I

Теория на игрите – 19.12.2013

Ако имаме k играчи

$J^1 = \{1, 2, \dots, s_1\}$ – чисти стратегии на 1ви играч

$\vdots$

$J^k = \{1, 2, \dots, s_k\}$ – чисти стратегии на k-ти играч

$\Pi_i(J_1, J_2, \dots, J_k)$, $i = 1, \dots, k$ – печалба в чисти стратегии

$J_1 \in J^1 \dots J_k \in J^k$

$$f_i(x_1, \dots, x_k) = \sum_{J_1=1}^{s_1} \sum_{J_2=1}^{s_2} \dots \sum_{J_k=1} x_1^{J_1} x_2^{J_2} \dots x_k^{J_k} \Pi_i(J_1, \dots, J_k), i = 1, \dots, k$$

вектор

фиксираме вектора x

$x = (x_1 \dots x_k)$

$\bar{x}_i = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_k)$

$(\bar{x}_i, x_i) = x$

$x_1 = (x_1^1, x_1^2, \dots, x_1^{s_1})$

$x_i^J \geq 0$, $J = 1, \dots, s$

$$\sum_{J=1}^{s_1} x_i^J = 1$$

$$f_i(x) = \sum_{J_i=1}^{s_i} x_i^J f(\bar{x}_i, J)$$

Равновесие по Неш е векторът

$x^* = (x_1^* \dots x_k^*)$, който съдържа

$f_i(\bar{x}_i^*, x_i) \leq f_i(x^*)$, $i = 1, \dots, k$

$f_i(\bar{x}_i^*, J_i) \leq f_i(x^*)$, $J = 1, \dots, s_i$, $i = 1, \dots, k$

чиста стратегия на i-тия играч

умножаваме неравенствата по x_i^J

$F: X_1 * X_2 * \ldots * X_k \to X_1 * X_2 \ldots * X_k$

$y = f(x)$

$$y_i^J = \frac{x_i^T + c_i^J}{1 + \sum_{\xi=1}^{S_i} c_i^\xi}$$

$$c_i^J = \max(0, f_i(\bar{x}_i, J) - f_i(x))$$

Има неподвижна точка

$$x^{*J} = \frac{x_i^{*T} + c_i^{*T}}{1 + \sum_{\xi=1}^{S_i} c_i^\xi} \qquad i = 1 \ldots k, \quad J = 1 \ldots S_i$$

$$c_i^J = \max(0, f_i(\bar{x}_i^*, J) - f_i(x^*))$$

1сл. $c_i^J = 0$, $J = 1 \ldots S_i$, $i = 1 \ldots k$ има равновесие по Неш

2сл. $c_{i_0}^{J_0} > 0$ $\quad f_{i_0}(\bar{x}^*_{i_0}, J_0) > f_{i_0}(x^*)$

$x_{i_0}^{*J} > 0 \to x_{i_0}^{*J} f_{i_0}(\bar{x}_{i_0}^*, J) > f_{i_0}(x^*) x_{i_0}^{*J}$ (като в случая с 2ма играчи)

~~Ако сумираме $f_{i_0}(x^*) > f_{i_0}(x^*)$~~

$u_\sigma, \sigma \in \Sigma$

$S \subset R^k \ni u^*$

$(S, u^*) \to \bar{u} \in R^k$

1. $\bar{u} \in S$, 2. $\bar{u} \geq x^*$

3. $\tilde{u}$ не същ. точка $\tilde{u} \in S, \tilde{u} \geq u$

4. $T \subset S$
$(S, u^*) \to \bar{u} \in T$
$(T, u^*) \to \bar{u}$

5. $u' = Lu$
$u'_i = \alpha_i u_i + \beta_i$
$i = 1 \ldots k$
$\alpha_i > 0$
$(S, u^*) \to \bar{u}$
$(L(S), L(u^*)) = L(\bar{u})$

6. симетрично (ако вземем пермутация на индексите)

$\pi: \{1, \ldots, k\} \to \{1, 2, \ldots, k\}$

$u \in S$, $\pi u \in S$

$(S, (u_1^*, \ldots, u_1^*)) = (\bar{v}, \ldots, \bar{v}_1)$

$\tilde{u} \in S, \tilde{u} > u^*$

$g(u) = \prod_{i=1}^{k} (u_i - u_i^*)$

$$\begin{cases} g(u) \to \max \\ u \in S \\ u \geq u^* \end{cases}$$

$g'(\bar{u}) = g(\bar{u}) \left(\frac{1}{\bar{u}_1 - u_1^*}, \frac{1}{\bar{u}_2 - u^*}, \dots, \frac{1}{\bar{u}_k - u_k^*} \right)$

$h(u) \leq h(\bar{u})$ – искаме да докажем, правим градиента и умножаване скаларно с u

$h(u) = \sum_{j=1}^{k} \frac{u_j}{\bar{u}_j - u_j^*}$

Допускане

$\hat{u} \in S \quad h(\hat{u}) > h(\bar{u})$

$\sum_{j=1}^{k} \frac{\hat{u}_j - \bar{u}_j}{\bar{u}_j - u_j^*}$

$u_\varepsilon = \bar{u} + \varepsilon(\hat{u} - \bar{u})$

$g(u_\varepsilon) = \prod_{i=1}^{k} [(\bar{u}_i - u_i^*) + \varepsilon(\hat{u}_i - \bar{u}_i)] = g(\bar{u}) + \varepsilon \sum_{i=1}^{k} \frac{g(\bar{u})}{\bar{u}_i - u_i^*} + O(\varepsilon^2)$

$= g(\bar{u}) + \varepsilon\, g(\bar{u})\, \underbrace{h(\hat{u} - \bar{u})}_{A} + O(\varepsilon^2) = g(\bar{u}) + \varepsilon \left(A + \frac{O(\varepsilon^2)}{\varepsilon} \right) > g(\bar{u})$

не може

Нека да погледнем това при 2ма играчи

$f(x,y) = x - x^2 - xy$

$g(x,y) = y - y^2 - xy$

$f(x,y) = x(1 - x - y)$

$x \in [0, \frac{1}{2}] \ni y$

$$\begin{cases} 2x + y = 1 \\ x + 2y = 1 \end{cases}$$

x	y	f	g
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$

Полагаме $\bar{x}=0 \to \max g_{/\bar{x}=0}$ $\bar{y}=\frac{1}{2} \Rightarrow f_{/\bar{y}=\frac{1}{2}} \to$ ma

Няма равновесие по Неш
$\bar{x}=\frac{1}{2} \to \max g_{/\bar{x}=\frac{1}{2}} \to \bar{y}=\frac{1}{2}$
$\to f_{/\bar{y}=\frac{1}{4}} \to \max$

това 1вия лист

$x=\frac{1}{2}(1-y)$

$y=\frac{1}{2}(1-x)$

$x=a(1-y),\ a\in(0,1)$

$y=b(1-x),\ b\in(0,1)$

$x=\frac{a(1-b)}{1-ab}\quad y=\frac{b(1-a)}{1-ab}$

$$f(x,y)=\frac{a(1-b)}{1-ab}-\frac{a^2(1-b)^2}{(1-ab)^2}-\frac{ab(1-b)(1-a)}{(1-ab)^2}=$$

$$=\frac{a(1-b)(1-ab)-a^2(1-b)^2-ab(1-b)(1-a)}{(1-ab)^2}=$$

$$=\frac{a(1-b)(1-\cancel{ab}-a+\cancel{ab}-b+ba)}{(1-ab)^2}=$$

$$=\frac{a(1-b)(-a-b+ba+1)}{(1-ab)^2}=\frac{a(1-a)(1-b)^2}{(1-ab)^2}=\varphi(a,b)$$

$$\psi(a,b)=\frac{b(1-b)(1-a)^2}{(1-ab)^2}$$

$$h(z)=\frac{z(1-z)}{(1-zb)^2}$$

…ъзинане интервала $(0,1)$ защото е смислено
търсим глобалния максимум

$$h'(z) = \frac{z - z^2}{1-2zb+b^2} = \frac{(1-2z)(1-2zb+b^2) - (z-z^2)(-2b)}{(1-2zb+b^2)^2}$$

$$= \frac{1-2z-\cancel{2zb}+4z^2b+b^2-2zb^2+\cancel{2zb}-2bz^2}{(1-2zb+b^2)^2} = \frac{b^2-2zb^2+2zb+1}{(1-2zb+b^2)^2} =$$

$$h'(z) = \frac{1-z(2-b)}{(1-bz)^3}$$

Анулира се $\bar{z} = \frac{1}{2-b} \in (0,1)$

$$h(\bar{z}) = \frac{1}{4(1-b)} > 0$$

α	β	φ	ψ
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{9}$	
$\frac{4}{5}$	$\frac{3}{4}$	...	

$b_n = 1 - \frac{1}{2n}$ $\quad a_n = 1 - \frac{1}{2n+1}$

Заместване в

$$x = \frac{a(1-b)}{1-ab} \qquad y = \frac{b(1-a)}{1-ab}$$

$\frac{n}{n+1}$ $\quad$ $\frac{n+1}{n+2}$

и после в f и g

$$x = \frac{\left(1-\frac{1}{2n+1}\right)\left(1-1+\frac{1}{2n}\right)}{1-\left(1-\frac{1}{2n}\right)\left(1-\frac{1}{2n+1}\right)} =$$

$$1-\left(1-\frac{1}{2n}\right)\left(1-\frac{1}{2n+1}\right)$$

$$\cancel{1-1}+\frac{1}{2n+1}+\frac{1}{2n}-\frac{1}{2n(2n+1)}$$

$$\frac{1}{2n}+\frac{1}{2n+1}-\frac{1}{2n(2n+1)} = \frac{2n+1+2n-1}{2n(2n+1)} = \frac{2}{2n+1}$$

$$x_n = \frac{\frac{1}{2n}\left(1-\frac{1}{2n+1}\right)}{\frac{2}{2n+1}} = \frac{2n}{2n(2n+1)} \cdot \frac{(2n+1)}{2} = \frac{1}{2}$$

$$y = \frac{\frac{1}{2n+1}\left(1-\frac{1}{2n}\right)}{} \cdot \frac{2n+1}{2} = \frac{2n-1}{2n(2n+1)} \cdot \frac{2n+1}{2} = \frac{2n-1}{4n} \to \frac{1}{2}$$