b}{a} < 1$$

II $a = b$

$$a \sqrt[n]{2} \rightarrow 1$$

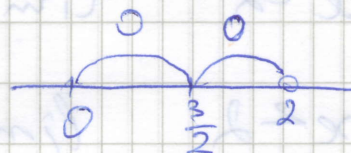
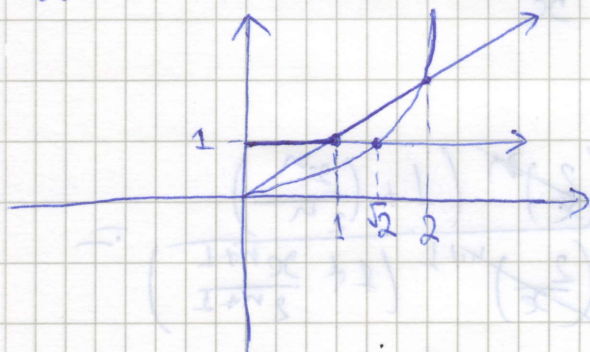
$$a = b$$

III $a < b$

$$b \sqrt[n]{(\frac{a}{b})^n + 1} \rightarrow 1$$

$$0 < \frac{a}{b} < 1$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{1 + x^n + (\frac{x^2}{2})^n} \rightarrow \max(1, x, \frac{x^2}{2})$$



$$\lim_{n \rightarrow \infty} \frac{2^n + x^n}{3^n + (2x)^n} \rightarrow \lim_{n \rightarrow \infty} \frac{2^n (1 + (\frac{x}{2})^n)}{3^n (1 + (\frac{2x}{3})^n)} = \lim_{n \rightarrow \infty} \frac{2^n}{3^n} = 1$$

I $x < \frac{3}{2}$

$$\lim_{n \rightarrow \infty} f(x) = 1$$

II $x = \frac{3}{2}$

$$\lim_{n \rightarrow \infty} f(x) = 1$$

III $2 > x > \frac{3}{2}$

$$\lim_{n \rightarrow \infty} f(x) = 0 \cdot \frac{1}{\infty} = 0$$

IV $x > 2$

$$\lim_{n \rightarrow \infty} f(x) = \frac{2^n (\frac{x}{2})^n [(\frac{2}{x})^n + 1]}{3^n (\frac{2x}{3})^n [(\frac{3}{2x})^n + 1]} =$$

$$\frac{(\frac{2}{x})^n + 1}{2 [(\frac{3}{2x})^n + 1]}$$

$$\lim_{n \rightarrow \infty} f_n(x) = 0$$

$$\lim_{n \rightarrow \infty} \frac{x^n + 2^n}{x^{n+1} + 2^{n+1}} = \lim_{n \rightarrow \infty} \frac{2^n \left(\left(\frac{x}{2}\right)^n + 1 \right)}{2^{n+1} \left(\left(\frac{x}{2}\right)^{n+1} + 1 \right)} = \lim_{n \rightarrow \infty} \frac{\left(\frac{x}{2}\right)^n + 1}{2 \left(\left(\frac{x}{2}\right)^{n+1} + 1 \right)} =$$

$x > 0$

~~I) $\frac{x}{2} < 1$ $x < 2$ $\lim_{n \rightarrow \infty} f_n(x) = \frac{1}{2}$~~

~~II) $x = 2$ $\lim_{n \rightarrow \infty} f_n(x) = \frac{1}{2}$~~

~~III) $x > 2$ $\lim_{n \rightarrow \infty} f_n(x) = \frac{1}{x}$~~

$$= \lim_{n \rightarrow \infty} \frac{x^n}{x^{n+1}} \cdot \frac{\left(1 + \left(\frac{2}{x}\right)^n\right)}{\left(1 + \left(\frac{2}{x}\right)^{n+1}\right)}$$

1) $\frac{x}{2} < 1$ $x < 2$ $\lim_{n \rightarrow \infty} f_n(x) = \frac{1}{x}$

2) $x = 2$ $\lim_{n \rightarrow \infty} f_n(x) = \frac{1}{2}$

3) $x > 2$ $\lim_{n \rightarrow \infty} f_n(x) = \frac{1}{x} \cdot \frac{\left(\frac{2}{x}\right)^n \left(1 + \left(\frac{x}{2}\right)^n\right)}{\left(\frac{2}{x}\right)^{n+1} \left(1 + \frac{x^{n+1}}{2^{n+1}}\right)} =$
 $= \frac{1}{2} \cdot \frac{1 + \left(\frac{x}{2}\right)^n}{1 + \left(\frac{x}{2}\right)^{n+1}} \rightarrow \frac{1}{2}$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)}$$

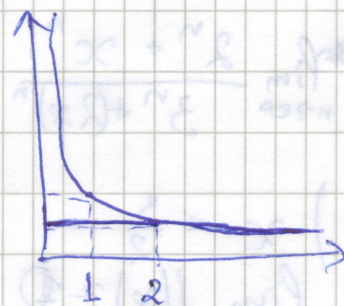
$$\frac{1}{(ax+b)(cx+d)(ex+f)} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{ex+f}$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} \Rightarrow A(x+1) + Bx = 1$$

$$A=1 \quad x \neq 0$$

$$B=-1 \quad x \neq -1$$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$



$$\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{n} - \frac{1}{n+1} =$$

$$= 1 - \underbrace{\left(\frac{1}{n+1}\right)}_{\rightarrow 0} = \frac{n+1-1}{n+1} = \frac{n}{n+1} \Rightarrow 1$$

→ (означава ипотеза - lim)

⇒ (означава следва)

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots + \frac{1}{n(n+1)(n+2)}$$

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2} \quad \frac{1}{x(x+1)(x+2)} = \frac{1}{2} \left(\frac{1}{x} - \frac{2}{x+1} + \frac{1}{x+2} \right)$$

$$A(x+1)(x+2) + B(x+2)x + C(x+1)x = 1$$

$$x=0$$

$$A = \frac{1}{2}$$

$$x=1$$

$$B = -1$$

$$x=-2$$

$$C = \frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{2} + \frac{1}{6} + \frac{1}{4} - \frac{1}{3} + \frac{1}{8} + \frac{1}{6} - \frac{1}{4} + \frac{1}{10} + \dots$$

$$* \frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2(n+2)} = \left(\frac{1}{2(n+2)} \right) \rightarrow 0 + \frac{1}{n} \Rightarrow 0$$

$$\lim_{n \rightarrow \infty} f_n(x) = 0$$

$$= \frac{1}{2} \left[\frac{1}{2} + \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2} \right] =$$

$$= \frac{1}{4}$$

$$S_n = \frac{1}{2} \left[1 - 1 + \frac{1}{3} \right]$$

$$\frac{1}{2} - \frac{2}{3} + \frac{1}{4}$$

$$\frac{1}{3} - \frac{2}{4} + \frac{1}{5}$$

$$\frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1}$$

$$\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}$$

Монотонни редици

$$\begin{cases} \{a_n\} \leq A \text{ за } \forall n \\ a_{n+1} \geq a_n \end{cases}$$

ограничена отгоре и сходяща

$$a_n \leq l$$

границата и (горна гран.)

огр. $\rightarrow$ с 1 точка на съставяне е сходяща
неогр. $\rightarrow \infty$ е точка на съставяне

$$\begin{cases} x \geq -1 \\ (1+x)^n \geq 1+nx \end{cases}$$

неравенство на Бернули

$$a_n \leq b_n \leq c_n \quad n > n_0$$

$\rightarrow a$

$b_n \rightarrow a$ и b_n е сходяща

$$\begin{matrix} x_n \rightarrow 0 \\ x_n < 0 \end{matrix} \quad \left(1 + \frac{x_n}{n}\right)^n$$

$$1 \geq \left(1 + \frac{x_n}{n}\right)^n \geq 1 + \frac{x_n}{n} \quad n > n_0$$

$\frac{x_n}{n} \rightarrow 0 \Rightarrow 1$

$$\begin{matrix} x_n > 0 \\ x_n \rightarrow 0 \end{matrix}$$

$$\left(1 + \frac{x_n}{n}\right)^n \frac{\left(1 - \frac{x_n}{n}\right)^n}{\left(1 - \frac{x_n}{n}\right)^n} =$$

$$= \frac{\left(1 - \frac{x_n^2}{n^2}\right)^n}{\left(1 - \frac{x_n}{n}\right)^n} \rightarrow 1$$

$$\frac{(-1)^n}{101}$$

$$n_0 \quad n > n_0 \quad x_n$$

$$-1 < \frac{x_n}{n}$$

$$\left(1 + \frac{x_n}{n}\right)^n \rightarrow 1$$

$$a_1, a_2, \dots, a_n \quad \frac{a_{n+1}}{a_n} = \frac{a+1}{a-1} > 1$$

$$a - \frac{1}{k} < a_n < a + \frac{1}{k}$$

$$x_n \rightarrow 0 \quad \left(1 + \frac{x_n}{n}\right)^n \rightarrow 1 \quad x_n \rightarrow 0 \quad n \rightarrow \infty$$

$\left(1 + \frac{x}{n}\right)^n$ — растяща

$\left(1 + \frac{x}{n}\right)^{n+1}$ — намаляваща
 $x > 0$

$$\frac{a_n}{a_{n+1}} = \frac{\left(1 + \frac{x}{n}\right)^n}{\left(1 + \frac{x}{n+1}\right)^{n+1}} = \frac{\left(1 + \frac{x}{n}\right)^n}{\left(1 + \frac{x}{n+1}\right)^n \left(1 + \frac{x}{n+1}\right)} = \frac{(n+1)^n}{(n+1+x)^n} \cdot \frac{(n+1)^n}{n} =$$

$$= \frac{(n+1)(n+1)}{n(n+1+x)} = \frac{(n+1)}{(n+1+x)} \left(\frac{(n+1)(n+1)}{n(n+1+x)} \right)^n =$$

$$\geq \frac{(n+1)}{(n+1+x)} \left(\frac{n^2 + 2n + 1}{n^2 + n + nx} \right)^n = \frac{(n+1)}{(n+1+x)} \left(1 + \frac{x}{n^2 + n + nx} \right)^n \geq$$

$$\geq \frac{n+1}{n+1+x} \left(1 + \frac{nx}{n^2 + n + nx} \right) = \frac{n+1}{n+1+x} \left(1 + \frac{nx}{n(n+1+x)} \right) =$$

$$= \frac{(n+1)(n+1+x) + (n+1)nx}{(n+1+x)(n+1+x)} = \frac{n^2 + n + nx + n + 1 + x + nx + x}{n^2 + n + nx + 1 + n + x + nx + x} = \frac{n^2 + 2nx + 2n + 2x + 1}{n^2 + 2nx + 2n + 2x + 1} =$$

$$= \frac{n^2 + 2n(x+1) + 1 + 2x}{n^2 + 2n(x+1) + x^2 + 2x + 1} =$$

$$\frac{a_{n+1}}{a_n} = \frac{\left(1 + \frac{x}{n+1}\right)^{n+1}}{\left(1 + \frac{x}{n}\right)^n} = \frac{(n+1)^{n+1}}{(n+1+x)^{n+1}} \cdot \frac{(n+1)^n}{n^n} = \frac{n+1+x}{n+1} \left(\frac{n^2 + n + x + nx}{n^2 + x + x + n} \right)^n =$$

$$= \frac{n+1+x}{n+1} \left(1 + \frac{-x}{n^2 + x + n} \right)^n \geq$$

$$\begin{aligned}
&\geq \frac{n+1+x}{n+1} \left(1 - \frac{x n}{n^2+xn+n+x} \right) = \frac{n+1+x}{n+1} \cdot \frac{n^2+xn+n+x-xn}{n^2+xn+n+x} = \\
&= \frac{(n+1+x)(n^2+n+x)}{(n+1)(n^2+xn+n+x)} = \frac{n^3+n^2+nx+n^2+n+x+xn^2+nx+x^2}{n^3+xn^2+n^2+nx+n^2+xn+n+x} = \\
&= \frac{n^3+2n^2+2nx+n^2x+n+x+x^2}{n^3+xn^2+2n^2+2xn+n+x} = \frac{n^3+n^2(1+1+x)+n(x+1+x)+x^2}{n^3+n^2(1+x+1)+n(x+x+1)+x^2}
\end{aligned}$$

$$\geq 1 \quad \frac{a_{n+1}(x)}{a_n(x)} \geq 1$$

$$\left(1 + \frac{x}{n}\right)^n \text{ — растущая} \quad \leftarrow \left(1 + \frac{1}{n}\right)^n = a_n$$

$$\left(1 + \frac{x}{n}\right)^{n+1} \text{ — убывающая} \quad \leftarrow \left(1 + \frac{1}{n}\right)^{n+1} = b_n$$

$$b_n = a_n \left(1 + \frac{1}{n}\right) \geq a_n$$

$$a_1 \leq a_2 \leq a_3 \leq \dots \leq a_n \leq b_n < b_{n-1} < \dots < b_1$$

$$\begin{array}{lll}
a_n \text{ е сходяща} & \text{и стр. монотонно} & a_n \rightarrow l \\
b_n \text{ е сходяща} & \text{и стр. монотонно} & b_n \rightarrow m \quad \underline{l=m}
\end{array}$$

$$\left(1 + \frac{1}{n}\right)^n \rightarrow e$$

$$\left(1 + \frac{1}{n}\right)^{n+1} \rightarrow e$$

$$\Rightarrow \left(1 + \frac{1}{n}\right)^n < e < \left(1 + \frac{1}{n}\right)^{n+1}$$

$$\left(\frac{3}{2}\right)^2 \leq e \leq \left(\frac{3}{2}\right)^3$$

$$2,25 \leq e \leq 3,375$$

$$\begin{aligned}
\left(1 + \frac{1}{n}\right)^n &\rightarrow e \\
\left(1 + \frac{k}{n}\right)^n &\rightarrow e^k \\
\left(1 + \frac{2}{n}\right)^n &\rightarrow e^2
\end{aligned}$$

$$\left(1 + \frac{2}{2n}\right)^{2n} = \left[\left(1 + \frac{1}{n}\right)^n\right]^2 \rightarrow e^2$$

$$\begin{aligned}
x_n &\rightarrow x_0 \\
\left(1 + \frac{x_n}{n}\right)^n &\rightarrow e^{x_0}
\end{aligned}$$

$$\left(\frac{n^2 - 3n + 2}{n^2 + 2n + 10} \right)^n = \left(\frac{n^2 + 2n + 10 - 5n - 8}{n^2 + 2n + 10} \right)^n = \left(1 + \frac{-5n - 8}{n^2 + 2n + 10} \right)^n =$$

$$\cancel{= \frac{1}{2}} = \left(1 + \frac{\overset{x_n \rightarrow -5}{-5n - 8}}{n^2 + 2n + 10} \right)^n \rightarrow e^{-5}$$

$$\frac{-5n^2 - 8n}{n^2 + 2n + 10} = \frac{n^2(-5 - \frac{8}{n})}{n^2(1 + \frac{2}{n} + \frac{10}{n^2})} \Rightarrow -5$$

$$\left(\frac{3n^2 - n - 1}{3n^2 + n - 2} \right)^n = \left(1 + \frac{\overset{-\frac{2}{3}}{n - 2n^2}}{3n^2 + n - 2} \right)^n \rightarrow \cancel{e^0} \rightarrow e^{-\frac{2}{3}}$$

$$\frac{n - 2n^2}{3n^2 + n - 2} = \frac{n^2(\frac{1}{n} - 2)}{n^2(3 + \frac{1}{n} - \frac{2}{n})} \rightarrow -\frac{2}{3}$$

$$a > 0 \quad \frac{n(\sqrt[n]{a} - 1)}{\frac{1}{n}} \quad \frac{1}{n+1} - \frac{1}{n} = \frac{1}{n(n+1)}$$

$$a^{\frac{1}{n(n+1)}} = b \quad \begin{aligned} &= (n+1)(b^n - 1) - n(b^{n+1} - 1) = \\ &= (n+1)(b - 1)(1 + b + b^2 + \dots + b^{n-1}) - \end{aligned}$$

$$1 - x^n = (1 - x)(1 + x + \dots + x^{n-1})$$

$$\begin{aligned} &-n(b-1)(1 + b + b^2 + \dots + b^n) = (b-1)((n+1)(1 + b + b^2 + \dots + b^{n-1}) - \\ &-n(1 + b + b^2 + \dots + b^n)) = \\ &= (b-1)(n+1 + (n+1)b + (n+1)b^2 + \dots + (n+1)b^{n-1} - \\ &-n - nb - nb^2 - \dots - nb^{n-1} - nb^n) = (b-1)(1 + b + b^2 + \dots + b^{n-1} - nb^n) \\ &= (b-1)(\underbrace{1 - b^n}_{1-b} + \underbrace{b - b^{n+1}}_{1-b} + \underbrace{b^2 - b^{n+2}}_{1-b} + \dots + \underbrace{b^{n-1} - b^{n+1}}_{1-b}) = \\ &= (b-1)^2 \left(\underbrace{1 + b + b^2 + \dots}_{1-b} \right) \end{aligned}$$

$a_{n+1} - a_n < 0$ монотонно убывает

ЧЗРУНА

Домашни: за групи ~~зетови~~ ^{вотрени} 31308 % 4.

① Да се покаже, че $\sum_{k=1}^n \cos kx = \frac{\sin \frac{nx}{2} \cos \frac{n+1}{2} x}{2 \sin \frac{x}{2}}$

② $\sum_{k=1}^n k \sin kx = \frac{(n+1) \sin nx - n \sin(n+1)x}{4 \sin^2 \frac{x}{2}}$

③ $\sum_{k=1}^n \cos(2k-1)x = \frac{\sin 2nx}{2 \sin x}$

④ $\sum_{k=1}^n \sin(2k-1)x = \frac{\sin^2 x}{\sin x}$

II заг. ① $\lim_{n \rightarrow \infty} \sqrt[n]{1 + \frac{1}{x^n} + \left(\frac{x^2}{4}\right)^n}$ ② $\lim_{n \rightarrow \infty} \sqrt[n]{2^n + \frac{1}{x^n} + \left(\frac{x^2}{9}\right)^n}$

③ $\lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{1}{2x}\right)^n + \left(\frac{x^2}{4}\right)^n}$ ④ $\lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{4}{3x}\right)^n + \left(\frac{x^2}{9}\right)^n}$

III заг. Да се намери границата на резултата

① $\left(\frac{2n^2 + n - 1}{2n^2 + 6n} \right)^{\frac{n}{2}}$ ② $\left(\frac{n^2 - 5n + 6}{n^2 + 3n + 2} \right)^{\frac{n}{3}}$

③ $\left(\frac{n^2 + 5n - 1}{n^2 - 2n + 5} \right)^{\frac{n}{3}}$ ④ $\left(\frac{n^2 - 2n + 2}{n^2 + 3n + 1} \right)^{\frac{4n}{3}}$

IV Да се намери границата

① $\lim_{a_n \rightarrow 7} \frac{\sqrt{a_n+2} - \sqrt[3]{a_n+20}}{a_n-7}$

③ $\lim_{a_n \rightarrow 0} \frac{\sqrt{1+a_n} - \sqrt[3]{1-a_n}}{\sqrt{1-a_n} - 1}$

② $\lim_{a_n \rightarrow 1} \frac{\sqrt[3]{a_n+7} - \sqrt{3+a_n}}{a_n-1}$

④ $\lim_{a_n \rightarrow 8} \frac{\sqrt[3]{a_n} - 2}{\sqrt[4]{8+a_n} - 2}$

V ① $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{3^{k-1}}$

② $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{(2k+3)(4k^2-1)}$

③ $\lim_{n \rightarrow \infty} \sum_{k=1}^n (-1)^{k-1} \frac{k}{2^{k-1}}$

④ $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k(9k^2-1)}$

VI Да се изследва редицата

① $a_{n+1} = \frac{2a_n^2 + 3a_n + 12}{a_n + 1}$

③ $a_{n+1} = \frac{3a_n^2 + a_n}{a_n^2 + a_n + 1}$

① $a_1 = 0$

② $a_1 = 3,5$

③ $a_1 = -1$

④ $a_1 = 2$

$\left(1 + \frac{x}{n}\right)^{n+1}$

$\frac{a_n}{a_{n+1}} \geq \frac{a_n}{a_{n+1}} \geq 1$

$0 \leq x \leq 1$

$x = 1$

редицата е намаляваща

$\left(1 + \frac{x}{n}\right)^n$

расходяща

e^x

$\left(1 + \frac{x_n}{n}\right)^n \xrightarrow{x_n \rightarrow x_0} e^{x_0}$

$$x_n = \sqrt[n]{x} - 1 \text{ монотонно } \rightarrow l(x)$$

$$\left(1 + \frac{x_n}{n}\right)^n \rightarrow e^{l(x)} = x \quad e^y = x \quad y = \ln x$$

$$0 < x < 1 \quad n(\sqrt[n]{x} - 1) = n\left(\sqrt[n]{\frac{1}{y}} - 1\right) = -\frac{1}{\sqrt[n]{y}} n(\sqrt[n]{y} - 1) \rightarrow -\ln y = -\ln \frac{1}{x} = \ln x$$

a_n — монотонна и ограничена $\rightarrow$ сходящаяся $\rightarrow l$
монотонно растущая и неограничена $\rightarrow +\infty$

$$a_{n+1} = \frac{2a_n^2 + a_n + 4}{a_n + 6}$$

$$a_1 = 5; 3; 0$$

$a_{n+1} = f(a_n)$ — рекуррентно заданная последовательность

$$a_{n+1} = f(a_n, a_n)$$

$$a_{n+1} - a_n = \frac{2a_n^2 + a_n + 4}{a_n + 6} - a_n = \frac{2a_n^2 + a_n + 4 - a_n(a_n + 6)}{a_n + 6} = \frac{a_n^2 - 5a_n + 4}{a_n + 6} = \frac{(a_n - 1)(a_n - 4)}{a_n + 6}$$

$$1 - a_{n+1} = \frac{2a_n^2 - 2}{a_n + 6} = \frac{2(a_n - 1)(a_n + 1)}{a_n + 6}$$

$$4 - a_{n+1} = \frac{4a_n + 4 - 2a_n^2 - a_n - 4}{a_n + 6} = \frac{-2a_n^2 + 3a_n}{a_n + 6} = \frac{2(4 - a_n)(a_n + 5)}{a_n + 6}$$

I.2.36

$$a_1 \geq 0$$

$$a_n \geq 0 \Rightarrow a_{n+1} = \frac{2a_n^2 + a_n + 4}{a_n + 6} > 0$$

за некое n $a_n \geq 0$ за $\forall n$

$$a_1 = 0 < 1$$

$$a_n < 1 \text{ за некое } n \Rightarrow 1 - a_{n+1} = \frac{2(1-a_n)(1+a_n)}{a_n+6} > 0$$

$$a_{n+1} < 1 : \text{за } \forall n \text{ } a_n < 1$$

$$a_{n+1} - a_n = \frac{(a_n-1)(a_n-4)}{a_n+6} \geq 0$$

растущая

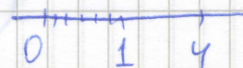
$$0 \leq a_n \leq 1$$

$$a_n \rightarrow l$$

$$a_{n+1} \rightarrow l$$

$$0 = l - l = \frac{(l-1)(l-4)}{l+6}$$

$$l_1 = 1 \vee l_2 = 4$$



$$a_1 = 3 > 1$$

$$a_n > 1 \text{ за некое } n \Rightarrow 1 - a_{n+1} < 0$$

$$a_{n+1} > 1$$

$$a_n > 1 \text{ за } \forall n$$

$$a_1 = 3 < 4$$

$$a_n < 4 \text{ за некое } n \Rightarrow 4 - a_{n+1} > 0$$

$$a_{n+1} < 4$$

$$a_n < 4 \text{ за } \forall n$$

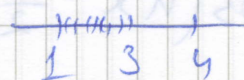
$$a_{n+1} - a_n = \frac{(a_n-1)(a_n-4)}{a_n+6} \leq 0$$

убывающая $1 \leq a_n \leq 4$

$$a_n \rightarrow l$$

$$a_{n+1} \rightarrow l$$

$$l_1 = 1 \vee l_2 = 4$$

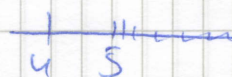


$$a_1 = 5 > 4$$

$$\text{Допускаем, что } a_n > 4 \Rightarrow a_{n+1} > 4 \Rightarrow a_n > 4 \text{ за } \forall n$$

$$\Rightarrow a_{n+1} > a_n$$

растущая



Обратные тригонометрические функции

- значения и диапазоны
- графики этих функций
- формулы

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

~~$$\sin(\alpha \pm \beta)$$~~

$$\cos(\alpha \pm \beta) =$$

$$\sin \alpha \cos \beta = \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$\sin \alpha \sin \beta =$$

$$\cos \alpha \cos \beta =$$

$$\begin{cases} \sin \alpha + \sin \beta = \\ \cos \alpha + \cos \beta = \end{cases}$$

$$\begin{cases} \sin \alpha - \sin \beta = \\ \cos \alpha - \cos \beta = \end{cases}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\begin{cases} \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} \\ \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2} \end{cases}$$

$$\cos^2 \alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\sin x + \sin 2x + \dots + \sin nx = \frac{A}{\sin \frac{x}{2}} = \frac{\cos \frac{x}{2} - \cos \frac{2n+1}{2} x}{2 \sin \frac{x}{2}}$$

$$A = \sin x - \sin \frac{x}{2} + \sin 2x - \sin \frac{x}{2} + \dots + \sin nx - \sin \frac{x}{2} =$$

$$= \frac{1}{2} \left(\cos \frac{x}{2} - \cos \frac{3x}{2} + \cos \frac{3x}{2} - \cos \frac{5x}{2} + \dots + \right.$$

$$\left. + \cos \left(nx - \frac{x}{2} \right) - \cos \left(nx + \frac{x}{2} \right) \right)$$

$$n \neq 1 \quad \sin x = \frac{\cos \frac{x}{2} - \cos \frac{3x}{2}}{2 \sin \frac{x}{2}} = \frac{2 \sin \frac{x}{2} \sin x}{2 \sin \frac{x}{2}} = \sin x \quad \text{е бреша}$$

$$x \neq 2k\pi$$

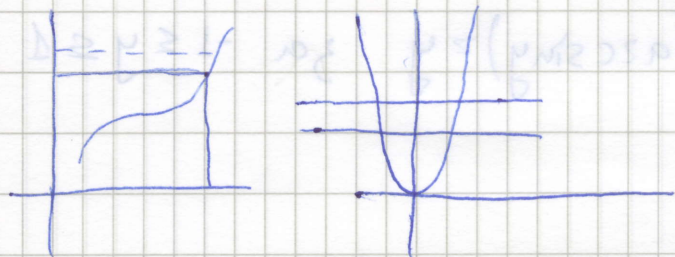
Допускаме, че за някое $k \in \mathbb{N}$ е в сила:

$$\sin x + \dots + \sin kx = \frac{\cos \frac{x}{2} - \cos \frac{2k+1}{2} x}{2 \sin \frac{x}{2}}$$

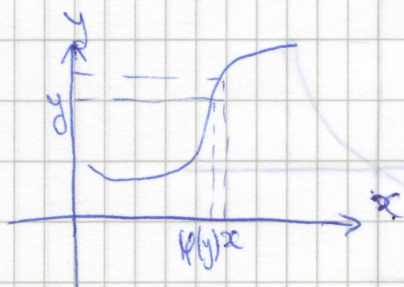
$$\begin{aligned} \sin x + \dots + \sin kx + \sin(k+1)x &= \frac{\cos \frac{x}{2} - \cos \frac{2k+1}{2} x}{2 \sin \frac{x}{2}} + \\ + \sin(k+1)x &= \frac{\cos \frac{x}{2} - \cos \frac{2k+1}{2} x + (\cos(k+1)x - \cos \frac{x}{2})}{2 \sin \frac{x}{2}} = \frac{\cos \frac{x}{2} - \cos \frac{2k+3}{2} x}{2 \sin \frac{x}{2}} \end{aligned}$$

тъй като $\Rightarrow$ вярно

Обратни тригонометрични функции



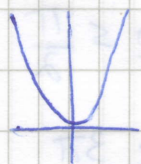
Една функция е обратима когато $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
 $f(x_1) = f(x_2)$, то $\Rightarrow x_1 = x_2$



$$\begin{aligned} f(x) &= y \\ \varphi(f(x)) &= z \Rightarrow f(\varphi(f(x))) = f(z) \\ f(x) &= f(z) \Rightarrow x = z \end{aligned}$$

$$\begin{cases} f(\varphi(y)) = y \\ \varphi(f(x)) = x \end{cases}$$

1) x^2



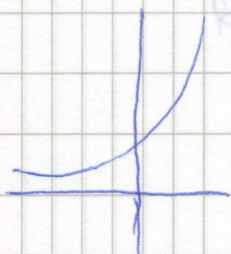
$x \geq 0 \Rightarrow y \geq 0$

$$\begin{aligned} x &= \sqrt{y} \quad \sqrt{x^2} = x \quad x \geq 0 \\ (\sqrt{y})^2 &= y \quad y \geq 0 \end{aligned}$$

2) $y = e^x > 0$

$$\begin{aligned} e^{\ln y} &= y \quad y > 0 \\ \ln e^x &= x \quad x \in \mathbb{R} \end{aligned}$$

$$\begin{aligned} y &= \frac{1}{x} \quad x = \frac{1}{y} \\ \frac{1}{\frac{1}{y}} &= y \end{aligned}$$



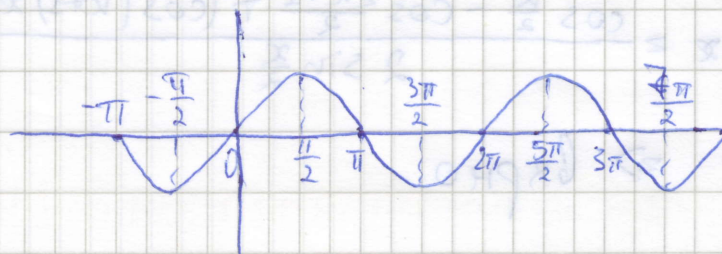
$$3) y = x^2 + 2x + 1 = (x+1)^2$$

$$\begin{aligned} x^2 - 1 & \\ |x+1| &= \sqrt{y} \\ x &= -1 \pm \sqrt{y} \end{aligned}$$

$$\begin{aligned} x &\leq -1 \\ -x-1 &= \sqrt{y} \\ x &= -1 - \sqrt{y} \end{aligned}$$



$$\begin{aligned} y &= \sin x \\ \sin x &= \sin y \\ -\frac{\pi}{2} &\leq x \leq \frac{\pi}{2} \\ -\frac{\pi}{2} &\leq y \leq \frac{\pi}{2} \\ x &= y \end{aligned}$$



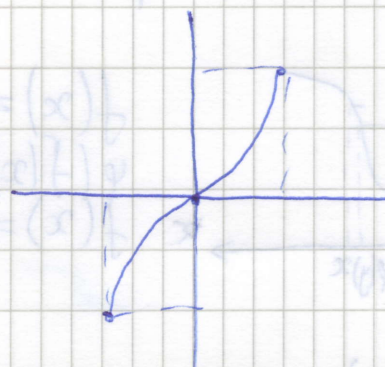
$$\begin{aligned} \sin x &= y \quad -1 \leq y \leq 1 \\ -\frac{\pi}{2} &\leq x \leq \frac{\pi}{2} \\ \arcsin y &= x \end{aligned}$$

$$\sin(\arcsin y) = y \quad \text{for } -1 \leq y \leq 1$$

$$\arcsin(\sin x) = x = d$$

$$\sqrt{x^2} = x$$

$$\left. \begin{aligned} -\frac{\pi}{2} &\leq x \leq \frac{\pi}{2} \\ -\frac{\pi}{2} &\leq d \leq \frac{\pi}{2} \\ \sin d &= \sin x \end{aligned} \right\} \Rightarrow d = x$$



$$\arcsin(\sin x) = x \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\arcsin(\sin x) = d \quad \frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$$

$$x - \frac{\pi}{2} = \frac{\pi}{2} - d$$

$$\sin d = \sin x$$

$$-\frac{\pi}{2} \leq d \leq \frac{\pi}{2}$$

$$x = \pi - d$$

$$\sin x = \sin(\pi - d) = \sin d$$

$$d = \pi - x$$

$$f\left(\pm \alpha \pm k \cdot \frac{\pi}{2}\right) = \begin{cases} \pm \cos f(\alpha) & - \text{четная} \\ \pm f(\alpha) & - \text{нечетная} \end{cases}$$

$$-\frac{\pi}{2} \geq -x \geq -\frac{3\pi}{2} \quad \sin\left(\frac{7\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\frac{\pi}{2} \geq -x + \pi \geq -\frac{\pi}{2} \quad \sin\left(\frac{3\pi}{2} - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\begin{cases} \sin \alpha = \sin x = \sin(\pi - x) \\ -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \\ -\frac{\pi}{2} \leq \pi - x \leq \frac{\pi}{2} \end{cases} \quad \alpha = \pi - x$$

$$\arcsin(\sin x) = \alpha$$

$$\frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$$

$$-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2} \leq x - 2\pi \leq \frac{\pi}{2}$$

$$\sin \alpha = \sin x = \sin(x - 2\pi) \quad \alpha = x - 2\pi \quad x = 2\pi + \alpha$$

- 1) $\arcsin(\sin x) = y$ $-1 \leq y \leq 1$
- 2) $\arcsin(\sin x) = x$ $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
- 3) $\arcsin(\sin x) = \pi - x$ $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$
- 4) $\arcsin(\sin x) = x - 2\pi$ $\frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$

$$\textcircled{1} \arcsin(2x^2 - 1) = \begin{cases} -\frac{\pi}{2} - 2\arcsin x & -1 \leq x < 0 \\ -\frac{\pi}{2} + 2\arcsin x & 0 \leq x \leq 1 \end{cases}$$

α β

$$-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2}$$

$$\cos 2\beta = 1 - 2\sin^2 \beta$$

$$\sin \alpha = 2x^2 - 1$$

$$\sin \beta = x$$

$$\sin \alpha = 2\sin^2 \beta - 1 = -\cos 2\beta + 1 - 1 = -\cos 2\beta = -\sin\left(\frac{\pi}{2} - 2\beta\right) = \sin\left(2\beta - \frac{\pi}{2}\right)$$

$$f\left(\frac{\pi}{2} - \alpha\right) = f \cos f$$

$$\sin d = \sin\left(2\beta - \frac{\pi}{2}\right)$$

$$-\frac{\pi}{2} \leq d \leq \frac{\pi}{2}$$

$$\text{I cn. } -1 \leq x \leq 0$$

$$-\frac{\pi}{2} \leq \beta \leq 0$$

$$\text{II cn. } 0 \leq x \leq 1$$

$$0 \leq \beta \leq \frac{\pi}{2}$$

$$\text{III cn. } 0 \leq \beta \leq \frac{\pi}{2}$$

$$0 \leq x \leq 1$$

$$0 \leq 2\beta \leq \pi$$

$$-\frac{\pi}{2} \leq 2\beta - \frac{\pi}{2} \leq \frac{\pi}{2}$$

$$d = 2\beta - \frac{\pi}{2}$$

$$\text{I cn. } d = -\frac{\pi}{2} - 2\beta$$

$$\sin d = \sin\left(-\frac{\pi}{2} - 2\beta\right)$$

$$\sin d = \sin\left(2\beta - \frac{\pi}{2}\right)$$

$$-\pi \leq \beta \leq \pi$$

$$\frac{3\pi}{2} \leq 2\beta - \frac{\pi}{2} \leq \frac{\pi}{2}$$

$$\sin d = \sin\left(-\frac{\pi}{2} - 2\beta\right) = -\cos 2\beta$$

$$\Rightarrow \sin\left(2\beta - \frac{\pi}{2}\right) = -\cos 2\beta$$

$$\cos\left(-\frac{\pi}{2} - 2\beta\right)$$

$$\text{I cn. } \sin d = \sin\left(2\beta - \frac{\pi}{2}\right)$$

$$-\frac{\pi}{2} \leq d \leq \frac{\pi}{2}$$

$$\sin d = \sin\left(-2\beta - \frac{\pi}{2}\right)$$

$$-\frac{\pi}{2} \leq 2\beta \leq \frac{\pi}{2}$$

Da ce gou., re 3a gou.

$$\cancel{\arcsin\left(\frac{1}{2}\sqrt{2-\sqrt{2x}}\right)}$$

$$\arcsin\left(\frac{1}{2}\sqrt{2-\sqrt{2x}}\right) = \frac{\pi}{8} + \frac{1}{4}\arcsin x$$

$$-1 \leq x \leq 1$$

1.2.42

$$2 \sin \frac{\varphi}{2} \cos \varphi = \cos \frac{3\varphi}{2}$$

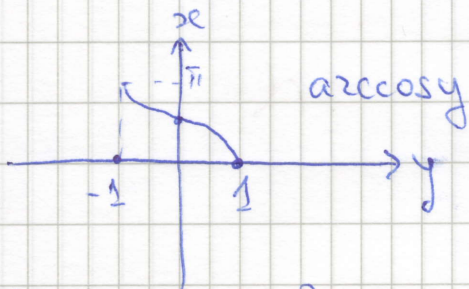
$$\sin\left(\frac{3\varphi}{2}\right) \neq \sin \frac{\varphi}{2} = \cos \frac{3\varphi}{2}$$

$$\sin \frac{3\varphi}{2} - \sin\left(\frac{\pi}{2} - \frac{3\varphi}{2}\right)$$

$$2 \sin\left(3\varphi - \frac{\pi}{2}\right)$$

$$\arccos(\cos x) = x$$

$$0 \leq x \leq \pi$$



$$\left. \begin{array}{l} \cos \alpha = \cos \beta \\ 0 \leq \alpha \leq \pi \\ 0 \leq \beta \leq \pi \end{array} \right\} \alpha = \beta$$

Da gilt, ze:

$$\arcsin x + \arccos x =$$

$$\overset{\parallel}{\alpha} \quad \overset{\parallel}{\beta}$$

$$\sin(\alpha + \beta)$$

$$\arcsin x = \alpha$$

$$-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$\sin \alpha = x$$

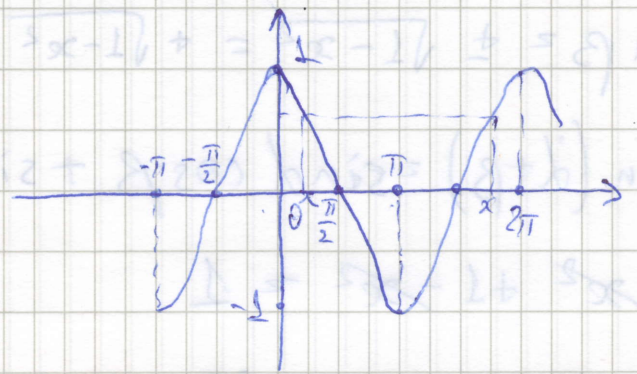
$$\arccos x = \beta$$

$$0 \leq \beta \leq \pi$$

$$\cos \beta = x$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos \alpha = \pm \sqrt{1 - x^2} = + \sqrt{1 - x^2}$$



$$\left. \begin{array}{l} 0 \leq \varphi \leq \pi \\ \cos x = y \end{array} \right\} \Leftrightarrow \arccos y = x$$

$$-1 \leq y \leq 1$$

$$0 \leq x \leq \pi$$

$$\arccos(\cos x) = \alpha$$

$$\pi \leq x \leq 2\pi = 2\pi - x$$

$$0 \leq \alpha \leq \pi$$

$$\cos \alpha = \cos x$$

$$x - \pi = \pi - \alpha$$

$$\alpha = 2\pi - x$$

$$0 \leq \alpha \leq \pi$$

$$0 \leq 2\pi - x \leq \pi$$

$$\pi \leq x \leq 2\pi$$

$$-2\pi \leq -x \leq -\pi$$

$$0 \leq 2\pi - x \leq \pi$$

$$\cos \alpha = \cos x = \cos(-x) = \cos(2\pi - x)$$

$$\cos(2\pi - x)$$

$$\alpha = 2\pi - x$$

$$\sin \beta = \pm \sqrt{1-x^2} = +\sqrt{1-x^2}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha = x \cdot x + \sqrt{1-x^2} \cdot \sqrt{1-x^2} = x^2 + 1 - x^2 = 1$$

$$-\frac{\pi}{2} \leq \alpha + \beta \leq \frac{3\pi}{2}$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{2}$$

$$\sin \alpha = \cos \beta$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \cos \beta$$

$$0 \leq \beta \leq \pi$$

$$\frac{\pi}{2} \leq -\alpha \leq \frac{\pi}{2}$$

$$0 \leq \frac{\pi}{2} - \alpha \leq \pi$$

$$\frac{\pi}{2} - \alpha = \beta$$

~~arccos~~

$$\cos(\arccos y) = y$$

$$\arccos(\cos x) = x$$

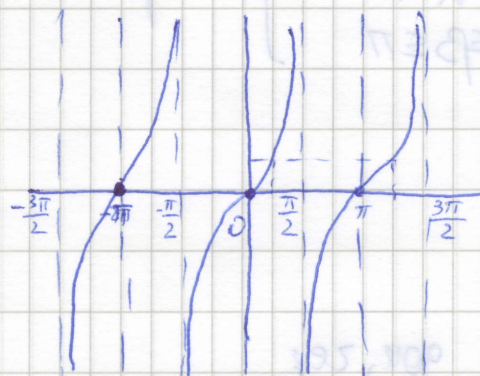
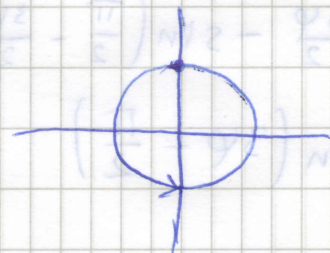
$$0 \leq x \leq \pi$$

$$\operatorname{tg}(\operatorname{arctg} y) = y$$

$$\operatorname{arctg}(\operatorname{tg} x) = x$$

$$\operatorname{arctg}(\operatorname{tg} x) = x - \pi$$

$$\frac{\pi}{2} < x < \frac{3\pi}{2}$$



$$\operatorname{tg} x = y \Rightarrow \operatorname{arctg} y = x$$

$$-\frac{\pi}{2} < x < \frac{\pi}{2} \Rightarrow -\infty < y < +\infty$$

$$\operatorname{arctg} y = x$$

$$-\infty < y < +\infty$$

$$\arcsin \frac{1}{\sqrt{1+x^2}}$$

$$\sin \alpha = \frac{1}{\sqrt{1+x^2}}$$

$$0 \leq \alpha \leq \frac{\pi}{2}$$

$$\arccos \frac{x}{\sqrt{1+x^2}}$$

$$\cos \beta = \frac{x}{\sqrt{1+x^2}}$$

$$0 \leq \beta \leq \pi$$

$$\cos \alpha = \pm \sqrt{1 - \frac{1}{1+x^2}} =$$

$$= \sqrt{\frac{x^2}{1+x^2}} = \frac{|x|}{\sqrt{1+x^2}}$$

$$\sin \beta = \pm \sqrt{1 - \frac{x^2}{1+x^2}} =$$

$$= \sqrt{\frac{1}{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\text{т.к. } x > 0 \quad \sin(\alpha + \beta) = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}} = \frac{2x}{\sqrt{1+x^2}}$$

$$\sin(\alpha + \beta) = 2 \sin \beta \cos \beta = \sin 2\beta$$

$$0 \leq \alpha + \beta \leq \pi$$

$$0 \leq 2\beta \leq \pi$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{x^2}{1+x^2} - \frac{1}{1+x^2} = \frac{x^2-1}{1+x^2}$$

$$\cos 2\beta = 2\cos^2 \beta - 1 = \frac{2x^2}{1+x^2} - 1 = \frac{2x^2-1-x^2}{1+x^2} = \frac{x^2-1}{1+x^2}$$

$$\cos(\alpha + \beta) = \cos 2\beta$$

$$\sin(\alpha + \beta) = \sin 2\beta$$

$$\cos(\alpha + \beta) = \cos \frac{2\beta}{2}$$

$$0 \leq \alpha + \beta \leq \pi$$

$$0 \leq 2\beta \leq \pi$$

$$\text{т.к. } x < 0$$

$$\cos \alpha = + \sqrt{1 - \frac{1}{1+x^2}} = \sqrt{\frac{x^2}{1+x^2}} = - \frac{x}{\sqrt{1+x^2}}$$

$$\cos(\alpha + \beta) = \frac{x^2}{1+x^2} - \frac{1}{1+x^2} = -1$$

$$\frac{\pi}{2} \leq \alpha + \beta \leq \frac{3\pi}{2}$$

$$\alpha + \beta = \pi$$

за том през, полагаме:
 $x = \cotg t$

$$\alpha = \arctan \frac{x-1}{x+1} \quad \arctan x = \beta \quad x \neq -1$$

$$\arctan \frac{x-1}{x+1} = \beta \quad \arctan x = \beta$$

$$-\frac{\pi}{2} < \alpha < \frac{\pi}{2} \quad -\frac{\pi}{2} < \beta < \frac{\pi}{2}$$

$$\tan \alpha = \frac{\tan \beta - 1}{\tan \beta + 1} = \frac{\frac{\sin \beta}{\cos \beta} - 1}{\frac{\sin \beta}{\cos \beta} + 1} = \frac{\frac{\sin \beta - \cos \beta}{\cos \beta}}{\frac{\sin \beta + \cos \beta}{\cos \beta}} = \frac{\sin \beta - \cos \beta}{\sin \beta + \cos \beta} = \frac{\frac{1}{\sqrt{2}} \sin \beta - \frac{1}{\sqrt{2}} \cos \beta}{\frac{1}{\sqrt{2}} \cos \beta + \frac{1}{\sqrt{2}} \cos \beta}$$

$$\frac{a \sin \alpha + b \cos \alpha}{\sqrt{a^2 + b^2}} \cdot \sqrt{a^2 + b^2}$$

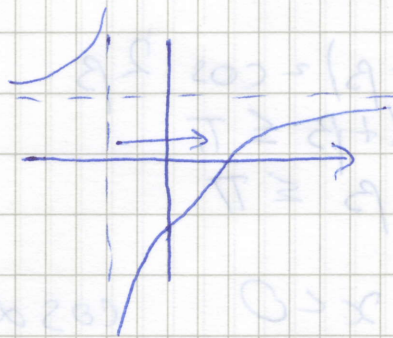
$$= \frac{\sin(\beta - \frac{\pi}{4})}{\cos(\beta - \frac{\pi}{4})} = \tan(\beta - \frac{\pi}{4}) = \tan \alpha$$

$$-\frac{\pi}{2} < \alpha < \frac{\pi}{2} \quad -\frac{3\pi}{2} < \beta - \frac{\pi}{4} < \frac{\pi}{4}$$

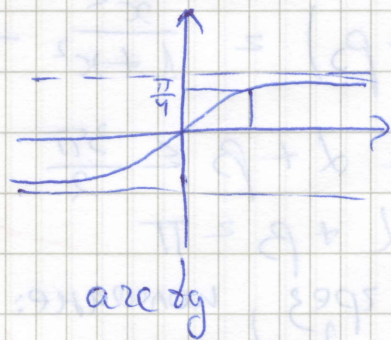
$$I) x > -1 \quad \frac{x-1}{x+1} < 1$$

$$-\frac{\pi}{2} < \alpha < \frac{\pi}{4} \quad -\frac{\pi}{4} < \beta < \frac{\pi}{2}$$

$$\alpha = \beta - \frac{\pi}{4}$$



$$II) x < -1$$



$$1 + 2x + 3x^2 + \dots + nx^{n-1} =$$

$$= 1 + x + x^2 + \dots + x^{n-1} + x + x^2 + \dots + x^{n-1} + x^2 + \dots + x^{n-1} + \dots + x^{n-1}$$

$$\frac{1-x^n}{1-x}$$

$$\frac{d}{dx} \frac{1-x^n}{1-x} = \frac{1-x^{n-1}}{1-x} - \frac{1-x^n}{(1-x)^2}$$

$$x^{n-1} \frac{1-x}{1-x} =$$

$$\frac{1-x^n}{1-x}$$

$$= \frac{1}{1-x} (1 + x + \dots + x^{n-1} - nx^n) = \frac{1}{1-x} \left(\frac{1-x^n}{1-x} - nx^n \right)$$

$$1 + 2x + 3x^2 + \dots + nx^{n-1} = \frac{A}{1-x}$$

$$1 + 2x + 3x^2 + \dots + nx^{n-1} - x - 2x^2 - \dots - (n-1)x^{n-1} - x^n = 1 + x + \dots + x^{n-1} - nx^n =$$

$$= \frac{1-x^n}{1-x} - nx^n = A$$

Граници на функции.

Непрерывность на функции.

Граници

Непрерывность

x_0 - гранична точка D
 $x_n \in D$ $x_n \neq x_0$ $x_n \rightarrow x_0 \Rightarrow$
 $\Rightarrow x_0$ е гранична точка

$x_0 \in D$

$$\frac{\sin x}{x}$$

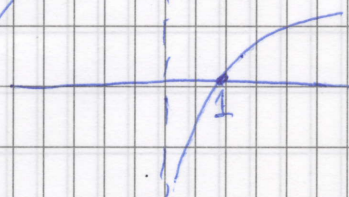


$$y = \sqrt{\ln(\sin x) + 1}$$

$$-\frac{3\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{\pi}{2} + 2k\pi$$

изолчрани точки



$$\sin x \geq 1 \quad \sin x \leq -1$$

Граници

$f(x)$ А x_0
Ако за $\forall$ редица

$$x_n \in D, x_n \neq x_0, x_n \rightarrow x_0 \Rightarrow f(x_n) \rightarrow A$$

Хаїне

Непреривчатост

За $\forall$ редица $x_n \in D, x_n \rightarrow x_0 \Rightarrow f(x_n) \rightarrow f(x_0)$

$$\Rightarrow f(x_n) \rightarrow A = f(x_0)$$

$x_0 \in D$

$$x_0, x_0, \dots \rightarrow x_0$$

$$f(x_0), f(x_0), \dots \rightarrow A$$

$$\rightarrow f(x_0)$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1} = -1$$

$$x_n \rightarrow 1, x_n \neq 1$$

$$f(x_n) = \frac{x_n^2 - 3x_n + 2}{x_n - 1} = \frac{(x_n - 1)(x_n - 2)}{x_n - 1} \rightarrow$$

$\rightarrow -1$

$$\frac{x^2 - 3x + 2}{x - 2}, \frac{x_n^2 - 3x_n + 2}{x_n - 2} \rightarrow$$

$$x_n \rightarrow 1$$

$$\rightarrow \frac{1^2 - 3 \cdot 1 + 2}{1 - 2} = f(1)$$

Коши

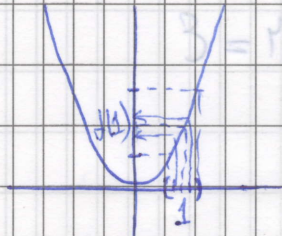
x_0 — гранична точка
Ако за $\forall \epsilon > 0$, може да се намери $\delta(\epsilon) > 0$, таква че $\forall x \in D, x \neq x_0$, за което е изн.

$$|x - x_0| < \delta \Rightarrow |f(x) - A| < \epsilon$$

$x_0 \in D$

Ако за $\forall \epsilon > 0$, може да се намери $\delta(\epsilon) > 0$, таква че $\forall x \in D$, за което е изн. $|x - x_0| < \delta \Rightarrow |f(x) - A| < \epsilon$

$$x^2$$

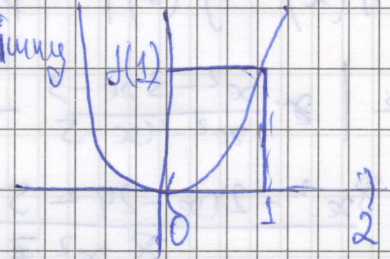


за интервал и с център $f(1)$, може да се намери интервал V около точка 1, $V \ni 1$, $x \in V \Rightarrow f(x) \in U$

$$|x^2 - 1^2| = |x - 1| |x + 1| \leq |x - 1| (|x| + |1|) \leq 3|x - 1|$$

$$|f(x) - f(x_0)| \leq |x - x_0| \text{ Ако } x_0 \text{ — условие на Липшиц}$$

$$0 < x < 2$$



$$|x^2 - 1^2| \leq 3|x - 1|$$

$$|x - 1| < 1$$

$$\varepsilon > 0 \Rightarrow \delta(\varepsilon) = \min\left(1, \frac{\varepsilon}{3}\right)$$

$$|x - 1| < \delta \Rightarrow \begin{matrix} |x - 1| < 1 \\ |x - 1| < \frac{\varepsilon}{3} \end{matrix} \Rightarrow |x^2 - 1| < 3|x - 1| < 3 \cdot \frac{\varepsilon}{3} = \varepsilon \Rightarrow$$

$\Rightarrow$ функцията е непрекъснатата в смисъл на Коши

$$f(x) = \frac{x^2 - 3x + 3}{x^2 - 2x + 1}$$

$$\begin{matrix} x_0 = 2 \\ x_n \rightarrow 2 \\ f(x_n) \rightarrow f(2) \end{matrix}$$

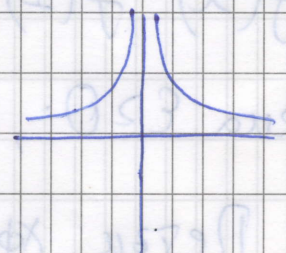
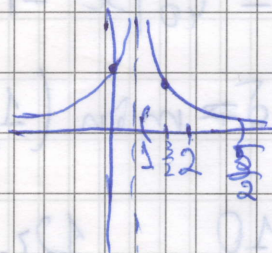
$$|f(x) - f(2)| = \left| \frac{x^2 - 3x + 3}{x^2 - 2x + 1} - 1 \right| = \left| \frac{1 - x + 2}{(x - 1)^2} \right| = \frac{|x - 2|}{(x - 1)^2} \leq \frac{1}{(x - 1)^2} |x - 2| \leq$$

$$|x - 2| \leq 4 \cdot \frac{1}{2}$$

$$\varepsilon > 0 \Rightarrow \delta = \min\left(\frac{1}{2}, \frac{\varepsilon}{4}\right) \Rightarrow |x - 2| < \delta \Rightarrow |x - 2| < \frac{1}{2} \Rightarrow |x - 2| < \frac{\varepsilon}{4}$$

$$\leq 4|x - 2|$$

$$|f(x) - f(2)| \leq 4 \cdot \frac{|x - 2|}{\frac{1}{2}} \Rightarrow |x - 2| < \frac{1}{2}$$



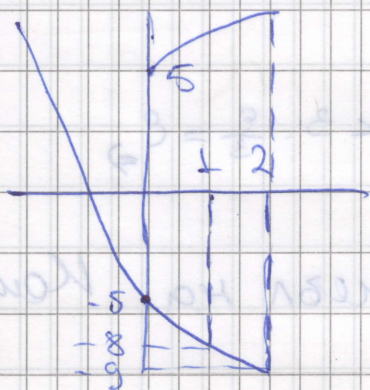
$$\Rightarrow |f(x) - f(2)| \leq 4 \cdot |x-2| \leq \frac{\varepsilon}{4} \cdot 4 = \varepsilon$$

$$f(x) = \frac{x^2 - 3x - 3}{x^2 - 4x - 5} \quad x=1$$

$$|f(x) - f(1)| = \left| \frac{x^2 - 3x - 3}{x^2 - 4x - 5} - \frac{-5}{-8} \right| = \left| \frac{x^2 - 3x - 3}{x^2 - 4x - 5} - \frac{5}{8} \right|$$

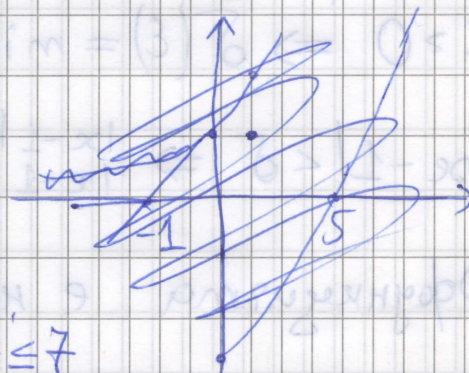
$$= \left| \frac{x^2 - 3x - 3}{x^2 - 4x - 5} - \frac{5}{8} \right| = \left| \frac{x^2 - 3x - 3}{x^2 - 4x - 5} - \frac{5}{8} \right|$$

$$= \left| \frac{8x^2 - 24x - 24 - 5x^2 + 20x + 25}{8x^2 - 32x - 40} \right| = \left| \frac{3x^2 - 4x + 1}{8(x^2 - 4x - 5)} \right|$$



$$|x-1| \leq 1$$

$$|3x-1| \leq |3x| + 1 \leq 7$$



$$= \left| \frac{(3x-1)(x-1)}{8(x^2-4x-5)} \right| \leq \frac{1}{8} \frac{|3x-1||x-1|}{(-x^2+4x+5)} \leq$$

$$\leq \frac{1}{40} |3x-1||x-1| \leq \frac{7}{40} |x-1|$$

$$|f(x) - f(1)| \leq \frac{7}{40} |x-1|$$

$$\text{Hence } \varepsilon > 0, \exists \delta = \min\left(1, \frac{40\varepsilon}{7}\right)$$

Первая 210

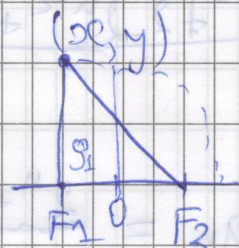
122-142

2.250

1. Zag.

①

② $m \in M \Rightarrow n+1 \in M$



$$\begin{matrix} \uparrow \\ A=B \\ \uparrow \\ A_1=B_1 \\ \vdots \\ \uparrow \\ A_n=B_n \end{matrix}$$

$$\begin{aligned} A &= A_1 = A_2 = \dots = B \\ A-B &= \dots = 0 \\ a &= b \Rightarrow a^2 = b^2 \\ a^2 &= b^2 \end{aligned}$$

$$g_1 = g_2 = \dots \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

1. Zag. $\lim \sqrt[n]{1 + \left(\frac{1}{2x}\right)^n + \left(\frac{x^2}{2}\right)^n} = \max\left(1, \frac{1}{2x}, \frac{x^2}{2}\right)$

$$\begin{aligned} \sqrt[n]{a_n} &\rightarrow 1 \\ a_n &\rightarrow 2 > 0 \end{aligned}$$

~~1. Zag.~~ $x_n \rightarrow x_0$

$$\left(1 + \frac{x_n}{n}\right)^n \rightarrow e^{x_0}$$

$$x_n \rightarrow \infty$$

$$\left(1 + \frac{1}{x_n}\right)^{x_n} \rightarrow e^{x_0}$$

$$\left(1 + \frac{a_n}{n}\right)^{\frac{n}{2}}$$

$$y_n \rightarrow x_0$$

$$\lim_{a_n \rightarrow 0} \frac{\sqrt{1+a_n} - 1}{a_n}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = A$$

$$a_n \rightarrow 0$$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{2^{k-1}}$$

$$S_n = 1 + \frac{2}{2} + \frac{3}{2^2} + \dots + \frac{n}{2^{n-1}}$$

$$\left(1 - \frac{1}{2}\right) S_n =$$

Теорема на Лопиталя

b_n е монотонна

$\approx$

$$\begin{aligned} a_n &\rightarrow 0 & \infty \\ b_n &\rightarrow 0 & \infty \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = e$$

$$\frac{a_n}{b_n} \rightarrow e$$

$$S_n = \frac{2^{n-1} + 2 \cdot 2^{n-2} + 3 \cdot 2^{n-3} + \dots + (n-1)2^1 + n}{2^{n-1}} \quad \frac{a_n}{b_n}$$

$$a_{n+1} - a_n = \frac{2^n + 2 \cdot 2^{n-1} + 3 \cdot 2^{n-2} + \dots + n \cdot 2 + n+1}{2^n + 2^{n-1} + \dots + 2 + 1} - \frac{2^{n-1} + 2 \cdot 2^{n-2} + \dots + (n-1)2 + n}{2^{n-1} + 2^{n-2} + \dots + 2 + 1}$$

$$\frac{a_{n+1} - a_n}{b_{n+1} - b_n} = \frac{2^{n+1} - 1}{2^n - 2^{n-1}} = \frac{2^{n+1}(1 - \frac{1}{2^{n+1}})}{2^n(1 - \frac{1}{2^n})} \rightarrow 4$$

~~lim~~ $\lim_{x \rightarrow 1} \left(\frac{\sqrt[3]{7+x^2} - \sqrt[3]{3+x}}{\sqrt[4]{2-x} - 1} \right)$ $a_n = f(n) \rightarrow A$ $f(x) \xrightarrow{x \rightarrow \infty} A$

$$\begin{aligned} \sqrt[3]{7+x^2} &= a & x \rightarrow 1 & a \rightarrow 2 & x \rightarrow 1 & A = a^5 + a^4b + a^3b^2 + \dots + b^5 \\ \sqrt[3]{3+x} &= b & & b \rightarrow 2 & & \rightarrow 2^5 \cdot 6 \\ \sqrt[4]{2-x} &= c & & c \rightarrow 1 & x \rightarrow 1 & B = c^3 + c^2 + c + 1 \rightarrow 4 \end{aligned}$$

$$C = \frac{a^6 - b^6}{c^4 - 1} \cdot \frac{B}{A} = \frac{(7+x^2)^3 - (3+x)^3}{1-x} \cdot \frac{B}{A} = \frac{B}{A} \cdot \frac{(7+(t+1)^2)^3 - (4+t)^3}{-t} = \frac{B}{A} \cdot \frac{(8+t+t^2)^2}{-1} = \frac{4}{28.6} - \frac{16}{-1} \Rightarrow \frac{1}{3}$$

$$\sum_{k=1}^n \frac{1}{k(9k^2-1)}$$

~~(3k+3)(3k-3)~~ $\sum_{k=1}^n \frac{1}{k(k^2-9)}$

$$a_{n+1} = \frac{3a_n^2 + a_n}{a_n^2 + a_n + 1}$$

$$a_1 = -1, a_2 = 2$$

$$a_{n+1} - a_n = \frac{(a_n - 2)(\dots)}{(\dots)}$$

$$a_1 = 2, a_n = 2 \Rightarrow a_{n+1} = 2 = 2, 2, \dots, 2 \rightarrow 2$$

$$a_2 = 2, -1, 2, \dots$$

индукция

опр. - тригоном.

график на функции и предел

м/у 6 - 15 задачи

~~График на функции~~

$$\frac{\sin x}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\frac{\sin x}{x} \xrightarrow{x \rightarrow \infty} 0$$

$$\frac{e^x - 1}{x} \xrightarrow{x \rightarrow 0} \frac{e^x - e^0}{x - 0} \rightarrow 1 \left[\frac{f(x) - f(x_0)}{x - x_0} \right] \xrightarrow{x \rightarrow x_0} f'(x_0)$$

$$\frac{\ln(1+x)}{x} \xrightarrow{x \rightarrow 0} 0$$

$$\frac{\ln y}{y-1} \xrightarrow{y \rightarrow 1} 1$$

Все функции являются непрерывными в целых дифференцируемых интервалах.

$\lg x$

$x \rightarrow x_0$

$\sin x \rightarrow \sin x_0$

$\arcsin x \rightarrow \arcsin x_0$

$e^x \rightarrow e^{x_0}$

$\ln x \rightarrow \ln x_0$

$\sqrt[n]{x} \rightarrow \sqrt[n]{x_0}$

$x^a \rightarrow x_0^a$

$$a^x = e^{x \ln a}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\frac{1^k + 2^k + \dots + n^k}{n^{k+1}} = \frac{\frac{n^{k+1}}{k+1} + P_k(n)}{n^{k+1}} \rightarrow \frac{1}{k+1}$$

$$S'_n = 1^3 + 3^3 + \dots + (2n+1)^3 = \frac{S'_n}{8n^4}$$

$$S_n'' = 2^3 + \dots + (2n)^3$$

$$S_n' + S_n'' = 1^3 + \dots + (2n+1)^3 = \frac{(2n+1)^4}{4} + P_3(n)$$

$$S_n'' = 8 \cdot (1^3 + \dots + n^3) = 8 \cdot \frac{n^4}{4} + P_3'(n)$$

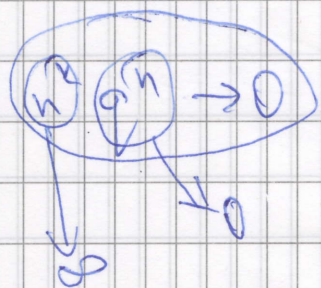
$$\frac{S'_n}{4} = \frac{\frac{(2n+1)^4}{4} - \frac{2n^4}{2} + P_3'(n)}{n^4} \rightarrow 2$$

Теорема на Уйона

$$\frac{S'_n}{n^4} \cdot \frac{S'_{n+1} - S'_n}{(n+1)^4 - n^4} \geq \frac{S'_n}{4n^3 + \dots} \rightarrow \frac{8}{4} = 2$$

$$1 + 2x + 3x^2 + \dots + nx^{n-1} \quad 0 < x < 1$$

$$= \frac{A}{1+x}$$



$$a_n \rightarrow \infty$$

$$b_n \rightarrow \infty$$

b_n е монотонна

$$\frac{a_{n+1} - a_n}{b_{n+1} - b_n} \rightarrow l$$

$$x = \frac{1}{t} \quad t \geq \frac{1}{x}$$

$$\frac{a_n}{b_n} \rightarrow l$$

$$x < 1 \quad t \geq 1$$

$$1 + \frac{2}{t} + \frac{3}{t^2} + \dots + \frac{n}{t^{n-1}}$$

~~$$\frac{\sin(x)}{x}$$~~

$$\frac{\sin(u(x))}{u(x)} \xrightarrow{u \rightarrow 0} 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos ax}{x^2} = \frac{a^2}{2}$$

$$\frac{e^u - 1}{u} \xrightarrow{u \rightarrow 0} 1$$

$$\frac{\ln(1+x)}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\frac{\ln y}{y-1}$$

$$\xrightarrow{y \rightarrow 1} 1$$

$$y = 1+x$$

$$\frac{\arcsin x}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\frac{\arctg x}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\frac{x^a}{x^n}$$

$$\begin{cases} a^x = e^{x \cdot \ln a} \\ \log_a x = \frac{\ln x}{\ln a} \end{cases}$$

$$\begin{aligned} 1 - \cos x &= 2 \sin^2 \frac{x}{2} \\ 1 + \cos x &= 2 \cos^2 \frac{x}{2} \end{aligned}$$

$$(1) \lim_{x \rightarrow -\infty} \frac{x^2 - x + 1}{2x^2 + x} = \lim_{x \rightarrow -\infty} \frac{x^2(1 - \frac{1}{x} + \frac{1}{x^2})}{x^2(2 + \frac{1}{x})} = \frac{1}{2}$$

$$(2) \lim_{x \rightarrow 0} (1+x)^5 = 1$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^5 - 1 - 5x}{x^2} = \left[\frac{0}{0} \right] =$$

~~$$(1+x)^5$$~~

$$= \frac{10x^2 + o(x^2)}{x^2} \rightarrow 10$$

$$\frac{f(x)}{g(x)} \sim o(x^2)$$

$$\frac{f(x)}{x^2} \rightarrow 0$$

$$(3) \lim_{x \rightarrow 0} \frac{\sqrt[n]{1+ax} \cdot \sqrt[n]{1+bx} - 1}{x} = \frac{(1+ax)^m (1+bx)^n - 1}{x \underbrace{(ab)^{nm-1} + (ab)^{n \cdot m - 1} + \dots + 1}}_{m \cdot n \quad A} =$$

$$z \frac{x(\alpha_m + \beta_n) + o(x)}{x - (A)} \rightarrow \frac{\alpha_m + \beta_n}{n, m}$$

$\swarrow$
 mn

$$(4) \lim_{x \rightarrow -\infty} \sqrt[3]{x^3 + 3x^2} + \sqrt{x^2 + 2x} = \lim_{x \rightarrow -\infty} \dots \rightarrow 0$$

$\begin{matrix} \text{u} & \text{u} \\ a & b \end{matrix}$

$$x = \frac{1}{x} \quad \sqrt{x^2} = -x$$

$\begin{matrix} a^6 + b^6 \\ a^6 - b^6 = \end{matrix}$

$$(a+b)(a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5) =$$

$$= a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + a^5b - a^4b^2 + a^3b^3 - a^2b^4 + ab^5 - b^6 = a^6 - b^6$$

$$(a - (-b)) = a^6 - b^6$$

$$\sqrt[3]{x^3 + 3x^2} + \sqrt{x^2 + 2x} = \frac{(x^3 + 3x^2)^{1/3} - (x^2 + 2x)^{1/2}}{A} = \frac{6x^5(6-6) + x^4(9-12) + x^3 + \dots}{x^5(1 - \dots)}$$

$a^6 + b^6$ - нима танабар овер
 $a^6 - b^6$ - нима танабар овер

$$(5) \lim_{x \rightarrow 0} \frac{\ln \cos bx}{\ln \cos ax} = \frac{\ln \cos bx}{\cos bx - 1} \cdot \frac{(-\cos bx + 1)x^2}{(-\cos ax + 1)x^2} \cdot \frac{\cos ax - 1}{\ln \cos ax} \Rightarrow \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} \cdot \frac{2}{a^2} = \frac{b^2}{a^2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos ax}{x^2} = \frac{a^2}{2} \quad \begin{aligned} 1 - \cos x &= 2 \sin^2 \frac{x}{2} \\ 1 + \cos x &= 2 \cos^2 \frac{x}{2} \end{aligned}$$

$$\frac{1 - \cos ax}{x^2} = \frac{a^2 \sin^2 ax}{(ax)^2 (1 + \cos ax)} \rightarrow \frac{a^2}{2}$$

$$\frac{b^2}{x} = \frac{b^2}{x^2} \cdot x = \frac{b^2}{a}$$

$$\textcircled{6} \lim_{x \rightarrow 0} \frac{a^{\cos x} - a}{x^2} = a \cdot \frac{e^{(\cos x - 1) \ln a} - 1}{(\cos x - 1) \ln a} = \frac{(\cos x - 1) \ln a}{x^2} \Rightarrow \frac{a \cdot \ln a}{2}$$

$$\textcircled{7} \lim_{x \rightarrow 1} (1 + \sin \pi x)^{\cot \pi x} = e^{\frac{\cos \pi x}{\sin \pi x} \cdot \ln(1 + \sin \pi x)} \rightarrow e^{-1}$$

$x \rightarrow 0$ (заупражение не име)

$$\textcircled{8} \lim_{x \rightarrow 0} \left(2e^{\frac{x}{x+1}} - 1 \right)^{\frac{x^2+1}{x}} = e^{\frac{x^2+1}{x} \cdot \ln(2e^{\frac{x}{x+1}} - 1)} \Rightarrow e^2$$

$$\begin{aligned} f(x) &\rightarrow B \neq 0, -1, \infty \\ g(x) &\rightarrow A \neq 0, \infty \\ f(x) &\rightarrow B \neq 0, -1, \infty \end{aligned}$$

$$e^{f(x)} = e^{g(x) \cdot \ln f(x)} \rightarrow e^{A \cdot \ln B}$$

$$0^0, 1^0, 1^\infty$$

$$\lim_{x \rightarrow 0} \frac{\ln(2e^{\frac{x}{x+1}} - 1)}{x} = \frac{\ln(2a-1)}{2a-2} \cdot \frac{1}{x}$$

$$\frac{2(e^{\frac{x}{x+1}} - 1)}{\frac{x}{x+1}} \Rightarrow 2$$

$$\frac{\sin nx}{\sin mx} = \frac{nx}{mx} = \frac{n}{m}$$

$$\textcircled{2} \arccos x = \arctg \sqrt{\frac{1-x}{1+x}} = \frac{1}{2} \arctg \frac{\sqrt{1-x^2}}{x} + \frac{\pi}{2}$$

$-1 \leq x < 0$

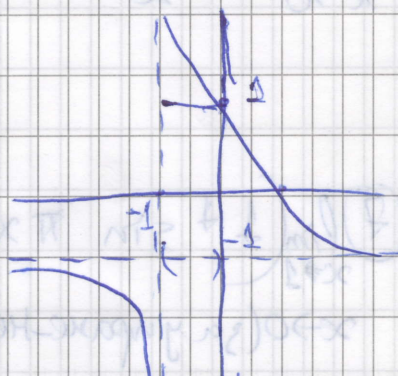
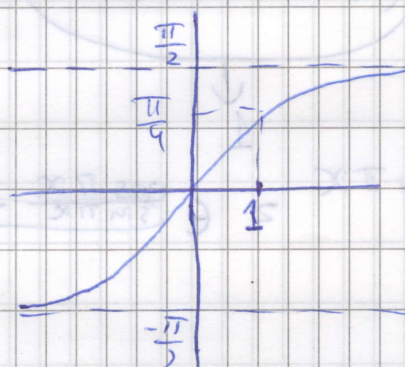
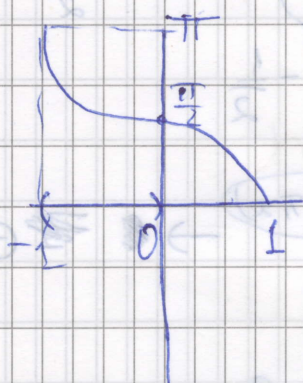
$$\cos \alpha = x$$

$\frac{\pi}{2} < \alpha < \pi$

$$\operatorname{tg} \beta = \sqrt{\frac{1-x}{1+x}}$$

$0 < \beta < \frac{\pi}{2}$

$$y = \frac{1-x}{1+x} = -1 + \frac{2}{1+x}$$



$$\left(\frac{\pi}{4} < \beta < \frac{\pi}{2} \right)$$

$$-1 < x < 0 \Rightarrow \frac{1-x}{1+x} > 1$$

$$\operatorname{tg} \gamma = \frac{\sqrt{1-x^2}}{x}$$

$-\frac{\pi}{2} < \gamma < 0$

$$\operatorname{tg} \beta = \sqrt{\frac{1-\cos \alpha}{1+\cos \alpha}} = \sqrt{\frac{2 \sin^2 \frac{\alpha}{2}}{2 \cos^2 \frac{\alpha}{2}}} = \left| \operatorname{tg} \frac{\alpha}{2} \right|$$

$$\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2} \Rightarrow \operatorname{tg} \frac{\alpha}{2}$$

$$\operatorname{tg} \beta = \operatorname{tg} \frac{\alpha}{2}$$

$\frac{\pi}{4} < \beta < \frac{\pi}{2}$

$$\beta = \frac{\alpha}{2}$$

$$\operatorname{tg} \gamma = \frac{\sqrt{1-x^2}}{x} = \frac{|\sin \alpha|}{\cos \alpha} = \operatorname{tg} \alpha = \operatorname{tg} (\alpha - \pi)$$

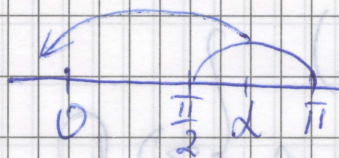
$$-\frac{\pi}{2} < \gamma < 0$$

$-\frac{\pi}{2} < \alpha - \pi < 0$

$$\gamma = \alpha - \pi$$

$$\alpha - \beta - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\cancel{\alpha} - \cancel{\beta} + \frac{\pi}{2} - \cancel{\alpha} = \frac{\pi}{2}$$



$$(3) a) \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^5 + n^4 + 2}}{\sqrt[3]{1 - n^6} + \sqrt[3]{n}} = 0$$

$$b) \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^6 + n^2}}{\sqrt[3]{n^6 - n^2} - \sqrt[3]{n}} = 1$$

$$c) \frac{1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + n(n+1)(n+2)}{\sqrt{9n^8 + 3n^2 + 1}} = \frac{\frac{1}{4} n(n+1)(n+2)(n+3)}{\sqrt{9n^8}} \rightarrow \frac{1}{4 \cdot 3} = \frac{1}{12}$$

$$(5) a) \lim_{x \rightarrow \frac{3\pi}{2}} \frac{\cos x}{\frac{3\pi}{2} - x} = x - \frac{3\pi}{2} = t \rightarrow 0$$

$$= \lim_{t \rightarrow 0} \frac{\cos(t + \frac{3\pi}{2})}{-t} = -\lim_{t \rightarrow 0} \frac{\sin t}{t} = -1$$

$$\cos(t + \frac{3\pi}{2}) = \sin t$$

$$\frac{\cos x - \cos x_0}{x - x_0} \xrightarrow{x \rightarrow x_0} -\sin x_0$$

$$= \frac{\cos x}{x - \frac{3\pi}{2}} = -\frac{\cos x - \cos \frac{3\pi}{2}}{x - \frac{3\pi}{2}} \rightarrow -\left(-\sin \frac{3\pi}{2}\right) = -1$$

$$b) \sin \sqrt{x+1} - \sin \sqrt{x} = \sin \frac{\sqrt{x+1} - \sqrt{x}}{2} \cos \frac{\sqrt{x+1} + \sqrt{x}}{2} =$$

$$\sqrt{x+1} - \sqrt{x} = \frac{1}{\sqrt{x+1} + \sqrt{x}} \rightarrow 0$$

$$= \frac{\sin \frac{\sqrt{x+1} - \sqrt{x}}{2}}{\frac{\sqrt{x+1} - \sqrt{x}}{2}} \cdot \frac{(\sqrt{x+1} + \sqrt{x})}{2} \cos \frac{\sqrt{x+1} + \sqrt{x}}{2}$$

$$\left. \begin{array}{l} f(x)g(x) \\ |g(x)| \leq A \\ f(x) \rightarrow 0 \end{array} \right\} \Rightarrow 0$$

$$0 \leq |\varphi(x)| \leq |\psi(x)| (\sqrt{x+1} - \sqrt{x}) \rightarrow 0$$

$$\left(\frac{x e^x + 1}{x \pi^x + 1} \right)^{\frac{1}{x^2}} = e^{\frac{1}{x^2} \ln \left(\frac{x e^x + 1}{x \pi^x + 1} \right)} \rightarrow e^{1 - \ln \pi} = \frac{e}{\pi}$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} \ln \frac{x e^x + 1}{x \pi^x + 1} = \frac{x e^x + 1 - 1 - x \pi^x}{(x \pi^x + 1)^2} = \pi^x \varphi(x) \frac{e^{x - x \ln \pi} - 1}{x(1 - \ln \pi)} \rightarrow 1$$

$\pi^x = e^{x \ln \pi}$

$$x(1 - \ln \pi) \Rightarrow 1 - \ln \pi$$

$$(\sin x + \cos x)^{\frac{1}{x}} =$$

Производны

$$(x^\alpha)' = \alpha x^{\alpha-1}$$

$$(e^x)' = e^x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\sin^2 x}$$

$$(\cot x)' = -\frac{1}{\sin^2 x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctg x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

$$a^x = e^{x \ln a}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\sqrt[n]{x^p} = x^{\frac{p}{n}}$$

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(Af)' = A \cdot f'$$

$$(A)' = 0$$

$$!!! (F(f(g)))' = F'(f(g)) f'(g) \cdot g'$$

Диференциране:

- 1) функцията да бъде приведена до таблични функции
- 2) прилагаме директно на таблицата
- 3) преработваме на производната

$$y = \sqrt{\sin^3 x + 2^{\frac{x \arccos x}{\sqrt{x}}}}$$

$$= \left((\sin x)^3 + e^{\frac{x \ln 2 \arccos x}{\sqrt{x}}} \right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2} \left((\sin x)^3 + e^{\frac{x \ln 2 \arccos x}{\sqrt{x}}} \right)^{-\frac{1}{2}}$$

$$\sqrt{\ln(\sin x)}$$

$$\frac{A}{f(x)} = A \cdot f^{-1}(x)$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$x \left[3(\sin x)^2 \cdot \cos x + e^{\frac{\ln 2 x \arccos x}{\sqrt{x}}} \cdot \ln 2 \cdot \left(\arccos x^{\frac{1}{2}} + x \cdot \frac{-\frac{1}{2} x^{-\frac{1}{2}}}{\sqrt{1-x^2}} \right) \right]$$

$$y = \sqrt[3]{x + \sqrt{\sin \frac{1}{x}}} \cdot 2^{\ln 3} =$$

$$= \left(x + \left(\sin \frac{1}{x} \right)^{\frac{1}{2}} \right)^{\frac{1}{3}} \cdot 2^{\ln 3} =$$

$$y = \sin^{\pi} \left(\frac{1}{\arctg \sqrt{x}} \right) + x^x =$$

$$= \sin^{\pi} (\arctg \sqrt{x})^{-1} + x^x =$$

$$y' = \frac{1}{3} \left(x + \left(\sin \frac{1}{x} \right)^{\frac{1}{2}} \right)^{-\frac{2}{3}} \cdot 2^{\ln 3} \cdot \left(1 + \frac{1}{2x} \left(\sin \frac{1}{x} \right)^{-\frac{1}{2}} \cdot \cos \frac{1}{x} (-1) x^{-2} \right)$$

$$= \left(\sin^{\pi} (\arctg x^{\frac{1}{2}}) \right)^{-1} + e^{x \ln x}$$

$$y' = \pi \left(\sin (\arctg x^{\frac{1}{2}})^{-1} \right)^{\pi-1} \cdot \cos (\arctg x^{\frac{1}{2}})^{-1} \cdot (-1) (\arctg x^{\frac{1}{2}})^{-2} \cdot$$

$$\cdot \frac{1}{1 + (x^{\frac{1}{2}})^2} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} + e^{x \ln x} \cdot \left(1 \cdot \ln x + x \cdot \frac{1}{x} \right)$$

$$y = \sin(\arcsin x^2) + x^{\sin x^2} =$$

$$= \sin(\arcsin(x)^2) + e^{\sin(x)^2 \cdot \ln x}$$

$$f(x)^{g(x)} = e^{g(x) \ln f(x)}$$

$$\sin x^2 = \sin(x^2)$$

$$\sin^2 x = (\sin x)^2$$

$$y' = \cos(\arcsin x^2) \cdot \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x + e^{\sin(x^2)^2 \cdot \ln x} \cdot \left(\cos x^2 \cdot \frac{1}{x} \cdot 2x \ln x + \sin^2 \cdot \frac{1}{x} \right)$$

$$y = \sqrt{x^3 \sqrt{x^4 \sqrt{x^3 x}}} + (x^2)^{\cos x} + \arccos^2(\cos x) =$$

$$= \left(x^3 \left(x \left(x - x^{\frac{1}{5}} \right)^{\frac{1}{4}} \right)^{\frac{1}{3}} \right)^{\frac{1}{2}} + (x^2)^{\cos x} + \arccos^2(\cos(x)) =$$

$$y' = \frac{1}{2} x^{-\frac{1}{2}} \left(x \left(x - x^{\frac{1}{5}} \right)^{\frac{1}{4}} \right)^{\frac{1}{3}} + \frac{1}{2} x^{-\frac{1}{2}} \left(x - x^{\frac{1}{5}} \right)^{\frac{1}{4}} +$$

$$+ \frac{1}{4} x^{-\frac{1}{2}} \left(x - x^{\frac{1}{5}} \right)^{-\frac{1}{4}} + \frac{1}{2} x^{-\frac{1}{2}} \left(x - x^{\frac{1}{5}} \right)^{\frac{1}{4}} +$$

$$+ \frac{1}{4} x^{-\frac{1}{2}} \left(x - x^{\frac{1}{5}} \right)^{-\frac{1}{4}} = x^{\frac{1}{2}} x^{\frac{1}{2} \cdot \frac{1}{3}} \cdot x^{\frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2}} x^{\frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2}} = x^{\frac{86}{120}} e^{2 \cos x \cdot \ln x} +$$

$$+ (\arccos(\cos x))^2$$

$$y' = \frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{6} x^{-\frac{1}{6}} + \frac{1}{24} x^{-\frac{23}{24}} + e^{2 \cos x \ln x} \cdot$$

$$\cdot (2(-\sin x) \cdot \ln x) + 2(\cos x) \cdot \frac{1}{x} + 2 \cdot \arccos(\cos x) \cdot \frac{-1}{\sqrt{1-\cos^2 x}} \cdot (-\sin x)$$

$$\arcsin(\sin x) = x$$

$$= \pi - x$$

$$= x - 2\pi$$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$$

$$\frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$$

$$() = 1.8$$

$$\left(\arctg(\tgg x) \right)' = \frac{x}{x-2\pi} = 1$$

$$y = \sqrt{x + \sqrt{x + 5\sqrt{x}}} + x^{x+x} + e^{\arctg \frac{1}{x}}$$

$$y = \frac{1}{e^{\ln \sin x}} + \frac{1+x^2}{1-\sin x} + (\sin x)^x = \frac{1}{\sin x} + \frac{1+x^2}{1-\sin x} + (\sin x)^x$$

$$y' = -\frac{1}{\sin^2 x} \cdot \cos x + \frac{2x}{1-\sin x} + \frac{1+x^2}{(1-\sin x)^2} \cdot (-\cos x) + (\sin x)^{x-1} \cdot \cos x$$

$$= -\frac{\cos x}{\sin^2 x} + \frac{2x(1-\sin x) - (1+x^2)(-\cos x)}{(1-\sin x)^2} + (\sin x)^{x-1} \cos x$$

$$y = \operatorname{arccotg}(\cos \frac{1}{\sqrt{x}}) + \sin x^x + \frac{1}{\ln x e^3} = \operatorname{arccotg}(\cos \frac{1}{\sqrt{x}}) + \sin(e^{x \ln x}) + \frac{1}{3} (e^{\ln x})^{-1}$$

$$y' = \frac{-1}{1+(\cos \frac{1}{\sqrt{x}})^2} \cdot (-\sin \frac{1}{\sqrt{x}}) \cdot (-\frac{1}{2}) x^{-\frac{3}{2}} + \cos(e^{x \ln x}) \cdot e^{x \ln x} \cdot (\ln x + 1)$$

$$= \left(\frac{1}{1+\cos^2 \frac{1}{\sqrt{x}}} \right) \cdot \frac{1}{2} x^{-\frac{3}{2}} \sin \frac{1}{\sqrt{x}} + \frac{1}{3} (-1) (\ln x)^2 \cdot \frac{1}{x}$$

$$y = \sqrt[5]{\frac{1}{\operatorname{arccos} \sqrt{1-x^2}}} + (\sqrt{x})^{\sqrt{x}} + \frac{\ln x}{\operatorname{arctg} x^2} = (\operatorname{arccos} \sqrt{1-x^2})^{-\frac{1}{5}} + (x^{\frac{1}{2}})^x + e^{\frac{x}{2} \ln x} \cdot \ln x \cdot \operatorname{arctg} x^2$$

$$= \frac{\ln x}{\operatorname{arctg} x^2}$$

$$y' = -\frac{1}{5} \operatorname{arccos} \sqrt{1-x^2}^{-\frac{6}{5}} \cdot \left(-\frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}} \right) \cdot \frac{1}{2} \cdot \frac{(-2x)}{\sqrt{1-x^2}} + (\sqrt{x})^{\sqrt{x}-1} \cdot \frac{1}{2} \cdot \left(\frac{1}{2} \cdot x^{-\frac{1}{2}} \ln x + x^{\frac{1}{2}} \cdot \frac{1}{x} + \frac{\frac{1}{2} \operatorname{arctg} x^2 - \ln x \cdot \frac{2x}{1+x^2}}{\operatorname{arctg}^2 x^2} \right)$$

$$y = \ln \left(\frac{1}{\operatorname{arccotg} \sqrt{x}} \right) + \underbrace{(\ln x)^{\ln x^2}}_{g(x)} + \underbrace{\frac{\sin x^2}{\sin^2 x}}_{h(x)}$$

$$\frac{f'(x)}{f(x)} = -\frac{1}{\operatorname{arccotg} x^{\frac{1}{5}}} - \left(-\frac{1}{1+x^{\frac{2}{5}}} \right) = \frac{1}{5} x^{-\frac{4}{5}}$$

$$g'(x) = e^{2 \ln x \ln(\ln x)} \cdot 2 \left(\frac{1}{x} \ln(\ln x) + \ln x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} \right)$$

$$h'(x) = \cos x^2 \cdot x^2 \cdot (2x \cdot \sin x)^{-2} = \sin x^2 \cdot (-2) \cdot (\sin x)^{-3} \cdot \cos x$$

$$z) y' = f'(x) + g'(x) + h'(x)$$

$$y(x) = x^x \quad \ln y(x) = x \ln x$$

$$\frac{y'(x)}{y(x)} = \ln x + \frac{x'}{x}$$

$$y' = x^x (\ln x + 1)$$

$$I y(x) = e^{x \ln x}$$

$$y = \frac{\operatorname{arccotg} \frac{1}{e^x + \cos x}}{x^2 - \sqrt{x}} + (\sqrt{x})^{\operatorname{arcsin} x} + e^{\ln \frac{1}{\operatorname{arccotg} \pi}} =$$

$$= \frac{\operatorname{arccotg}(e^x + \cos^{-1})}{x^2 - x^{\frac{1}{2}}} + e^{\frac{1}{2} \operatorname{arcsin} x} \cdot \ln x^{\frac{1}{2}} + C =$$

$$y' = \underline{\hspace{10em}}$$

$$y = \frac{\operatorname{tg}^2 \frac{1}{\arctg x^2}}{\sin x + \cos x} + (\sqrt{x}) e^x + \cos(\arccos \frac{1}{\sqrt{1-x^2}})$$

$$y = \frac{(\operatorname{tg}(\arctg x^2))^{-1}}{\sin x + \cos x} + e^{\frac{x}{2}} \ln x + \frac{1}{\sqrt{1-x^2}} (1-x^2)^{-\frac{1}{2}}$$

$$y' = \frac{2(\operatorname{tg}(\arctg x^2))^{-2} \cdot \frac{1}{\cos^2(\arctg(x^2))^{-1}} \cdot (-1)(\arctg(x^2))^{-2} \cdot \frac{2x}{1+x^2}}{(\sin x + \cos x)^2}$$

$$x(\sin x + \cos x) - (p)(\cos x - \sin x) +$$

$$\left. \begin{array}{l} f(x) \\ x_0 \\ x_n \rightarrow x_0 \\ x_n \neq x_0 \\ x_n \in D \end{array} \right\} f(x_n) \rightarrow A$$

$$\begin{array}{l} x_n > x_0 \\ x_n < x_0 \end{array} \quad \text{— граница}$$

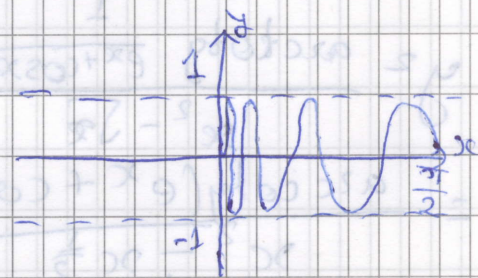
$$\begin{array}{ccc} x_n' & x_n'' & x' > x_0 \\ \downarrow & \downarrow & x'' > x_0 \\ 0 & 0 & \end{array}$$

$$\begin{array}{l} f(x_n') \rightarrow A \\ f(x_n'') \rightarrow B \end{array} \quad A \neq B$$

$$x_1', x_2'', x_1'', x_2' \rightarrow x_0$$

$$\sin \frac{1}{x} \\ x \rightarrow 0$$

$$\begin{array}{l} x_n = \\ \frac{1}{x_n} = n \frac{\pi}{2} \end{array} \quad \sin \frac{1}{x_n} = \begin{cases} 1 \\ 0 \\ -1 \\ 0 \end{cases}$$



$$x_n = \frac{2}{n\pi} \rightarrow 0 \quad \Rightarrow \text{няма граница}$$

$$x \cdot \sin \frac{1}{x}$$

$$x \rightarrow 0$$

$$f(x) \rightarrow A \neq 0$$

$$x \rightarrow x_0$$

$$|f(x)| \rightarrow |A|$$

$$f(x) \rightarrow 0$$

$$|f(x)| \rightarrow 0$$

$$f(x) \leq g(x) \leq h(x)$$

$$x \rightarrow x_0$$

$$g(x) \rightarrow A$$

$$0 \leq |x \cdot \sin \frac{1}{x}| \leq |x|$$

0

$$|x \sin \frac{1}{x}| \rightarrow 0$$

$$x \rightarrow 0$$

$$f(x) = \begin{cases} x \cdot \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f(x) \rightarrow f(x_0)$$

$$x \rightarrow x_0$$

$f(x)$ е непрекъсн.

$$\frac{f(x) - f(x_0)}{x - x_0}$$

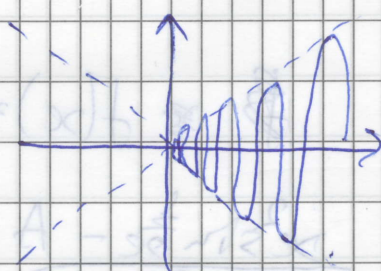
$$x \rightarrow x_0$$

$$x < x_0$$

дифференциално
частно

$$A - f'_A(x_0)$$

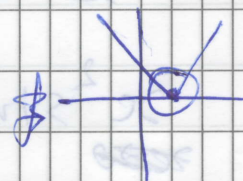
лява производна
($x < x_0$) граница



$$|x|$$

$$f(x) = f(0)$$

$$f(x) = |x|$$



$$\frac{|x| - 0}{x - 0} = \frac{|x|}{x} = 1 \rightarrow 1$$

$$\frac{|x|}{x}$$

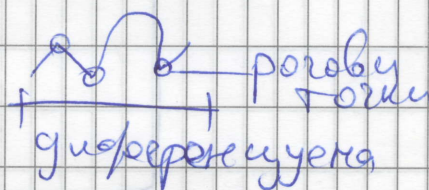
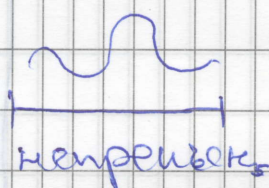
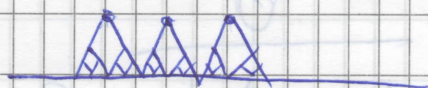
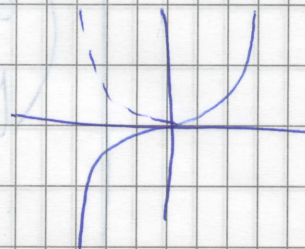
$$x \rightarrow x_0$$

$$x < x_0$$

$$-1 \rightarrow -1$$

$$\lim_{x \rightarrow 0} \frac{x|x|}{x-0} = f(x) = x|x|$$

$$\frac{x|x| - 0}{x - 0} = \frac{x|x|}{x} = |x| \rightarrow 0$$



$$f(x) = x \sin \frac{1}{x}$$

$$f(0) = 0$$

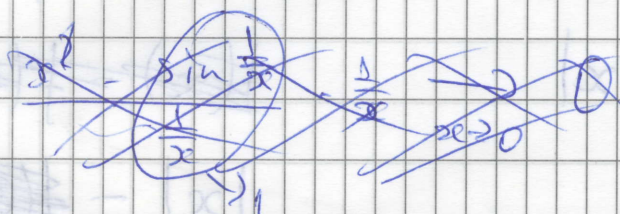
$$\frac{x \sin \frac{1}{x} - 0}{x - 0} = \frac{x \sin \frac{1}{x}}{x} = \sin \frac{1}{x}$$

КСМА произведет в точке 0.

$$f(x) = x^2 \sin \frac{1}{x} \quad f(0) = A$$

$$\frac{x^2 \sin \frac{1}{x} - A}{x - 0} = \frac{x^2 \sin \frac{1}{x} - A}{x}$$

$$x^2 \sin \frac{1}{x} \rightarrow 0 \quad x \rightarrow 0$$

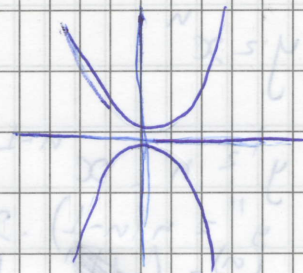


$$0 \leq |x^2 \sin \frac{1}{x}| \leq x^2 \rightarrow 0$$

$$0 \leq \left| \frac{\sin x}{x} \right| \leq \frac{1}{x} \rightarrow 0$$

1.2.68

$$\frac{x^2 \sin \frac{1}{x} - 0}{x - 0} = x \sin \frac{1}{x} \rightarrow 0$$



лѣва граница $b \text{ т. } x_0$
 непрек. отлѣво $b \text{ т. } x_0$
 диффер. отлѣво $b \text{ т. } x_0$

$$y = \frac{1}{x+a} = (x+a)^{-1}$$

$$y' = -1(x+a)^{-2}$$

$$y'' = (-1)(-2)(x+a)^{-3}$$

$$y''' = (-1)(-2)(-3)(x+a)^{-4}$$

$$y^{(n)} = (-1)^n n! (x+a)^{-(n+1)}$$

$$y = \ln(1+x)$$

$$y' = \frac{1}{1+x} = (1+x)^{-1}$$

$$y^{(n)} = (-1)^{n-1} (n-1)! (1+x)^{-n}$$

$$y = \frac{1}{x^2-3x+2} = (x^2-3x+2)^{-1} = \frac{1}{x-2} - \frac{1}{x-1} = \frac{A}{x-2} + \frac{B}{x-1}$$

$$y^{(n)} = (-1)^n n! (x^2-3x+2)^{-(n+1)} (2x-3)$$

~~$$y^{(n)} = (-1)^n n! (x^2-3x+2)^{-(n+1)}$$~~

$$A(x-1) + B(x-2) = 1$$

$$x_1 = 1 \quad B = -1$$

$$x_2 = 2 \quad A = 1$$

$$y = \frac{1}{x-2} - \frac{1}{x-1}$$

$$y^{(n)} = (-1)^n n! (x-2)^{-n-1} - (-1)^n n! (x-1)^{-n-1}$$

$$y = x^n$$

$$P_n(x)$$

$$y' = n \cdot x^{n-1}$$

$$y'' = n(n-1) \cdot x^{n-2}$$

$$y^{(n)} = \cancel{n!} n! (x)^0 = n!$$

$$y = \sin x$$

$$y' = \cos x$$

$$y'' = -\sin x$$

$$y''' = -\cos x$$

$$y^{(iv)} = \cos x$$

$$y^{(v)} = \sin x$$

$$y = \sin x$$

$$y^{(4k)} = \sin x$$

$$y^{(4k+1)} = \cos x$$

$$y^{(4k+2)} = -\sin x$$

$$y^{(4k+3)} = -\cos x$$

$$y = \sin x$$

$$y' = \cos x =$$

$$= \sin\left(x + \frac{\pi}{2}\right)$$

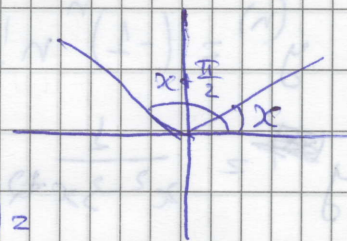
$$y = \cos x \text{ za D.R.}$$

$$y' = \cos\left(x + \frac{\pi}{2}\right) =$$

$$= \sin(x + \pi)$$

$$y'' = \cos(x + \pi) = \sin\left(x + \frac{3\pi}{2}\right)$$

$$y^{(n)} = \sin\left(x + \frac{n\pi}{2}\right)$$



$$y = \sin(ax + b)$$

$$y^{(n)} = \sin\left(ax + b + \frac{n\pi}{2}\right) \cdot a^n$$

$$y = e^x \cdot \sin x$$

$$y' = e^x \cdot \sin(x + \frac{\pi}{2}) + e^x \cos x =$$

$$= e^x \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right)$$

$$= e^x \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$$

$$y = \frac{1}{x+a}$$

$$y^{(n)} = \frac{(-1)^n n!}{(x+a)^{n+1}}$$

$$y = \frac{1}{(x-1)(x+2)}$$

$$y = \frac{1}{1+x^2}$$

$$y = \frac{1}{(x+i)(x-i)} = \frac{A}{x+i} + \frac{B}{x-i} = \frac{1}{2i} \left(\frac{1}{x-i} - \frac{1}{x+i} \right)$$

$$= \frac{A(x-i)}{(x+i)(x-i)} + \frac{B(x+i)}{(x+i)(x-i)} \quad y^{(n)} = \frac{(-1)^n n!}{2i} \left(\frac{1}{(x-i)^{n+1}} - \frac{1}{(x+i)^{n+1}} \right) =$$

$$= \frac{(-1)^n n!}{2i (x^2+1)^{n+1}} ((x+i)^{n+1} - (x-i)^{n+1}) = \frac{(-1)^n n!}{2i (x^2+1)^{\frac{n+1}{2}}} (\cos(n+1)\varphi + i \sin(n+1)\varphi)$$

$$\frac{x+i}{x-i}$$

$$\frac{1}{x-i}$$

$$x \pm i = \sqrt{x^2+1} (\cos \varphi \pm i \sin \varphi)$$

$$\frac{x}{1} = \cot \varphi \quad \varphi = \operatorname{arccot} x$$

$$x \pm i$$

Формула на де Мовер

$$(z(\cos \varphi + i \sin \varphi))^n = z^n (\cos n\varphi + i \sin n\varphi)$$

$$(x^2+1)^{\frac{n+1}{2}} =$$

$$\rightarrow = \frac{(-1)^n n!}{2i (x^2+1)^{\frac{n+1}{2}}} ((\cos(n+1)\varphi + i \sin(n+1)\varphi) - (\cos(n+1)\varphi - i \sin(n+1)\varphi))$$

$$= \frac{(-1)^n n!}{(x^2+1)^{\frac{n+1}{2}}} \sin((n+1)\varphi)$$

$$y = \frac{1}{(x-1)(x+2)}$$

$$y = \frac{1}{1+x^2}$$

$$y^{(n)} = \frac{(-1)^n n!}{(x^2+1)^{\frac{n+1}{2}}} - \sin((n+1) \arccot x)$$

$$y = \arctan x$$

$$y' = \frac{1}{1+x^2} = (1+x^2)^{-1}$$

$$y^{(n)} = \left(\frac{1}{1+x^2} \right)^{n-1}$$

$$(uv)^{(n)} = \binom{n}{0} u^{(n)} v^{(0)} + \binom{n}{1} u^{(n-1)} v^{(1)} + \binom{n}{2} u^{(n-2)} v^{(2)} + \dots + \binom{n}{n-1} u^{(1)} v^{(n-1)} + \binom{n}{n} u^{(0)} v^{(n)}$$

$$u^{(0)} = u$$

$$y = e^x \cos x$$

$$y' = e^x \cos x - e^x \sin x = e^x (\cos x - \sin x) \cdot \frac{1}{\sqrt{2}} =$$

$$= \sqrt{2} e^x (\cos x - \cos \frac{\pi}{4} - \sin x - \sin \frac{\pi}{4})$$

$$\frac{a \sin x + b \cos x}{\sqrt{a^2 + b^2}} \cdot \sqrt{a^2 + b^2}$$

$$y'' = (\sqrt{2})^2 e^x \cos(x + 2 \frac{\pi}{4})$$

$$y^{(n)} = (\sqrt{2})^n e^x \cos(x + n \frac{\pi}{4})$$

$$y = x^2 e^x$$

$$y' = 2x \cdot e^x + x^2 \cdot e^x = x e^x (2 + x)$$

$$y'' = e^x (2 + x) +$$

$$(e^x x^2)^{(n)} = 1 e^x \cdot x^2 + n \cdot e^x \cdot 2x + \frac{n(n-1)}{2} e^x \cdot 2 + 0 \dots 0$$

1.2.72

$$y(x) = \arctan x$$

$$y'(x) = \frac{1}{1+x^2}$$

$$(1+x^2) y'(x) = 1$$

$$1 y^{(n+1)}(x) (1+x^2) + n y^{(n)}(x) \cdot 2x + \frac{n(n-1)}{2} y^{(n-1)}(x) \cdot 2 = 0$$

$$y^{(n+1)}(0) = -n(n-1) y^{(n-1)}(0)$$

$$n = 2k$$

$$y^{(n-1)}(0) = - (n-2)(n-3) y^{(n-3)}(0)$$

$$y^{(n-3)}(0) = - (n-4)(n-5) y^{(n-5)}(0)$$

$$\dots$$

$$y^{(3)}(0) = -2 \cdot 1 \cdot y'(0)$$

$$y^{(2k+1)}(0) = (-1)^k (2k)! \cdot y'(0)$$

$$y^{(2k+1)}(0) = (-1)^k \cdot (2k)! \cdot$$

$$n = 2k-1$$

$$y^{(2k)}(0) = (-1)^{k-1} \cdot (2k-1) \cdot \dots \cdot 3 \cdot 2 \cdot y''(0) = 0$$

$$y''(0) = -1 (1+x^2)^{-2} = 0$$

$$y = \arcsin x \quad y' = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-\frac{1}{2}}$$

$$y'' = \frac{x}{(1-x^2)^{\frac{3}{2}}} = \frac{x}{(1-x^2)} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$(1-x^2)y''(x) = xy'$$

Докажи за след 2 случая

① $y^{(n)}(0) = ?$

$y = \arcsin x$

Да се повторят разсъжденията от узаот 9

② Докажи за x_n ~~$\left(\frac{\ln x}{x}\right)^{(n)} = \frac{(-1)^{n-1} n!}{x^{n+1}} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln x\right)$~~

$$\left(\frac{\ln x}{x}\right)^{(n)} = \frac{(-1)^{n-1} n!}{x^{n+1}} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln x\right)$$

③ $\left(\frac{1}{(x-1)(x-2)^2}\right)^{(n)}$

Теорема на Лопитал

$$\begin{aligned} f(x) &\rightarrow 0 \\ g(x) &\rightarrow 0 \\ g'(x) &> 0 \end{aligned}$$

$$\frac{f(x)}{g(x)} \xrightarrow{x \rightarrow x_0} \frac{f'(x)}{g'(x)}$$

$$\frac{f'(x)}{g'(x)} \xrightarrow{x \rightarrow x_0} l$$

$$\frac{f(x)}{g(x)} \xrightarrow{x \rightarrow x_0} l$$

$$\frac{e^x}{x} \xrightarrow{x \rightarrow 0} \pm \infty$$

$$\frac{e^x - 1}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\frac{e^x}{1} \rightarrow 1$$

$$\frac{e^x}{1} \rightarrow 1$$

$$\frac{e^x - 1}{x} \xrightarrow{x \rightarrow 0} 1 \Rightarrow (e^x)' = e^x$$

$$\frac{\sin x}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\frac{\cos x}{1} \rightarrow 1$$

$$f(x) \rightarrow \infty$$

$$g(x) \rightarrow \infty$$

$$g'(x) \neq 0$$

$$\frac{f'(x)}{g'(x)} \xrightarrow{x \rightarrow x_0} l$$

$$\frac{f''(x)}{g''(x)}$$

$$\frac{f(x)}{g(x)} \rightarrow e$$

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1-x} - 1}{\sqrt[4]{2-x} - \sqrt[4]{2+x}} = \frac{-\frac{1}{3}(1-x)^{-\frac{2}{3}}}{-\frac{1}{4}(2-x)^{-\frac{3}{4}} - \frac{1}{4}(2+x)^{-\frac{3}{4}}} \rightarrow$$

$$\rightarrow \frac{-\frac{1}{3}}{-\frac{2}{4}(2)^{-\frac{3}{4}}} \rightarrow \frac{2\sqrt[4]{2^3}}{3}$$

$$\lim_{x \rightarrow \infty} (x - x^2 \ln(1 + \frac{1}{x})) \rightarrow \frac{1}{2} \quad \frac{1}{x} = t$$

$$\frac{1}{t} - (\frac{1}{t})^2 \ln(1+t) = \frac{t - \ln(1+t)}{t^2} = \frac{f(x)}{g(x)}$$

$$\frac{f'(x)}{g'(x)} = \frac{1 - \frac{1}{1+t} \cdot 1}{2t} = \frac{1+t-1}{2t(1+t)} = \frac{1}{2(1+t)} \rightarrow \frac{1}{2}$$

$a^x = e^{x \ln a}$

$$\left(\frac{e^x - 1}{x}\right)^{\frac{1}{e^x - 1}} = \cancel{e^{\frac{1}{e^x - 1} \cdot \ln(e^x - 1)}} = e^{\frac{1}{e^x - 1} \cdot \ln(e^x - 1)} \rightarrow e^{\frac{1}{2}}$$

$$\frac{f(x)}{g(x)} = \frac{\ln \frac{e^x - 1}{x}}{e^x - 1} \rightarrow \frac{1}{2} \quad \frac{f(x)}{g'(x)} = \frac{\ln(e^x - 1) - \ln x}{e^x} = \frac{\frac{e^x}{e^x - 1} - \frac{1}{x}}{e^x} = \frac{e^x - x - e^x + 1}{e^x(e^x - 1)x} \rightarrow \frac{1}{2}$$

$$= \frac{e^x(x-1)}{e^x(e^x-1)x}$$

$$\frac{f''(x)}{g''(x)} = \frac{(xe^x - e^x + 1)'}{(xe^x - x)'} = \frac{e^x + xe^x - e^x}{e^x + e^x - 1} \rightarrow \frac{1}{2}$$

$$\frac{f'''(x)}{g'''(x)} = \frac{(xe^x)'}{(e^x + xe^x - 1)'} = \frac{e^x + xe^x}{e^x + e^x + xe^x} \rightarrow \frac{1}{2}$$

11.811 20122

$f \rightarrow 0, f \rightarrow \infty$

$g \rightarrow 0, g \rightarrow \infty$

$$x \rightarrow x_0 \quad \left. \begin{array}{l} g'(x) \neq 0 \\ \frac{f'(x)}{g'(x)} \rightarrow l \end{array} \right\} \Rightarrow \frac{f(x)}{g(x)} \rightarrow l$$

$$\lim_{x \rightarrow \infty} \frac{e^{x^2}}{e^x} \rightarrow \frac{\infty}{\infty} \quad \begin{array}{l} f = x^2 \rightarrow \infty \\ g = x \rightarrow \infty \end{array} \quad \begin{array}{l} f' = 2x \\ g' = 1 \end{array} \quad \frac{f'}{g'} \rightarrow \infty$$

$$\frac{e^{x^2} - 2x}{e^{2x}}$$

$$\frac{x^2}{\ln x} \rightarrow \frac{\infty}{\infty} \quad \begin{array}{l} \text{хат-диржа } \phi-3 \\ \text{хат-диржа } \phi-3 \end{array}$$

$$\frac{x}{e^x} \rightarrow 0 \quad \text{хат-диржа } \phi-3$$

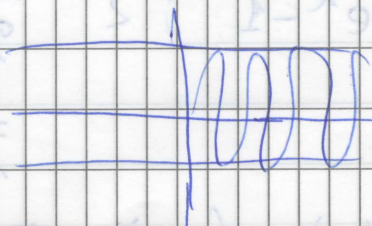
$$\frac{2(e^{x^2} \cdot 2x^2 + e^{x^2})}{e^x} \quad x \rightarrow \infty \quad x^2 - x \rightarrow \infty$$

$$\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x} \Rightarrow \frac{0}{0} \quad 0 \leq |x^2 \sin \frac{1}{x}| \leq x^2$$

$$\frac{2x - \sin \frac{1}{x} + x^2 \cdot \cos \frac{1}{x} (-\frac{1}{x^2})}{\cos x} \rightarrow \text{хат-диржа } \phi-3 \quad x \sin \frac{1}{x} \rightarrow 0$$

$$\cos \frac{1}{x} = \cos(x)^{-1} \text{ - хат-диржа } \phi-3 \quad x \rightarrow 0$$

$$\frac{x}{\sin x} \cdot x \cdot \sin \frac{1}{x} \rightarrow 0$$



$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}} = 1$$

~~$$\frac{1}{x} \frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$~~

$$\frac{1}{x} \frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{\frac{x}{\sqrt{x^2+1}}}{1} = \frac{x}{\sqrt{x^2+1}}$$

$$\frac{1 - \cos ax}{x^2} \rightarrow \frac{a^2}{2}$$

$$a_n = \left(\sin \frac{x}{n} + \cos \frac{x}{n} \right)^{\frac{n}{x}} \rightarrow e^x$$

$$\varphi(t) = (\sin t + \cos t)^{\frac{1}{t}} = e^{\frac{\ln(\sin t + \cos t)}{t}} \Rightarrow e$$

$$\frac{\ln(\sin t + \cos t)}{t} = \frac{\ln(\sin t + \cos t)}{\sin t + \cos t - 1} \cdot \frac{\sin t + \cos t - 1}{t}$$

$$\Rightarrow \left(\frac{\sin t}{t} - \frac{1 - \cos t}{t^2} \right) t \Rightarrow 1$$

$$\frac{\cos t - \sin t}{\sin t + \cos t} \rightarrow 1$$

Заманно:

$$\textcircled{4} \lim_{x \rightarrow 1} \left(\frac{\ln x}{x-1} \right)^{\frac{1}{x}}$$

$$\textcircled{5} ka \leq b$$

$$q_0 = 2 \lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}}$$

$$\begin{aligned} a) & x \rightarrow 0 \\ b) & x \rightarrow \infty \\ c) & x \rightarrow -\infty \end{aligned}$$

$$\lim_{\substack{x \rightarrow 0 \\ x > 0}} x^x \rightarrow e^{x \ln x} \rightarrow 1$$

$$\lim_{\substack{x \rightarrow 0 \\ x > 0}} x \ln x = \lim_{t \rightarrow \infty} \frac{\ln t}{t} = -\lim_{t \rightarrow \infty} \frac{1/t}{1} = -\lim_{t \rightarrow \infty} \frac{1}{t} = 0$$

$\ln x = \frac{1}{x}$

Формула на Тейлор

$$f(x) = f(a)$$

$$\varphi(x) = \frac{f(x) - f(a)}{x - a} - f'(a)$$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

$$\varphi(x) \rightarrow 0$$

$$f(x) = f(a) + (x-a)f'(a) + \underbrace{(x-a)\varphi(x)}_{o(x-a)}$$

$$f(x) = f(a) + (x-a)f'(a) + o(x-a)$$

~~$$f(x) = f(a) + (x-a)f'(a) + o(x-a)$$~~

~~$$\frac{f(x) - f(a) - (x-a)f'(a)}{(x-a)^2}$$~~

$$o(x-a) = \frac{(x-a)\varphi(x)}{\varphi(x-a)} \rightarrow 0$$

$$\frac{f(x) - f(a) - (x-a)f'(a)}{(x-a)^2} \rightarrow \frac{f''(a)}{2}$$

$$\frac{f'(x) - f'(a)}{2(x-a)} \rightarrow \frac{f''(a)}{2}$$

$$\frac{f(x) - f(a) - (x-a)f'(a) - \frac{(x-a)^2}{2}f''(a)}{(x-a)^3} \rightarrow 0$$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + o((x-a)^2)$$

1.2.78

$$\psi(x) = \frac{f(x) - f(a) - (x-a)f'(a) - \frac{(x-a)^2}{2}f''(a)}{(x-a)^3} = -\frac{f'''(a)}{3!} \rightarrow 0$$

$$\frac{f''(x) - f''(a)}{3(x-a)} \rightarrow \frac{f'''(x) - f'''(a)}{3-2 \cdot (x-a)} \rightarrow \frac{f'''(a)}{3!}$$

$$\frac{f}{g} \quad \frac{f'}{g'} \quad \frac{f''}{g''} \quad \frac{f'''}{g'''} \quad \dots \quad \frac{f^{(n)}}{g^{(n)}} \rightarrow 0$$

$$\psi(x)(x-a)^3 = o(x-a)^3$$

$$f(x) = f(a) + \frac{x-a}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + o(x-a)^3$$

$$f(x) = f(a) + \frac{(x-a)f'(a)}{1!} + \dots + \frac{(x-a)^n f^{(n)}(a)}{n!} + o(x-a)^n$$

$$(\sin x)^n = \sin\left(x + n \frac{\pi}{2}\right) \quad \text{---}$$

$+\sin x$	0	$\sin x = x - \frac{x^3}{3!} + o(x^4)$
$+\cos x$	1	$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + o(x^2)$
$-\sin x$	0	
$-\cos x$	-1	
$+\sin x$	0	

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$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a) + R_n(x)$$

$R_n(x)$ - полином на Тейлор
формула на Тейлор

$$f(x) - P_n(x) = R_n(x)$$

$$R_n(x) = o((x-a)^n) \quad \text{форма на Пеано}$$

$$\psi = o(\rho) \quad \frac{\psi}{\rho} \rightarrow 0$$

$$\frac{(x-a)^\epsilon \cdot o((x-a)^n)}{(x-a)^{n+\epsilon}} \rightarrow 0$$

$$\frac{o((x-a)^n)}{(x-a)^n} = \frac{o(x-a)^n}{(x-a)^n}$$

$$\frac{o(x-a)^n}{(x-a)^{n+\epsilon}} \rightarrow 0$$

$$\frac{o(\rho)}{o(\rho)} \quad \begin{matrix} n \\ n-\epsilon \end{matrix}$$

$$\frac{\psi}{\rho} \rightarrow 0$$

форма на Лагранж : $R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}(\xi)$
между a, x

~~$R_n(x)$~~ при $a=0$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + R_n$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{(-1)^{n-1}}{(2n-1)!} \cdot \frac{x^{2n-1}}{(2n-1)!} + R_{2n}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{(-1)^{n-1}}{(2n-2)!} \cdot \frac{x^{2n-2}}{(2n-2)!} + R_{2n+1}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{(-1)^{n-1}}{n} \cdot \frac{x^n}{(2n-2)!} + R_n$$

$$(1+x)^{\alpha} = \binom{\alpha}{0} + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \dots + \binom{\alpha}{n}x^n + R_n$$

$$\frac{1}{1+x} = \cancel{1-x+x^2-x^3+\dots} = 1-x+x^2+\dots+(-1)^{n-1}x^{n-1}+R_n$$

$$\arctg x = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{2n-1} + \frac{1}{1+x} = \frac{1(-x)^n}{1+x} + R_n$$

$$R_n = \frac{-(-x)^n}{1+x}$$

$$\textcircled{1} \lim_{x \rightarrow \infty} x - x^2 \ln\left(1 + \frac{1}{x}\right)$$

$$x \rightarrow \infty$$

$$t = \frac{1}{x}$$

$$t \rightarrow 0 \quad \frac{t - \ln(1+t)}{t^2} = \frac{t - (t + o(t))}{t^2} =$$

$$= \frac{o(t)}{t^2} = \frac{t - (t - \frac{t^2}{2} + o(t^2))}{t^2} = \frac{\frac{1}{2}t^2 + o(t^2)}{t^2} \rightarrow \frac{1}{2}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{\ln \cos ax}{\ln \cos bx} = \frac{\ln \cos ax}{\cos ax - 1} - \frac{\cos ax - 1}{\ln \cos bx} \cdot \frac{\ln y}{y - 1} \xrightarrow{y \rightarrow 1} 1$$

$$\lim_{x \rightarrow 0} \frac{\ln \cos ax}{\ln \cos bx} = \frac{-\sin ax \cdot a}{\cos ax (-\sin bx \cdot b)} = \frac{-a}{-b} = \frac{a}{b}$$

$$\cos ax = 1 - \frac{a^2 x^2}{2!} + o(x^2)$$

$$\cos bx = 1 - \frac{b^2 x^2}{2!} + o(x^2)$$

$$\frac{\ln \cos ax}{\ln \cos bx} = \frac{\ln\left(1 - \frac{a^2 x^2}{2} + o(x^2)\right)}{\ln\left(1 - \frac{b^2 x^2}{2} + o(x^2)\right)} = \frac{-\frac{a^2 x^2}{2} + o(x^2)}{-\frac{b^2 x^2}{2} + o(x^2)} =$$

$$= \frac{a^2 + \frac{o(x^2)}{x^2}}{b^2 + \frac{o(x^2)}{x^2}} \rightarrow \frac{a^2}{b^2}$$

$$\textcircled{3} \lim_{x \rightarrow 0} \frac{e^{\sqrt{1+x}} - e^{1+\frac{x}{2}}}{x^2} = e^{\frac{1+x}{2}} \cdot \frac{e^{-\frac{x^2}{8} + o(x^2)} - 1}{x^2} = \boxed{\frac{e^x - 1}{x} \rightarrow 0}$$

$$\begin{aligned} \sqrt{1+x} &= \binom{\frac{1}{2}}{0} + \binom{\frac{1}{2}}{1}x + \binom{\frac{1}{2}}{2}x^2 + o(x^2) = 1 + \frac{1}{2}x + \frac{\frac{1}{2} \cdot (-\frac{1}{2})}{2 \cdot 1}x^2 + o(x^2) \\ &= e^{\frac{1+x}{2}} \cdot \frac{1 - \frac{x^2}{8} + o(x^2) - 1}{x^2} \rightarrow e \cdot \left(-\frac{1}{8}\right) \end{aligned}$$

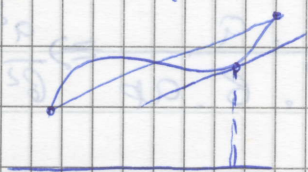
arctg

18. XII. 20122. $f(x)$ 1) дефинирана в $[a, b]$ (интервал за действително)
- може и отворен, но интервал
непривзснат
2) диференцируема в (a, b)

Теорема за крайните нараствания (Лагранж):

Ако са изпълнени горните условия

$$\Rightarrow \exists \xi \in (a, b) \quad f(b) - f(a) = f'(\xi)(b-a)$$



① Да се док. неравенството: $\frac{x-y}{\cos^2 x} \leq \operatorname{tg} x - \operatorname{tg} y \leq \frac{x-y}{\cos^2 y}$

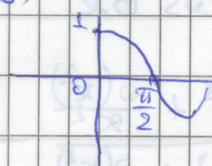
$$0 < x < y < \frac{\pi}{2}$$

$$\operatorname{tg} x$$

$$\frac{x-y}{\cos^2 x} > \operatorname{tg} x - \operatorname{tg} y = \frac{1}{\cos^2 \xi}(x-y) > \frac{1}{\cos^2 y}(x-y)$$

$$0 < x < \xi < y < \frac{\pi}{2}$$

$\cos^2 x$ - монотонно намаляваща
расч. $f(x) > g(y)$ расч. $g(y)$



$f(x)$	p	p	n	n
$g(y)$	p	n	p	n
$g(f(x))$	p	n	n	p

$$x_1 < x_2$$

$$f(x_1) < f(x_2)$$

$$g(f(x_1)) > g(f(x_2))$$

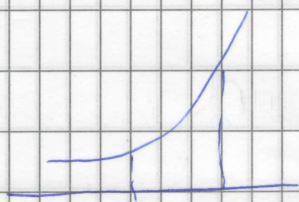
$$\cos^2 x > \cos^2 z > \cos^2 y$$

$$\frac{1}{\cos^2 x} < \frac{1}{\cos^2 z} < \frac{1}{\cos^2 y}$$

~~Условие~~ на Либмичу

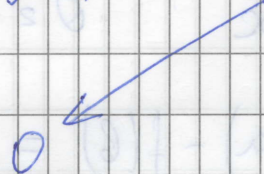
$$x, y \in M$$

$$|f(x) - f(y)| \leq A|x - y|$$



$$x_n \rightarrow x_0$$

$$0 \leq |f(x_n) - f(x_0)| \leq A|x_n - x_0|$$



$$x^n \quad 0 < a < b$$

$$a < x < y < b$$

$$f(x) - f(y) = f'(\xi)(x - y)$$

$$|x^n - y^n| = n \xi^{n-1} |x - y| \leq n b^{n-1} |x - y|$$

константа на Либмичу

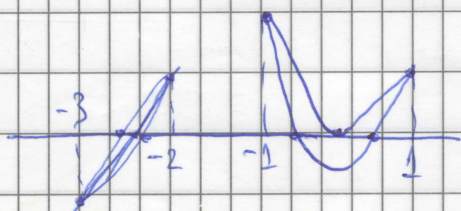
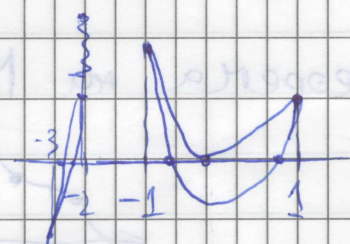
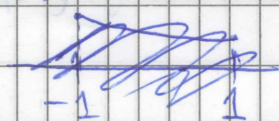
Теорема на Рол: Ако условията за $f(x)$ са изпълнени (от предишната теорема) теоремата на Ларанже)
 $\Rightarrow f(a) = f(b) \Rightarrow f'(\xi) = 0$

$$(2) f(x) \geq 0$$

$$f'(x) = 0$$

$$x^3 - 3x + 5 = 0$$

$$3x^2 - 3 = 0$$



$$\begin{matrix} f(x) \\ g(x) \\ [a, b] \end{matrix} \quad \varphi(x) = \begin{vmatrix} f(x) & g(x) & 1 \\ f(a) & g(a) & 1 \\ f(b) & g(b) & 1 \end{vmatrix} \quad \varphi(a) = \varphi(b) = 0$$

$$\exists \xi \quad \varphi'(\xi) = 0$$

$$\varphi(x) = f(x)(g(a) - g(b)) - g(x)(f(a) - f(b)) + f(a)g(b) - g(a)f(b) = 0$$

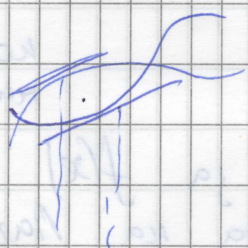
$$\varphi'(x) = f'(x)(g(a) - g(b)) - g'(x)(f(a) - f(b))$$

$$g(x) = x \quad 0 = \varphi'(\xi) = f'(\xi)(g(a) - g(b)) - (f(a) - f(b))$$

$$f(a) - f(b) = f'(\xi)(a - b)$$

$$f'(\xi)(g(a) - g(b)) = g'(\xi)(f(a) - f(b))$$

$$\frac{f(a) - f(b)}{g(a) - g(b)} = \frac{f'(\xi)}{g'(\xi)} \quad \left(\begin{array}{l} \text{Обобщенная теорема на} \\ \text{Лагранже (Лагранж-Коши)} \end{array} \right)$$



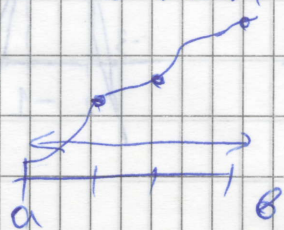
$$\frac{f(a) - f(b)}{g(a) - g(b)} = \frac{f'(\xi_f)(b-a)}{g'(\xi_g)(b-a)}$$

$$\text{Теорема на Коши: } \frac{f(a) - f(b)}{g(a) - g(b)} = \frac{f'(\xi)}{g'(\xi)}$$

Теорема на Мон

$$f'(x) \geq 0$$

$f(x)$ е строго
растваща



~~Основна теорема на~~
~~интегралното~~
~~смятане~~
 ОТИ ИС $f'(x) \geq 0$ $x \in (a, b)$

$$\arcsin \frac{1}{\sqrt{1+x^2}} - \arccos \frac{x}{\sqrt{1+x^2}} = \begin{cases} 0 & x \geq 0 \\ \pi - 2 \arccos \frac{x}{\sqrt{1+x^2}} & x < 0 \end{cases}$$

$$f = \arcsin \frac{1}{\sqrt{1+x^2}} \quad f' = \frac{1}{\sqrt{1-\frac{1}{1+x^2}}} \cdot \left(-\frac{1}{2}\right) (1+x^2)^{-\frac{3}{2}} \cdot 2x =$$

$$g = \arccos \frac{x}{\sqrt{1+x^2}} \quad g' = \frac{-x}{\sqrt{x^2} (1+x^2)}$$

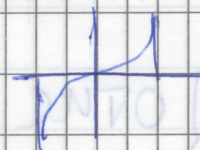
~~$$g' = \frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}} \cdot \frac{-\frac{1}{2}}{(1+x^2)^{\frac{3}{2}}} \cdot 2x =$$~~

$$g' = \frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}} \cdot \frac{-\frac{x^2}{1+x^2}}{(1+x^2)^{\frac{3}{2}}} = \frac{-1}{1+x^2}$$

$$-1 \leq \frac{1}{\sqrt{1+x^2}} \leq 1 \quad \arcsin t$$

$$0 < \frac{1}{\sqrt{1+x^2}} \leq 1 \quad \text{непривзката}$$

$$\frac{1}{\sqrt{1-t^2}}$$



$$\frac{x}{\sqrt{1+x^2}} \leq 1 \quad \text{гоор. и непр.}$$

$$x > 0 \quad \text{набавяване}$$

$$x^2 \leq 1+x^2$$

$$x^2 \geq 1+x^2 \text{ не е извърш.}$$

1) $x \geq 0$ ~~непривзката~~ в затв. интервал
 диференцируема в отв. интервал

$$f'(x) = \frac{1}{1+x^2} \geq g'(x)$$

$$(f+g)' = 0$$

$$f-g = C$$

$$x \geq 0$$

$$x=0$$

$$f(0) - g(0) = \arcsin 1 - \arccos 0 = 0$$

$$x \rightarrow \infty$$

$$\arcsin 0 - \arccos 1 = C$$

$$\Rightarrow C = 0$$

$$x=1$$

$$\arcsin \frac{1}{\sqrt{2}} - \arccos \frac{1}{\sqrt{2}} = C = 0$$

$$2) x \leq 0$$

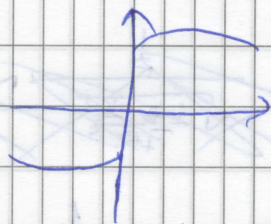
$$f' = \frac{1}{1+x^2}$$

$$g' = -\frac{1}{1+x^2}$$

$$(f+g)' = 0$$

$$f+g = C = \pi$$

$$\frac{\pi}{2} + \frac{\pi}{2} = C$$



Доказателство: ⑥ Да се покаже че

$$\arccos \frac{x + \sqrt{1-x^2}}{\sqrt{2}} = \begin{cases} -\arccos x + \frac{\pi}{4} & x \geq \frac{1}{\sqrt{2}} \\ \arccos x - \frac{\pi}{4} & x < \frac{1}{\sqrt{2}} \end{cases}$$

1) отиде

2) както се справим от график.

⑦ $\arcsin \frac{2x}{1+x^2}$ през $\arctg x = g$

$$f' = \frac{2x}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}} \cdot \frac{4x^2}{\sqrt{1+x^2} - \sqrt{1-x^2}} \cdot 2 = \frac{2(1-x^2)}{\sqrt{(1-x^2)^2} (1+x^2)} = \frac{-2}{1+x^2}$$

$$f' = \frac{2}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}} \cdot \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{2(1-x^2)}{\sqrt{(1-x^2)^2} (1+x^2)} = \frac{-2}{1+x^2}$$



①.286

$$4) x \leq -1$$

$$g' = \frac{1}{1+x^2}$$

$$(2g + f)' = 0$$

$$2g + f = C$$

$$x \rightarrow -\infty$$

$$-\pi + 0 = C$$

$$2g + f = -\pi$$

20. 811 20122.

$f(x)$ гедо. невр. в $[a, b]$

$f'(x)$ съществува безкраен брой точки $\left\{ \begin{array}{l} f(x) \text{ е строго} \\ \text{растваща} \\ \text{в интерв.} \end{array} \right.$

$f'(x) \geq 0$ в краен брой точки ($f'(x) = 0$) $[a, b]$

$f(x)$ е непрекъсната в затворен интервал $[a, b] \Rightarrow f(x)$ приема $\max$ и $\min$

$f(a) \cdot f(b) < 0 \Rightarrow f(x_0) = 0 \quad x_0 \in [a, b]$
непрекъсната $[a, b]$

Рон: $f^2(a) = f(b)$
 $f'(x) = 0$

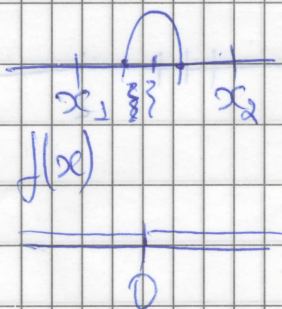


$$x^2 - 2 = 0$$

$$f(x) = x^2 - 2$$

$$f'(x) = 2x$$

$$f''(x) = 2$$

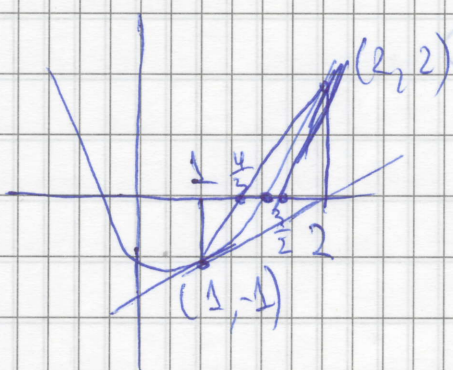


$$f(0) = -2$$

$$f(1) = -1$$

$$f(2) = 2$$

39. 2.87



$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\begin{aligned} f(2) &= 2 \\ f(1) &= -1 \end{aligned}$$

$$\frac{y+1}{3} = \frac{x-1}{1}$$

$$y = 3x - 4$$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$$\begin{aligned} x &= t \cdot 1 + 2 \\ y &= t \cdot 3 + 2 \end{aligned} \quad \text{вектор по направлению}$$

$$\begin{aligned} \text{Т.н.} & \quad 1, 3 \\ & \quad 3, -1 \end{aligned}$$

$$\begin{aligned} 3x - y + c &= 0 \\ c &= -4 \end{aligned}$$

$$\frac{3x - y - 4}{\sqrt{10}} = 0$$

$$3x - y = 4$$

$$\begin{aligned} f(x) \\ f'(x) \end{aligned}$$

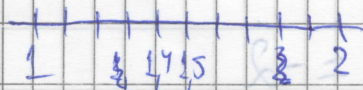
$$y - y_0 = \operatorname{tg} \alpha (x - x_0)$$

$$f'(x_0) = \operatorname{tg} \alpha$$

$$\begin{aligned} y - 2 &= 4(x - 2) \\ 4x - 8 + 2 &= 0 \\ 4x &= -2 \\ x &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} f\left(\frac{4}{3}\right) &< 0 \\ f\left(\frac{3}{2}\right) &> 0 \end{aligned}$$

$$\begin{aligned} x^2 - 2 &= 0 \\ \frac{4}{3} < x_0 < \frac{3}{2} \end{aligned}$$



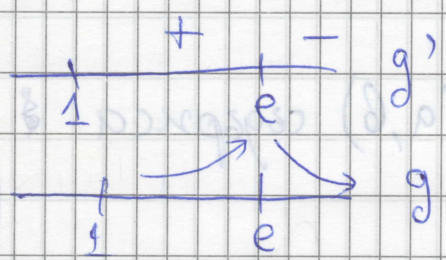
① $\frac{\ln^2 x}{2} \leq \frac{x-1}{e} \quad x \geq 1$ $\sqrt{\ln^2 x} = |\ln x|$

$f(x) = \frac{\ln^2 x}{2} - \frac{x-1}{e} \quad g(x) = \frac{\ln x}{\sqrt{2}} - \frac{\sqrt{x-1}}{\sqrt{e}}$

$f'(x) = \frac{1}{x} - \frac{1}{e} = \frac{e \ln x - x}{ex}$

$\ln x = \frac{1}{e}$

$g(x) = e \ln x - x$
 $g'(x) = \frac{e}{x} - 1 = \frac{e-x}{x}$

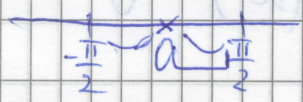


$0 = g(e) \geq g(x) \quad x \geq 1$
 $f'(x) \leq 0$

$0 = f(1) \geq f(x) \quad \forall x \geq 1$

② $\cos x \leq \left(\frac{\sin x}{x}\right)^3 \quad 0 < |x| \leq \frac{\pi}{2}$

$0 < x < \frac{\pi}{2}$



$f(x) = x - \frac{\sin x}{\sqrt[3]{\cos x}}$

$0 \leq x \leq \frac{\pi}{2}$

$f'(x) = 1 - \frac{1}{3}(\cos x)^{-\frac{4}{3}}(-\sin x) =$

$= \frac{\frac{1}{3}(\cos x)^{-\frac{4}{3}}(-\sin x) - \cos x}{\frac{1}{3}(\cos x)^{-\frac{4}{3}}(-\sin x)}$

$f'(x) = 1 - \frac{\cos \sqrt[3]{\cos x} - \sin \frac{1}{3} \sqrt[3]{\cos^2 x} (-\sin x)}{\sqrt[3]{\cos^2 x}} =$

$t = \cos^2 x > 0$

$\sqrt[3]{t} = u > 0$

$= 1 - \frac{3 \cdot \frac{t}{3} + 1 - t}{3 \cdot \sqrt[3]{t^2}} = 1 - \frac{2t+1}{3 \cdot \sqrt[3]{t^2}} = \frac{3u^2 + 1 - 2u^3}{3 \cdot u^2} =$

$$z = \frac{-(u-1)(2u^2 - 2u - 1)}{3 \sqrt[3]{t^2}} = \frac{-(u-1)^2(u+\frac{1}{2})}{3 \sqrt[3]{t^2}} < 0 \quad x^2 \in \mathbb{N} \quad (2)$$

$f(x)$ ~~е~~ строго намаляваща

$$0 = f(0) > f(x)$$

08.1.2013г.

$f(x)$ дефинирана и непрекъсната в интерв. $[a, b]$

$a \xrightarrow{\quad} b$

$f'(x)$ диференцируема в интервал (a, b) съдържа ∞ $\Rightarrow$
и непрекъсната

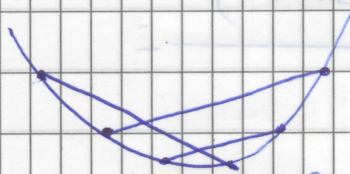
Безкраен брой точки

$$f'(x) \geq 0 \quad = 0 \text{ в краен брой точки}$$

$\Rightarrow f(x)$ е строго растяща

$a \xrightarrow{\quad} b$

f е непрекъсн.
 f' съществува освен краищата
 f'' съществува освен краищата
 $f''(x) \geq 0$ } $f(x)$ е изпъкнала



геометричен смисъл
изпъкнала ф-я: ако графика
винаги е под хордата
(не излиза над хордата)

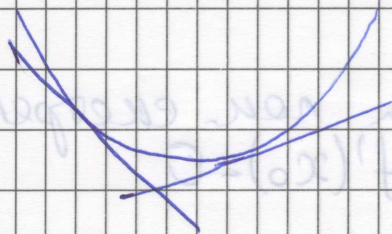
аналитичен смисъл
 $x_1, x_2 \in [p, q]$
 $p+q=1$
 $p \geq 0$
 $q \geq 0$
неравенство на Енсен $\Rightarrow$ ф-ята е непрекъсн.
 $f(px_1 + qx_2) \leq pf(x_1) + qf(x_2)$

I.2.00

$$f\left(\frac{x_1+x_2}{2}\right) \leq \frac{f(x_1)+f(x_2)}{2} \Leftrightarrow \text{неравенство на Ексен}$$

непрерывната

а + + + + + в



извъкнала ф-я: ^{наг} всяка допирателна
ф-я е наг чрз. Изисква производна
в допирната точка.

Вдлъбната е аналогично и обратно.

$$\textcircled{1} f(x) = 0$$

отделяне на корени (приблизително пресмятане)

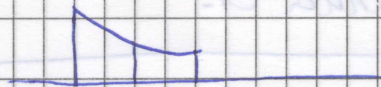


$$\textcircled{2} f(x) \leq g(x)$$

$$\textcircled{3} \text{най-голяма и най-малка стойности}$$

помощта ф-я и гертасм
схематично
графика на ф-я

$$\textcircled{4}$$



$$\textcircled{5} \text{намерете локалните екстремуми}$$

$$\textcircled{6} \text{намерете абсолютните екстремуми}$$

⑤ x_0  (1) x_0 — локален екстремум

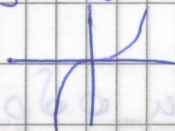
1) $U \subset D$

$x_0 \in U$

2) $f(x_0) \geq f(x)$
 $x \in U$

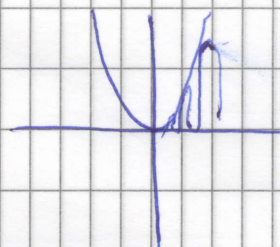
Теорема: 1) Ако в x_0 има локален екстремум и $f(x)$ е диференцируема, то $f'(x_0) = 0$
(f' съществува)

$y = x^3$
 $y' = 3x^2$



③' = 5 (3') 1) точки, в които няма произв. (критични точки)
2) $f'(x) = 0$ (необходими условия)

$x^2 \sin^2 \frac{1}{x} \geq 0$



(достатъчни условия)
1) в x_0 $f'(x)$ сменя знака $f'(x_0)$ сменя знака
2) $f''(x_0) > 0$ минимум
 $f''(x_0) < 0$ максимум

$f(p)$ — списък от числа

най-голямо $p \rightarrow$ максимум най-голяма ст.

най-малко $p \rightarrow$ минимум най-малка ст.

1 зад. $y = x(x+1)^2(x-1)^3$

локален екстремум

най-голяма и най-малка стойности в $[-2, 5]$
компактно множество

Компактно множество:

M — затворено множ.

$x_n \in M$
 $x_n \rightarrow x_0$

x_n (последователност)
(сходяща)

$\Rightarrow x_0 \in M$

ограничено множество, ако можем да посочим
красен интервал $[A, B]$

$$x \in M \Rightarrow A \leq x \leq B$$

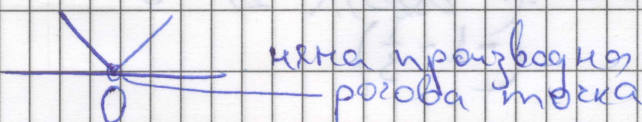
компактно множество: ограничено и затворено
множество



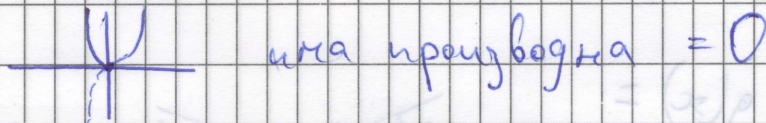
1 зад. $y' = \frac{1}{2}(x+1)^2 + x \cdot 2(x+1)$ y'

1. елементарна ф-я : $|f(x)|$ $f(x) = 0$ възможно е
да няма производна

$$y = |x|$$

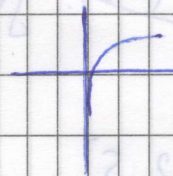


$$y = |x^3|$$



2. $y = x^\alpha$ ($0 < \alpha < 1$)

$$(f(x))^\alpha$$
 ($0 < \alpha < 1$)



$f(x) = 0$ възможно е да няма производна

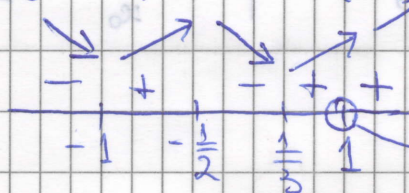
$$\sqrt{x}$$

няма произв. в точка (0) α
има произв. в точка (0) = 0

arcsin f(x) $f(x) = \pm 1$ възможно е да няма притоци

$$\frac{1}{\sqrt{1-f^2(x)}} \cdot f'(x)$$

$$\begin{aligned} y' &= (x+1)^2(x-1)^3 + 2x(x+1)(x-1)^3 + 3x(x+1)^2(x-1)^2 = \\ &= (x+1)(x-1)^2(x^2-1+2x^2-2x+3x^2+3x) = \\ &= (x+1)(x-1)^2(6x^2+x-1) = 6(x+1)(x-1)^2(x-\frac{1}{3})(x+\frac{1}{2}) \end{aligned}$$



от квадрата не се сменя знака

$$+1; \frac{1}{3}; -\frac{1}{2}$$

$$f''(-1) = 6 \cdot 2^2 \cdot \left(\frac{1}{3} \right) \cdot \left(-\frac{1}{2} \right) > 0$$

$$f(x) \cdot g(x) =$$

$$x_0$$

$$f'(x_0) = f'(x_0) \cdot g(x_0) + f(x_0) \cdot g'(x_0)$$

$$-1; -\frac{1}{2}; \frac{1}{3}; 1 \quad [-2, 5]$$

$$f(-2) = 27 \cdot 2 = 54$$

$$f(-1) = 0$$

$$f(-\frac{1}{2}) = -\frac{1}{2} - \frac{1}{4} - \left(-\frac{3}{2}\right)^3 =$$

$$f(\frac{1}{3}) = \frac{1}{3} \cdot \left(\frac{1}{3}\right)^2 - \left(-\frac{2}{3}\right)^3 = \checkmark$$

$$f(5) = 5 \cdot 36 - 4^3$$

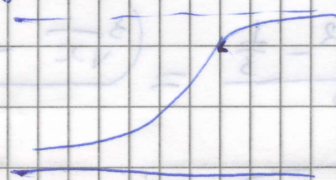
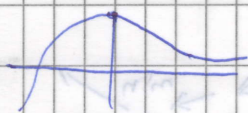
$$1.294$$

10.1 (2012-13) ... $x^2 - x = y$... (2)

1. Абсолютни екстремуми
2. Локални екстремуми
3. Графики на функции

- ① 1. непрекънатост
2. диференцируемост
3. $f'(x) = 0$

P_i $f(P_i)$



$$\lim_{x \rightarrow \infty} f(x) = +\infty$$

$$\lim_{x \rightarrow \infty} f(x) = A$$

② $f'(x)$ знака

③ Γ_f Нагертайте Γ_f (графиката на f')

$$f'(x) = 0$$

1. критични точки
2. таблица
3. асимптоти

$$\begin{array}{l} x \rightarrow a \\ y \rightarrow \infty \\ x = a \end{array}$$

$$\begin{array}{l} x \rightarrow \infty \\ y \rightarrow b \\ y = b \end{array}$$

$$\begin{array}{l} x \rightarrow \infty \\ y \rightarrow \infty \\ f(x) \end{array}$$

$$\frac{f(x)}{x} \xrightarrow{x \rightarrow \infty} K$$

$$f(x) - K \cdot x \xrightarrow{x \rightarrow \infty} b \text{ (наклонена асимптота)}$$

$$y = Kx + b$$

1-2-96

$f(x) - kx \geq b$ - пресечни точки на асимптотата и кривата

4. графика

Задачи:

(1) $y = \sqrt[3]{(x-1)(2-x)^2}$

НГС НМС $\forall (-\infty, +\infty)$
НГС НМС $\forall [0, 2]$

непрерывна $(-\infty, +\infty)$
дифференцируема $\neq 1, 2$

Локс Екстр-
 Γ_f

~~$f(x)$~~

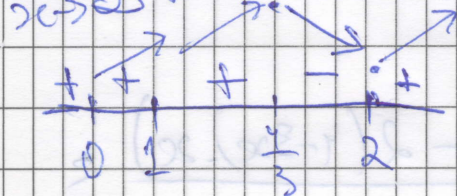
$-\infty, +\infty, 1, 2, \frac{1}{3}$

$y' = \frac{1}{3} \cdot (x-1)^{-\frac{2}{3}} (2-x)^{-\frac{1}{3}} \cdot \cancel{(2-x)^2 + 2(x-1)(-1)(2-x)}$

$= \frac{(2-x)(2-x-2x+2)}{3 \sqrt[3]{(x-1)^2(2-x)^4}} = \frac{(2-x)(4-3x)}{3 \sqrt[3]{(x-1)^2(2-x)^4}} = \frac{\sqrt[3]{(2-x)^3(4-3x)}}{3 \sqrt[3]{(x-1)^2 \cdot \sqrt[3]{(2-x)^4}}} =$

$= \frac{1}{3} \frac{(4-3x)}{\sqrt[3]{(x-1)^2(2-x)}}$

$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} x \sqrt[3]{(1-\frac{1}{x})(\frac{2}{x}-1)^2}$



$f(0) = \sqrt{-4}$

$\checkmark$

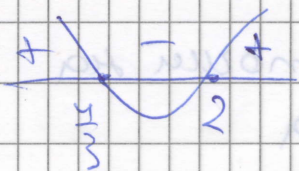
$f(1) = 0$

$f(\frac{1}{3}) = \frac{\sqrt[3]{4}}{2}$

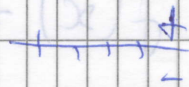
$f(2) = 0$

$\sqrt[3]{\frac{4}{8}} = \frac{\sqrt[3]{4}}{2} \checkmark$

$$\frac{P(x)}{Q(x)}$$



$$\frac{(-)(-)(-)}{(-)(-)(-)} < 0$$



$$f(a) - f(x_0) = f'(\xi_a)(a - x_0)$$

$$\frac{f(a) - f(x_0)}{a - x_0} = f'(\xi_a) \rightarrow A$$

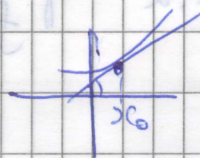
$$\lim_{x \rightarrow x_0} f'(x) = A$$

$A \rightarrow$ число — точка графика

$A \rightarrow$ не число — не точка графика

$$x \rightarrow 1$$

$$|y'| \rightarrow |\infty|$$



$$f'(x_0) = \pm \infty$$

(вертикальная дотирательная)

$$f'(x_0) = 0$$

(горизонтальная дотирательная)

$$y' = \frac{1}{3} \sqrt[3]{\frac{(4-3x)^3}{(x-1)^2(2-x)}}$$

$$y'' = \frac{1}{3} - \frac{1}{3} \frac{3(4-3x)^2 \cdot 3(x-1)^2(2-x) - (4-3x)^3 \cdot 2(x-1) \cdot 1}{(x-1)^4(2-x)^2}$$

$$= \frac{1}{9} \frac{(4-3x)^2 (3(x-1)^2(2-x) - 2(x-1)(4-3x) \cdot 2x)}{(x-1)^4(2-x)^2}$$

$$= \frac{1}{9} \frac{(4-3x)^2 (x-1) (3(x-1)(2-x) - 2(4-3x) \cdot 2x)}{(x-1)^4(2-x)^2}$$

$$= \frac{1}{9} \frac{(4-3x)^2 (3(2x-x^2-2+x) + 6x^2 - 8x)}{(x-1)^3(2-x)^2}$$

1.2.98

$$z \frac{1}{3} \frac{(4-3x)^2(3x^2+x-2)}{(x-1)^3(2-x)^2} =$$

$$y'' = \frac{1}{9} \frac{\sqrt[3]{(x-1)^4(2-x)^2}}{(4-3x)^6} \cdot \frac{3(-3)(x-1)(2-x) - \sqrt[3]{2(4-3x)(2-x)}}{(x-1)^3(2-x)^2}$$

$$= \frac{1}{9} \frac{\sqrt[3]{(x-1)^4(2-x)^2}}{(4-3x)^6} \cdot \frac{-9(2x-x^2-2+x) - (8-6x)(2-x) - x+1}{(x-1)^3(2-x)^2}$$

$$= \frac{1}{9} \frac{\sqrt[3]{(x-1)^4(2-x)^2}}{(4-3x)^6} \cdot \frac{-27x+9x^2+18-(16-8x+12x+6x^2x+1)}{(x-1)^3(2-x)^2}$$

$$= \frac{1}{9} \frac{\sqrt[3]{(x-1)^4(2-x)^2}}{(4-3x)^6} \cdot \frac{9x^2-27x+18-16-4x-6x^2+x}{(x-1)^3(2-x)^2}$$

$$= \frac{1}{9} \frac{\sqrt[3]{(x-1)^4(2-x)^2}}{(4-3x)^6} \cdot \frac{3x^2-30x+1}{(x-1)^3(2-x)^2} =$$

$$= \frac{\sqrt[3]{(x-1)^4(2-x)^2}(-2)}{9(x-1)^3(2-x)^2}$$

* $g = x - \frac{1}{3}$

x	$-\infty$	1	$\frac{4}{3}$	2	$+\infty$
y	$-\infty$	0	$\frac{2\sqrt{5}}{3}$	0	$+\infty$
y'	$+$	$+$	0	$-$	$+$
y''	$+$	$-$	$-$	$-$	$-$

$\frac{y}{x} = \frac{\sqrt[3]{(x-1)(2-x)^2}}{x} \rightarrow 1$

$y - x = \sqrt[3]{(x-1)(2-x)^2} - x$

$$\lim_{x \rightarrow \infty} (\sqrt[3]{(x-1)(2-x)^2} - x) = \frac{1}{3} \frac{(4-3x)^2}{(x-1)^3(2-x)^2} \cdot \frac{3(-3)(x-1)(2-x) - \sqrt[3]{2(4-3x)(2-x)}}{(x-1)^3(2-x)^2}$$

$$= \frac{1}{3} \frac{(4-3x)^2}{(x-1)^3(2-x)^2} \cdot \frac{-9(2x-x^2-2+x) - (8-6x)(2-x) - x+1}{(x-1)^3(2-x)^2}$$

$$\frac{1}{x} = t \quad t \rightarrow 0$$

$$\lim_{t \rightarrow 0} \frac{\sqrt[3]{(1-t)(2t-1)^2} - 1}{t} = \frac{\cancel{(1-t)^{\frac{1}{3}}} \cancel{(1-2t)^{\frac{2}{3}}} - 1}{t}$$

$$\frac{(1-t)^{\frac{1}{3}} (1-2t)^{\frac{2}{3}} - 1}{t} \approx \frac{\left(1 - \frac{1}{3}t\right) \left(1 - \frac{2}{3} \cdot 2t\right) - 1}{t} =$$

$$= \frac{-\frac{5}{3}t + o(t^2)}{t} \rightarrow -\frac{5}{3}$$

$$y - x = \sqrt[3]{\cancel{(x-1)(2-x)^2}} - x = -\frac{5}{3}$$

$$(x-1)(2-x)^2 = \left(x - \frac{5}{3}\right)^3$$

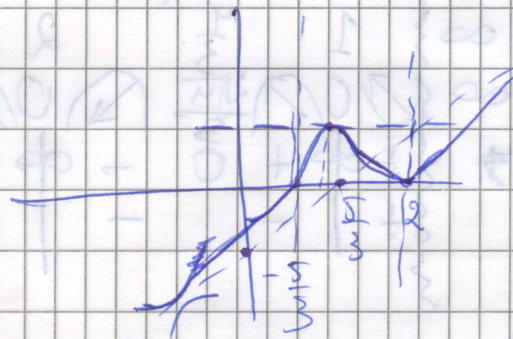
$$(x-1)(4-4x+x^2) = \left(x - \frac{5}{3}\right)^3$$

$$\cancel{x^3} - 4x^2 + 4x - x^2 + 4x - 4 = \cancel{x^3} - 3x^2 - \frac{5}{3} + \frac{8}{3}x - \frac{125}{27}$$

$$-\frac{125}{27} = \frac{89}{9}$$

$$\cancel{\frac{1}{3}x} = \frac{89}{9}$$

$$x = -\frac{89}{3}$$



$$y = \frac{1}{(x-1)(x-2)^2}$$

$$y^{(n)} = ?$$

$$y' = \frac{1}{(x-1)(x-2)^2} = \frac{1}{(x-2)^2(x-2+2x-2)} = \frac{1}{(x-2)^2(3x-4)}$$

$$= \frac{-\frac{1}{2}(2(x-2) \cdot 2(3x-4))}{(x-2)^2(3x-4)^2} = -\frac{4(x-2)(3x-4)}{(x-2)^4(3x-4)^2} = -\frac{4}{(x-2)(3x-4)}$$

$$y'' = \frac{-4}{(x-2)(3x-4)} = \frac{-4 \cdot (x-2)(3x-4)}{(x-2)^2(3x-4)^2} = \frac{-4}{(x-2)(3x-4)}$$

$$y''' = \frac{12}{(x-2)^2(3x-4)^2}$$

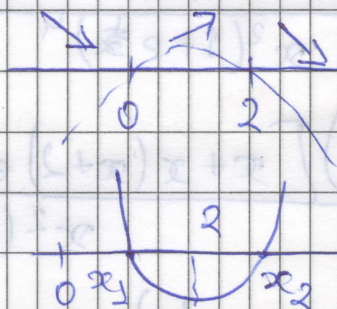
$$y''' = \left(\frac{12}{(x-2)^2(3x-4)^2} \right)' = \frac{-12 \cdot 2(x-2) \cdot 2(3x-4)}{(x-2)^4(3x-4)^4} = -\frac{48}{(x-2)^5(3x-4)^3}$$

$$y^{(4)} = \left(-\frac{48}{(x-2)^5(3x-4)^3} \right)' = \frac{48 \cdot 5(x-2)^4 \cdot 3(3x-4)^2}{(x-2)^9(3x-4)^6} = \frac{48 \cdot 9}{(x-2)(3x-4)^4}$$

$$y^{(n)} = \frac{(-1)^{n-1} \cdot 48}{(x-2)^n(3x-4)^n}$$

$$\lim_{x \rightarrow 1} \left(\frac{f(x)}{g(x)} \right) \frac{1}{x-1} =$$

15.1.2013r.



$$x^2 \rightarrow \infty$$

$$x \rightarrow -\infty$$

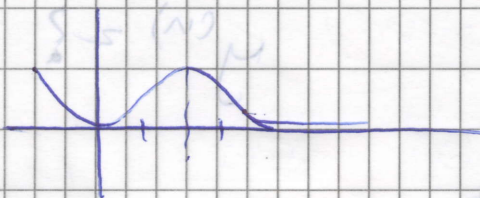
$$e^{-x} \rightarrow \infty$$

$$\frac{x^2}{e^x} \rightarrow 0 = k$$

$$\frac{x^2}{e^{x-x}} \rightarrow 0 = k$$

1.2.101

$$y - \ln x = y$$



$x \rightarrow \infty$	0	x_1	2	x_2	$+\infty$
$y \rightarrow \infty$	$\searrow$	0	$\nearrow$	$\frac{1}{e^2}$	$\searrow$
y'	—	0	+	+	0
y''	+	+	+	+	0

$$\textcircled{1} \text{geg: } y = \frac{xc}{1+e^{\frac{1}{x}}} \quad y' = \frac{1(1+e^{\frac{1}{x}}) + x \cdot \frac{1}{x^2} \cdot (-\frac{1}{x^2})}{(1+e^{\frac{1}{x}})^2} =$$

$$= \frac{1+e^{-x} - \frac{1}{x^2}}{(1+e^{\frac{1}{x}})^2} = \frac{1+e^{-x}}{(1+e^{\frac{1}{x}})^2} = \frac{1+e^{\frac{1}{x}} - x \cdot e^{\frac{1}{x}} (\frac{1}{x^2})}{(1+e^{\frac{1}{x}})^2} =$$

$$= \frac{1+e^{\frac{1}{x}} - \frac{x}{x^2}}{(1+e^{\frac{1}{x}})^2} = \frac{x+e^{\frac{1}{x}}(1+\frac{1}{x})}{x(1+e^{\frac{1}{x}})^2}$$

$$y'' = \frac{(1+e^{\frac{1}{x}})((x+1) + e^{\frac{1}{x}} \cdot 1) - x(1+e^{\frac{1}{x}})^2 \cdot \frac{1}{x^2}}{x^2(1+e^{\frac{1}{x}})^4} =$$

$$= \frac{(x+e^{\frac{1}{x}}(x+1))(1+e^{\frac{1}{x}})^2 - x \cdot 2(1+e^{\frac{1}{x}}) \cdot \frac{1}{x^2}}{x^2(1+e^{\frac{1}{x}})^4} =$$

$$= \frac{((1+e^{\frac{1}{x}}(x+2))x(1+e^{\frac{1}{x}})^2) - (x+e^{\frac{1}{x}} + 1)((1+e^{\frac{1}{x}})^2 + 2xe^{\frac{1}{x}})}{x^2(1+e^{\frac{1}{x}})^4}$$

$$\frac{x(1+e^{\frac{1}{x}})}{x^2(1+e^{\frac{1}{x}})^4} = \frac{(1+e^{\frac{1}{x}})[x + x(x+2)e^{\frac{1}{x}} - (x+(x+1)e^{\frac{1}{x}})]}{x^2(1+e^{\frac{1}{x}})^4}$$

$$= \frac{x(1+e^{\frac{1}{x}} + 2xe^{\frac{1}{x}})}{x^2(1+e^{\frac{1}{x}})^4} =$$

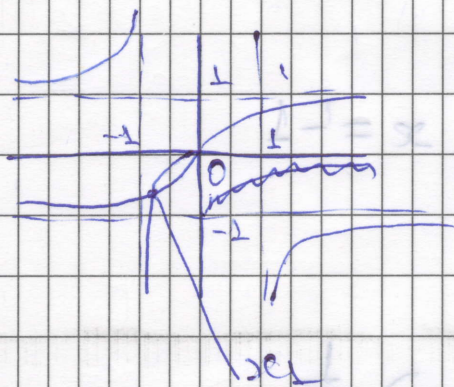
9.2.102

$$= \frac{(e^{\frac{1}{x}} - 1) \cdot e^{\frac{1}{x}}}{x(1 + e^{\frac{1}{x}})^2}$$

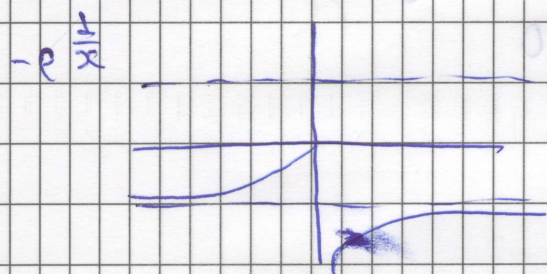
$$-\infty, 0, +\infty$$

$$x + (x+1) \cdot e^{\frac{1}{x}}$$

$$\frac{x}{x+1} = -e^{\frac{1}{x}}$$



$$\frac{x}{x+1}$$



$$y' = \frac{e^{\frac{1}{x}}}{x^2} > 0$$

$$e^{\frac{1}{x}} \rightarrow 1 \Rightarrow -e^{\frac{1}{x}} \rightarrow -1$$

$$x \rightarrow 0$$

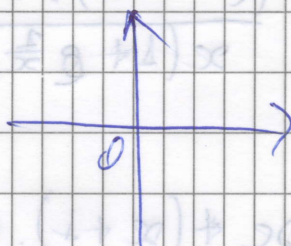
$$x < 0 \quad \frac{1}{x} \rightarrow -\infty$$

$$y = x + (x+1)e^{\frac{1}{x}} \quad -1 < x < 0$$

$$y' = 1 + 1 \cdot e^{\frac{1}{x}} - (x+1) \cdot e^{\frac{1}{x}} \cdot \frac{1}{x^2}$$

$$\Rightarrow y' = 1 + e^{\frac{1}{x}} - \frac{x^2 - x - 1}{x^2}$$

x	$-\infty$	-1	x_1	0	$+\infty$
$f(x)$	$-\infty$	$-$	0	0	$+\infty$
			0	$\frac{1}{2}$	0



$$y = \frac{2x}{1+e^{\frac{1}{x}}} \quad \frac{2x}{x^2+1} + e^{\frac{1}{x}}$$

$$y = x + (x+1)e^{\frac{1}{x}} < 0 \quad \text{wpy} \quad x = -1$$

$$\left(\frac{1}{x}\right)' = \frac{1'x - \frac{1}{x} \cdot x'}{x^2} = -\frac{1}{x^2}$$

$$\frac{f(x)}{x} = \frac{2x}{1+e^{\frac{1}{x}}} \cdot \frac{1}{x} = \frac{1}{1+e^{\frac{1}{x}}} \rightarrow \frac{1}{2}$$

$$y - x = \frac{x-1}{1+e^{\frac{1}{x}}} - \frac{1}{2}x$$

$$\lim \left(\frac{2x-2-x-x \cdot e^{\frac{1}{x}}}{2+2e^{\frac{1}{x}}} \right)$$

$$\frac{1}{x} \xrightarrow{x \rightarrow \infty} 0 \Rightarrow x = \frac{1}{t} \Rightarrow \frac{\frac{1}{t}-1}{e^t+1} - \frac{1}{2t} =$$

$$= \frac{\frac{1-t}{t}}{e^t+1} - \frac{1}{2t} = \frac{\frac{1-t}{t} \cdot 2t - e^t - 1}{2t(e^t+1)} = \frac{2-2t-e^t-1}{2t(e^t+1)} =$$

$$= \frac{1-2t-e^t}{2t(e^t+1)} = \frac{1-2t-1-t+o(t)}{2t(e^t+1)} =$$

$$= \frac{-3t+o(t)}{2t(e^t+1)} \rightarrow -\frac{3}{4}$$

$$y = \frac{1}{2}x - \frac{3}{4}$$

$$\begin{matrix} x \rightarrow 0 \\ x \leftarrow 0 \end{matrix} \rightarrow \frac{1}{x} \rightarrow \pm \infty$$

$$e^{\frac{1}{x}} = e^{-\infty} \rightarrow 0$$

$$\begin{matrix} x \rightarrow 0 \\ x > 0 \end{matrix}$$

$$e^{\frac{1}{x}} \rightarrow +\infty$$

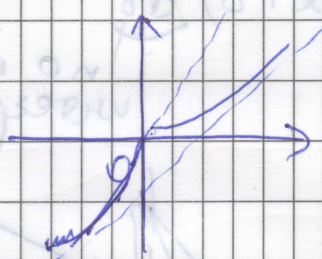
$$y = \frac{x}{1+e^{\frac{1}{x}}} \quad \lim_{x \rightarrow 0} \frac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} = ?$$

$$y' = \frac{1 + te^{t(1+\frac{1}{t})}}{(1+e^t)^2} \rightarrow \frac{2}{4} = \frac{1}{2}$$

$$t \rightarrow +\infty$$

$$t \rightarrow 0$$

$$e^{\frac{1}{x}}$$



Графика на функцията?

$$y = \frac{x(x-2)}{x+2}$$

асимптота $y', y'' = ?$

пресечни точки?

17.1.2013г.

12г. - контролно 104 загла

1. производни, графики на функции (Менделеев, Пиако, Шейнгар)
2. неравенства (НМС, НГС, по екстрем.)
3. графики на функции.

$$\int f(x) dx = \int f(x) d(x+a)$$

$$\int f(x) dx = \frac{1}{a} \int f(x) da$$

$$\int f dg = \int f dG$$

$$\int f dg = \int f g' dx$$

$$a \int f dx = \int a f dx = \int f da$$

$$\int (f+g) dx = \int f dx + \int g dx$$

$$\int f dg = fg - \int g df \text{ (по частям)}$$

$$\int dx = x + C$$

$$\int x^a dx = \frac{x^{a+1} + 1}{a+1} + C \quad a \neq -1$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{dx}{\cos^2 x} = \tan x + C$$

$$\int \frac{dx}{\sin^2 x} = -\cot x + C$$

$$\int e^x dx = e^x + C$$

2.106

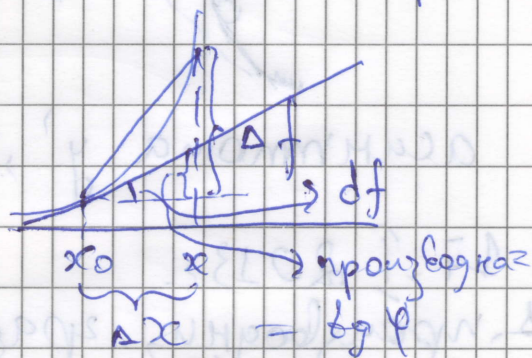
$$G = \int g dx$$

Вкасяне под знака на диференциала (интегриране)

изкарване от знака на диференциала (диференциране)

$$\int (x+t) dt$$

не изваб интегриране



$$df = f'(x_0) dx$$

$$\int \frac{dx}{1+x^2} = \arctg x + C = -\operatorname{arccotg} x + C_1$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \operatorname{arcsin} x + C = -\operatorname{arccos} x + C_1$$

$$\int \frac{dx}{\sqrt{x^2+a}} = \ln \left(x + \sqrt{x^2+a} \right), \quad a \neq 0$$

$$\textcircled{1} \int \cos(f(x)) df(x) = \sin(f(x))$$

Интеграл — неопределен интеграл +

$F(x)$ — ~~примитив~~ ^{на} ф-я $f(x)$, ако
 ~~$F' = f$~~ $F' = f$

неопределен интеграл — совокупность от
 примитивов функции (функций)

определен интеграл — число

$$\operatorname{arcsin} x$$

$$\operatorname{arcsin} x + \operatorname{arccos} x = \frac{\pi}{2}$$

$$-\operatorname{arccos} x = \operatorname{arcsin} x - \frac{\pi}{2}$$

$$\operatorname{arctg} x + \operatorname{arctg} \frac{1}{x} = \frac{\pi}{2}$$

$$\frac{1}{2} \int \sin 2x dx = -\frac{1}{2} \cos 2x$$

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx+1}{(x+1)^2+1} = \operatorname{arctg}(x+1)$$

$$\int \frac{x dx}{(1+x^2)} = \frac{1}{2} \int \frac{dx^2+1}{1+x^2} = \ln \{1+x^2\} + C$$

$$\int \frac{x \arctg x^2}{1+x^4} dx = \int \frac{\arctg x^2}{1+(x^2)^2} dx^2 = \int \arctg x^2 \cdot \frac{d \arctg x^2}{x^2}$$

$$= \frac{1}{2} \arctg^2 x^2$$

$$\frac{1}{2} \int \frac{\sin 2x \cdot dx \cdot 2}{\sqrt{9 \sin^2 x + 16 \cos^2 x}} = \frac{1}{2} \int \frac{d \cos 2x}{\sqrt{\frac{9}{2}(1-\cos 2x) + \frac{16}{2}(1+\cos 2x)}} =$$

$$= \frac{2}{2 \cdot 7} \int \frac{d \cos 2x \cdot \frac{7}{2} + \frac{25}{2}}{\sqrt{\frac{25}{2} + \frac{7}{2} \cos 2x}} =$$

$$= -\frac{2}{7} \sqrt{\frac{25}{2} + \frac{7}{2} \cos 2x}$$

$$\int \frac{2 \sin x \cos x dx}{\sqrt{9 \sin^2 x + 16 \cos^2 x}} =$$

$$= \int \frac{2 \sin x \cdot d \sin x}{\sqrt{9 \sin^2 x + 16 \cos^2 x}} =$$

$$= \frac{2}{2} \int \frac{d \sin^2 x}{\sqrt{9 \sin^2 x + 16 \cos^2 x}} =$$

$$= \frac{1}{2} \int \frac{d(\sin^2 x + 16)}{\sqrt{16 - 7 \sin^2 x}} = \frac{2}{7} \sqrt{16 - 7 \sin^2 x} = \int \sin x d \sin x = \frac{1}{2} \int d \sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\begin{aligned} \sin \alpha \sin \beta &= \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \sin \alpha \cos \beta &= \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)] \end{aligned}$$

$$\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}} du =$$

$$= 2\sqrt{u}$$

$$\int \sin x \cos x dx =$$

$$\int \sin x d \sin x = \frac{1}{2} \int d \sin^2 x$$

$$\int \frac{dx}{\sqrt{x^2+x+1}} = \int \frac{dx}{\sqrt{(x+\frac{1}{2})^2 + \frac{3}{4}}} = \int \frac{dx+\frac{1}{2}}{\sqrt{(x+\frac{1}{2})^2 + \frac{3}{4}}} = \ln(x+\frac{1}{2}) + \sqrt{x^2+x+1}$$

Преобразование (субституция) на Хорнер.

$$\int \frac{dx}{x\sqrt{x^2+1}} = \int \frac{dx}{x^2\sqrt{1+\frac{1}{x^2}}} = \int \frac{\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^2}}} = -\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

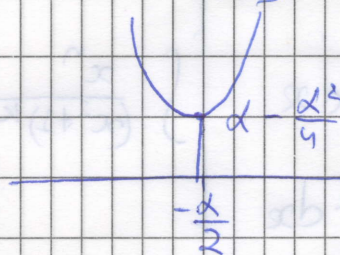
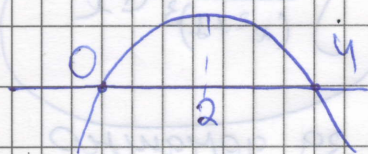
$$= -\ln\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$$

$$\int x \sin x dx = -\int x d\cos x = -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x$$

$$\int x e^x = \int x d e^x = x e^x - \int e^x dx = x e^x - e^x$$

22.1.2013. $\max_{x \in [0,4]} [\min_{x \in \mathbb{R}} (x^2 + dx + d)] = 1$



$$\frac{f'}{g'}$$

$$\frac{f''}{g''}$$

$$\frac{\sin x}{x}$$

1.2.109

$$\int f dg = fg - \int g df$$

$$\int R(x) f(x) dx =$$

~~R(x) f(x)~~

$f'(x)$ - алгебрична ф-я
 $f'(x)$ - трансцендентна

$$\frac{R(x) dx}{f(x) dx}$$

Загет	Интеграл - интегрирани	по загет
$\int \arctg x dx$	$\int x^n \sin x$	$\int e^{ax} \cos bx$
$\int x^n \arctg x dx$	$\int x^n \cos x$	когато и функцията да вие параме ни върши работата.
$\int x^n \arcsin x dx$	$\int x^n \lg x$	
$\int x^n \ln x dx$	$\int x^n e^x$	

~~Загет~~

Загет

$$\int \sqrt{x^2 - a^2} dx$$

$$\int \sqrt{a^2 - x^2} dx$$

$$\int \frac{x^n}{(x^2+1)^k} dx$$

($n=0,1,2$)
 $k \geq 3$
 за допълнително

$$\int \frac{x^n}{(x^2+1)^3} dx$$

за допълнително
 за четвъртък

20 задачник

$$\int \frac{x^n}{(x^2+1)^k} dx \begin{cases} \rightarrow n=0 \\ \rightarrow n=2k \\ \rightarrow n=2k+1 \end{cases}$$

$$\int \frac{1+x^2 - x^2}{x^{2k+1} (1+x^2)^k} dx = \int \frac{1}{x^{2k+1}} dx - \int \frac{x^2}{(1+x^2)^k} dx$$

$x^2 = t$

$$\int \sin^n x \cdot \cos^m x dx$$

n, m - четные числа

$$\begin{aligned} \textcircled{1} \int x \arcsin x dx &= \frac{x^2}{2} \arcsin x - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx \\ &= \frac{x^2 \arcsin x}{2} - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx = \end{aligned}$$

$$= \frac{x^2 \arcsin x}{2} - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx$$

Выводим само x

$$\int \frac{x^2}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int x(1-x^2)^{-\frac{1}{2}} d(x^2+1) =$$

матри операция с
слож. зритель

$$\sqrt{1-x^2} = \frac{1-x^2}{\sqrt{1-x^2}}$$

$$= - \int x d\sqrt{1-x^2} = -x\sqrt{1-x^2} + \int \sqrt{1-x^2} dx =$$

$$= -x\sqrt{1-x^2} + \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x^2}{\sqrt{1-x^2}} dx$$

$\arcsin x$ -1

$$2 \int = -x \sqrt{1-x^2} + \arcsin x$$

$$\begin{aligned} \textcircled{2} \int \sqrt{x^2+3} dx &= x\sqrt{x^2+3} - \int x d\sqrt{x^2+3} = \\ &= x\sqrt{x^2+3} - \int \frac{x^3 dx}{\sqrt{x^2+3}} = x\sqrt{x^2+3} - \int \sqrt{x^2+3} dx + 3 \int \frac{dx}{\sqrt{x^2+3}} = \\ &\quad \left((\sqrt{x^2+3})' = \frac{x}{\sqrt{x^2+3}} \right) \end{aligned}$$

$$= x\sqrt{x^2+3} - \int + \ln |x + \sqrt{x^2+3}|$$

$$\left(\begin{array}{l} \int \sqrt{x^2-a^2} dx \\ \int \sqrt{a^2-x^2} dx \\ \int \sqrt{x^2+a^2} dx \end{array} \right) \quad \int \sqrt{1-x^2} dx \Leftrightarrow \int \frac{x^2 dx}{\sqrt{x^2-a^2}}$$

Субституция на Роркер

$$\textcircled{3} \int \sqrt{3x^2+2x+1} dx = \sqrt{3} \int \sqrt{x^2+2x\frac{1}{3}+\frac{1}{3}} - \frac{1}{3} + \frac{1}{3} dx =$$

матрицу
тогда
уважат

$$= \sqrt{3} \int \sqrt{\left(x+\frac{1}{3}\right)^2 + \frac{2}{9}} dx + \frac{1}{3}$$

$$\int \sqrt{t^2 + \frac{2}{9}} dt$$

$$\textcircled{4} \int \sqrt{-2x^2+3x+5} dx = \sqrt{2} \int \sqrt{-x^2+2x \cdot \frac{3}{4} + \frac{9}{16} + \frac{9}{16} + \frac{5}{2}} dx =$$

$$= \sqrt{2} \int \sqrt{\frac{49}{16} - (x - \frac{3}{4})^2} dx = \frac{7^2 \sqrt{2}}{8^2} \int \sqrt{1 - (\frac{4x-3}{7})^2} d(\frac{4x-3}{7})$$

$$\int \sqrt{1-t^2} dt$$

$$\int \frac{dx}{(ax+b)^a} \quad \int \frac{dx}{ax^2+bx+c} \Rightarrow \int \frac{dt}{d^2t^2+e} \Rightarrow \int \frac{du}{u^2+1}$$

$$\int \frac{P_n(x)}{(ax^2+bx+c)^k} \Rightarrow \sum \int \frac{t^s}{(t^2+1)^e}$$

$$\int \frac{x^3}{(1+x^2)^5} = \frac{1}{2} \int \frac{x^2 dx^2}{(1+x^2)^5} = \frac{1}{2} \int \frac{t+1-1}{(1+t)^5} dt = \frac{1}{2} \int \frac{dt}{(1+t)^4} - \frac{1}{2} \int \frac{dt}{(1+t)^5}$$

$$P(x) = \sum \frac{(x+1)^k}{k!} \cdot P^k(-1)$$

$$\int \frac{x^5}{(x+1)^3}$$

$$x^5 = -1 + \frac{(x+1)}{1!} + 5 + \frac{(x+1)^2}{2!} \cdot 20 + \frac{(x+1)^3}{3!} \cdot 60 - \frac{(x+1)^4}{4!} \cdot 120 + \frac{(x+1)^5}{5!} \cdot 120$$

$$\int \frac{x^6}{(1+x^2)^3} = \frac{1}{2} \int x^5 (1+x^2)^{-3} dx^2 + 1 = \frac{1}{2} \int x^5 d(1+x^2)^{-2} =$$

$$= \frac{x^5}{2(1+x^2)^2} - \frac{1 \cdot 5}{2} \int \frac{x^4 dx}{(1+x^2)^2}$$

$$\int \frac{x^4 dx}{(1+x^2)^2} = \frac{1}{2} \int x^3 (1+x^2)^{-2} dx^{2+1} = \frac{1}{2} \int x^3 d \frac{1}{1+x^2} =$$

$$= -\frac{x^3}{(1+x^2) \cdot 2} + \frac{1}{2} \int \frac{x^4 dx}{1+x^2} = -\frac{x^3}{2(1+x^2)} + \frac{3}{2} \int \frac{x^2 dx}{1+x^2} =$$

$$\int \frac{1+x^2-x^2 dx}{(1+x^2)^4} =$$

$$\int \sqrt{ax^2+bx+c} = \int$$

$b \neq 0$ за домашно
 $a \neq c \neq 1$ с произволни
 a, b, c
 $a \neq 0$

$$\int x \arctg x dx$$

за домашно

$$\int \frac{P(x)}{Q(x)} dx =$$

Стъпки:

1) Ако числ. е от степен k , а знаменателя l
 $k < l$

2) Разлагаме знаменателя на неразложими множители

$(x-a)^k (cx^2+bx+c)^l$ - от втора степен

1.2.14

3) Разложение на элементарные функции

24.1.2013 г. $F(x) = \int f(x) dx$
 $x = \varphi(t) \quad t = \psi(x)$
 $x \in [a, b] \quad t \in [\alpha, \beta]$

$$dx = \varphi'(t) dt$$

$$G(t) = \int f(\varphi(t)) \varphi'(t) dt$$

$$F(x) = G(\psi(x)) + C$$

$\sqrt{a^2 - x^2}$	$x = a \sin t$	$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$	$\operatorname{sh} x = \frac{e^x - e^{-x}}{2} = t$
$\sqrt{x^2 - a^2}$	$x = a \operatorname{arcsin} t$	$\operatorname{gh} x = \frac{-e^x - e^{-x}}{2}$	$t = \ln x + \sqrt{x^2 + 1} $
$\sqrt{x^2 + a^2}$	$x = a \operatorname{sh} t$	$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$	

$$\int \sqrt{1-x^2} dx = \frac{x\sqrt{1-x^2} + \operatorname{arcsin} x}{2}$$

$x = \sin t$
 $-1 \leq x \leq 1$
 $t = \operatorname{arcsin} x$
 $[-\frac{\pi}{2}; \frac{\pi}{2}]$

$$\int \frac{x^2}{\sqrt{1+x^2}}$$

$$\int \sqrt{1-x^2} dx = \int |\cos t| \cdot \cos t dt = \int \cos^2 t dt =$$

$$= \int \frac{1 + \cos 2t}{2} dt = \frac{1}{2} t + \frac{1}{4} \sin 2t =$$

$$= \frac{1}{2} \operatorname{arcsin} x + \frac{1}{2} \sin 2 \operatorname{arcsin} x$$

$$\int \cos^2 t dt = \int \cos t d \sin t = \sin t \cdot \cos t - \int \sin t d \cos t =$$

$$= \sin t \cdot \cos t + \int (1 - \cos^2 t) dt = \sin t \cdot \cos t + t - \int$$

1.2.195

$$\int \frac{dx}{\sqrt{1+x^2}} = \int \frac{\cosh t dt}{\underbrace{\sqrt{1+\sinh^2(t)}}_{\cosh t}} = \int dt = t = \ln|x + \sqrt{x^2+1}|$$

$$x = \sinh(t)$$

Рационализация дробей

$$\int R(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{p_1}{q_1}}, \left(\frac{ax+b}{cx+d}\right)^{\frac{p_2}{q_2}} \dots) dx$$

$$\frac{ax+b}{cx+d} = t^k$$

k — НОК на $q_1, q_2, \dots, q_n$

$$\int \sqrt{1-x^2} dx = \int \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot (1-x) dx$$

$$\text{non: } \frac{1+x}{1-x} = t^2$$

$$x =$$

$$dx =$$

$$1-x^2 =$$

$$t^2 - t^2 x = 1+x$$

$$(t^2+1)x = \cancel{(t^2-1)} (t^2-1)$$

$$x = \frac{t^2-1}{t^2+1}$$

$$1-x = \frac{t^2+1 - t^2+1}{t^2+1} = \frac{2}{t^2+1}$$

$$dx = \frac{2t}{(t^2+1)^2}$$

$$\int t \cdot \frac{1+x}{t^2} \cdot dx = \int t \cdot \frac{\frac{2t^2}{t^2+1}}{t^2} \cdot \frac{2}{(t^2+1)^2} dt = 8 \int \frac{t^3}{(1+t^2)^3} dt$$

$$\int x^n (ax^m + b)^p$$

1сл.) p - цяло число m и n - не са цели числа
 свехеда се до предимката

2сл.) $\frac{n+1}{m} + 1$ $ax + b = t^k$ k - знаменател на p
 m - външното

k - знаменател на p p - не е цяло число

3сл.) $\frac{n+1}{m} + p$ x^m извехеда се пред скоби x^m
 и се свехеда до горния

$$\int \sqrt{1-x^2} dx = \int x^0 (1-x^2)^{\frac{1}{2}} = \int x (x^{-2} - 1)^{\frac{1}{2}} dx =$$

$$\frac{0+1}{2} + \frac{1}{2} = 1$$

$$\frac{1+1}{-2}$$

$$x^2 - 1 = t^2$$

$$x = (1+t^2)^{-\frac{1}{2}}$$

$$x' = dx = -\frac{1}{2} (1+t^2)^{-\frac{3}{2}} \cdot 2t \cdot dt$$

$$= -\int \frac{1 \cdot t}{(1+t^2)^{\frac{1}{2}} (1+t^2)^{\frac{3}{2}}} dt = -\int \frac{t^2}{(1+t^2)^2} dt$$

$$\int R(\sin x, \cos x) dx$$

$$R(-\sin x, -\cos x) = R(\sin x, \cos x)$$

$$\operatorname{tg} x = t$$

$$\sin x = \frac{t}{\sqrt{1+t^2}}$$

$$\cos x = \frac{1}{\sqrt{1+t^2}}$$

$$x = \arctan t$$

$$x = \arctan t$$

$$\operatorname{tg} \frac{x}{2} = t$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2} \quad x = 2 \arctan t$$

$$\int \frac{dx}{\sin x (2 + \cos x - 2 \sin x)} = \int \frac{(1+t^2)^2 (1+t^2) \cdot 2}{2t(t^2-4t+3)(1+t^2)} = \int \frac{1+t^2}{t(t-1)(t-3)}$$

$$2 + \cos x - 2 \sin x = 2 + \frac{1-t^2}{1+t^2} - \frac{4t}{1+t^2} = \frac{2+t^2+1-t^2-4t}{1+t^2} =$$

$$= \frac{t^2-4t+3}{1+t^2} = \frac{(t^2-1)(t-3)}{1+t^2}$$

$$\frac{1+t^2}{t(t-1)(t-3)} = \frac{A}{t} + \frac{B}{t-1} + \frac{C}{t-3}$$

$$\int \frac{dx}{2 + \cos^2 x}$$

Integralin ka O'liner

$$\int R(x, \sqrt{ax^2+bx+c}) dx$$

$$\sqrt{ax^2+bx+c} = \begin{cases} \pm \sqrt{a}x \pm t & a > 0 \\ \pm xt \pm \sqrt{c} & c > 0 \\ (x-x_0)t & D > 0 \end{cases}$$

$$\sqrt{x^2+x+1} = -x+t$$

$$\sqrt{-x^2-x-1}$$

$$\int \sqrt{1-x^2} dx =$$

$$\sqrt{1-x^2} = (1-x)t$$

$$\sqrt{1-x^2} = xt+1$$

$$x = \frac{-2t}{t^2+1} \quad t =$$

$$xt+1 = \frac{-t^2+1}{t^2+1}$$

$$\sqrt{1-x^2} = \frac{-t^2+1}{t^2+1}$$

$$dx =$$

1.2.118

$$x = \frac{\sqrt{1-x^2} - 1}{t}$$

$$t = \frac{\sqrt{1-x^2} - 1}{x}$$

$$\sqrt{1-x^2} = xt + 1$$

$$dx = \frac{\frac{1}{2}(\sqrt{1-x^2})^{-1/2} \cdot (-x) - (\sqrt{1-x^2} - 1)}{t^2} dt$$

$$\sqrt{1-x^2} = \frac{1}{2\sqrt{1-x^2}}$$

$$\int (xt + 1) \cdot \frac{\frac{1}{2}(\sqrt{1-x^2})^{-1/2} \cdot (-x) - (\sqrt{1-x^2} - 1)}{t^2} dt =$$

$$\int \sqrt{1-x^2} dx = 2 \int \frac{1-t^2}{t^2+1} \cdot \frac{t^2-1}{(t^2+1)^2} dt = 2 \int \frac{dt}{t^2+1} + 2 \int \frac{4t}{(t^2+1)^3} dt$$

$$\frac{d}{dx} = \text{norma еден и еден}$$

$$-(t^2+1)^2 + 4t^2 - 4t^2$$