Теория на игрите – 05.12.2013г.

Зад:

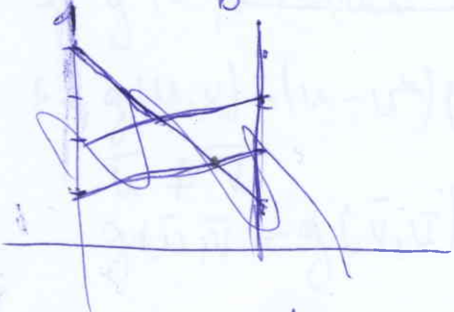
	B	
A	5/1	0/2
	3/3	3/4
	0/4	5/1

$$= \overset{A}{\begin{pmatrix} \underline{5} & 0 \\ 3 & 3 \\ 0 & \underline{5} \end{pmatrix}} \overset{B}{\begin{pmatrix} 1 & \underline{2} \\ 3 & \underline{4} \\ \underline{4} & 1 \end{pmatrix}}$$

изпит 04.02, 08.02, 12.02

08.02 – 09:00 – 13:00 писмен

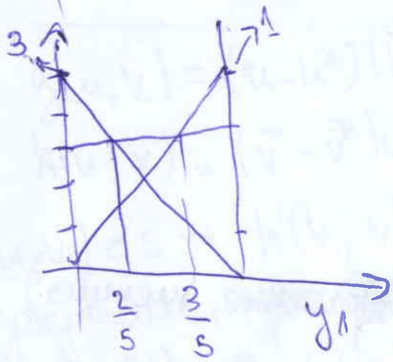
12.02 – 10:00



учите са 2, гледаме A

x – 3

$P(2,\bar{y}) > P(1,\bar{y})$ $i=1,3$



$P(x,\bar{y}) \le P(\bar{x},\bar{y}) = V$

$\bar{x} = (\bar{x}_1, \bar{x}_2, \bar{x}_3)$

Ако $P(1,\bar{y}) < V$ $\quad \bar{x}_1 = 0$

$\bar{x}_1 = 1, \quad \bar{x}_3 = 0$

$\bar{y}_1 = \frac{2}{5}, \quad \bar{y}_2 = \frac{3}{5}$

$$\begin{array}{c} \bar{x}_2 \\ \bar{x}_3 \end{array}\begin{pmatrix} 3 & 4 \\ 4 & 1 \end{pmatrix} \qquad \left| \begin{array}{l} 3\bar{x}_2 + 4\bar{x}_3 = V_2 \\ 4\bar{x}_2 + \bar{x}_3 = V_2 \\ \bar{x}_2 + \bar{x}_3 = 1 \end{array} \right.$$

$-x_2 + 3x_3 = 0$

$x_3 - 1 + 3x_3 = 0$

$x_3 = \frac{1}{4}$

$$\left| \begin{array}{l} \bar{x}_1 = 0 \\ \bar{x}_2 = \frac{3}{4} \\ \bar{x}_3 = \frac{1}{4} \end{array} \right.$$

$V_2 = \frac{13}{4}$

$$\begin{array}{c|cc|c} \bar{x}_1 & 1 & 2 & 1 \\ \bar{x}_2 & 3 & 4 & 3 \\ \hline & 3 & 4 & \end{array}$$

$$\begin{cases} \bar{x}_1 + 3\bar{x}_2 = v_2 \\ 2\bar{x}_1 + 4\bar{x}_2 = v \\ \bar{x}_1 + \bar{x}_2 = 1 \end{cases}$$

Сега не гледаме това в чисти стратегии, а таблица

5.) $(S, (u^*, v^*)) \to (\bar{u}, \bar{v})$

1. $\bar{u} \geq u^*, \bar{v} \geq v^*$
2. $(\bar{u}, \bar{v}) \in S$
3. $(\tilde{u}, \tilde{v}) \in S$

   $\tilde{u} > \bar{u}, \tilde{v} > \bar{v}$

4. $T \subset S$ T-затворено изпъкнало

   $(S, (u^*, v^*)) \to (\bar{u}^*, \bar{v}) \in T$

   $(T, (u^*, v^*)) \to (\bar{u}, \bar{v})$

5. $u' = \alpha_1 u + \beta_1 \quad \alpha_1 > 0$ (линейно изображение, функция)

   $v' = \alpha_2 v + \beta_2 \quad \alpha_2 > 0$

   $(L(S), L(u^*, v^*)) \leftarrow (S, (u^*, v^*)) \to (\bar{u}, \bar{v})$ (ако това е изпълнено)

   $\Downarrow$ (следва)

   $L(\bar{u}, \bar{v})$

6. $\begin{cases} u^* = v^* \\ (u, v) \in S \to (v, u) \in S \end{cases}$

   $\to (S, (u^*, u^*)) \to (\bar{u}, \bar{u})$

$\tilde{u} > u^*, \tilde{v} > v^*$

$$\begin{cases} g(u,v) \to max \\ u \geq u^* \\ v \geq v^* \\ (u,v) \in S \end{cases} \qquad g(u,v) = (u - u^*)(v - v^*)$$

Тази задача има единствено решение $(\bar{u}, \bar{v})$

1., 2., 3. – очевидно изпълнени

4.

5. $g'(u', v') = (u' - u^{*\prime})(v' - v^{*\prime}) = \alpha_1 \alpha_2 (u - u^*)(v - v^*) = \alpha_1 \alpha_2 g(u,v)$

6. $g(u,v) = (u - u^*)(v - v^*) = g(v,u)$

$\bar{u} \neq \bar{v}$

$g(\bar{u}, \bar{v}) = g(\bar{v}, \bar{u})$

$g(u,v) = (u - u^*)(v - v^*)$

$h(u,v) = (\bar{v} - v^*)u + (\bar{u} - u^*)v$

заместване

$(u,v) \in S \Rightarrow h(u,v) \leq h(\bar{u}, \bar{v})$

допускаме противното, а именно

$(\tilde{u}, \tilde{v}) \in S : h(\tilde{u}, \tilde{v}) > h(\bar{u}, \bar{v}) \Rightarrow h(\tilde{u} - \bar{u}, \tilde{v} - \bar{v}) > 0$

$(\bar{v} - v^*)(\tilde{u} - \bar{u}) + (\bar{u} - u^*)(\tilde{v} - \bar{v}) > 0$

$g((\bar{u}, \bar{v}) + \alpha(\tilde{u} - \bar{u}, \tilde{v} - \bar{v}))$

$\overset{x_0}{} \xrightarrow{x_0 + \alpha d} d$

$f(x_0 + \alpha d) = f(x_0) + \alpha f'(x_0) d + O(\alpha) =$

$= \alpha \left(f'(x_0) d + \frac{d\alpha}{\alpha} \right)$

$(\bar{u}, \bar{v})$ – точката x_0

$(\tilde{u} - \bar{u}, \tilde{v} - \bar{v})$ – направление d

$g((\bar{u}, \bar{v}) + \alpha(\tilde{u} - \bar{u}, \tilde{v} - \bar{v})) = (\bar{u} + \alpha(\tilde{u} - \bar{u}) - u^*)(\bar{v} + \alpha(\tilde{v} - \bar{v}) - v^*) =$

$= (\bar{u} - u^* + \alpha(\tilde{u} - \bar{u}))(\bar{v} - v^* + \alpha(\tilde{v} - \bar{v})) =$

$= g(\bar{u}, \bar{v}) + \alpha . h(\tilde{u} - \bar{u}, \tilde{v} - \bar{v}) + \alpha^2 (\tilde{u} - \bar{u})(\tilde{v} - \bar{v}) =$

$= g(\bar{u}, \bar{v}) + \alpha [h(\tilde{u} - \bar{u}, \tilde{v} - \bar{v}) + \alpha(\tilde{u} - \bar{u})(\tilde{v} - \bar{v})]$ $\quad A + \alpha B > 0$

$$\left.\begin{array}{l} g(u,v)\to\max \\ u\ge u^*,\ v\ge v^* \\ (u,v)\in S\end{array}\right\} \to (\bar u,\bar v)$$

Цел: да докажем, че $(\bar u,\bar v)$ изпълнява 6-те свойства

$$S\subseteq H^- = \{(u,v): h(u,v)\le h(\bar u,\bar v)\}$$

$$(\bar u-u^*)v + (\bar v - v^*)u \le (\bar u-u^*)\bar v + (\bar v - v^*)\bar u$$

$$(\bar u-u^*)(v-v^*) + (\bar u-u^*)v^* + (\bar v-v^*)(u-u^*) + (\bar v - v^*)u^* \le$$

$$(\bar u-u^*)\bar v + (\bar v - v^*)\bar u$$

$$(\bar u-u^*)(v-v^*)+(\bar v-v^*)(u-u^*)\le 2(\bar u-u^*)(\bar v-v^*)$$

$$\frac{u-u^*}{\bar u-u^*}+\frac{v-v^*}{\bar v-v^*}\le 2$$

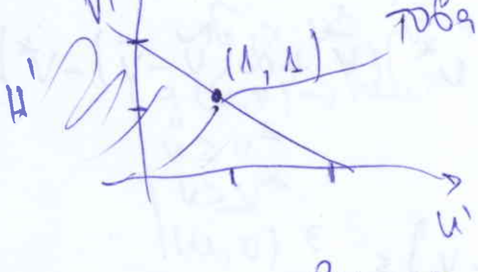
$L:\ u'=\dfrac{u-u^*}{\bar u-u^*}\quad v'=\dfrac{v-v^*}{\bar v-v^*}$ – дефинира се лин. изобр. (*)

коеф: ——

$$L(H^-)=\{(u',v'): u'+v'\le 2\}$$

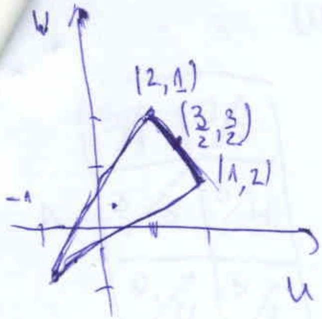
$$L(u^*,v^*)=(0,0)$$

$$L(\bar u,\bar v)=(1,1)$$



това е единствената точка, която удовлетворява $L(S)$ (*) ако е деф. по начин (*)

$\Rightarrow$ единствената точка, която удовлетворява всички 6 свойства, е $(\bar u,\bar v)$



$(\overset{*}{u}, \overset{*}{v}) = \left(\frac{1}{5}, \frac{1}{5}\right)$

$(\bar{u}, \bar{v}) = \left(\frac{3}{2}, \frac{3}{2}\right)$ – ако видим, че множеството е симетрично

$g = \left(u - \frac{1}{5}\right)\left(v - \frac{1}{5}\right) = uv - \frac{u}{5} - \frac{v}{5} + \frac{1}{25} = 0$

$\left(u - \frac{1}{5}\right) =$

$\left(u - \frac{1}{5}\right)\left(v - \frac{1}{5}\right) = 0$

$uv - \frac{u}{5} - \frac{v}{5} + \frac{1}{25} = 0$