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$\mu(s)$ – специф. скорост на растеж

$$\begin{cases} \dot{s}(t) = u(\overset{\text{влиза}}{s^{i}} - s(t)) - k\mu s(t) x(t) \\ \dot{x}(t) = (\mu(s(t)) - \alpha u) x(t) \\ \dot{p}(t) = \rho_n \mu(s(t)) x(t) - \beta u p(t) \end{cases}$$

P – продукт

u – управление, обратна връзка;

$$s(0) = s_0 > 0$$
$$x(0) = x_0 > 0$$
$$p(0) = p_0 > 0$$

$$u(t) = \frac{k\mu(s(t))x(t)}{s^{i} - \bar{s}}$$

$0 < \bar{s} < s^{i}$

равновесна точка

Пречистване

анаеробни

отделя се газ

x – не може да се мери в реално време

обем на газа, който се отделя

$$\boxed{y(t) = \lambda \mu(s(t))\, x(t)}$$

$0 < \lambda < 1$

знаем

$s(t)$

$s(t)$ се мери лесно

адаптивно у-е

$$\boxed{\begin{gathered}\dot{\gamma}(t) = -c\,(s(t) - \bar{s})(M - \gamma(t)) \\ (\gamma(t) - m)\, y(t) \\ 0 < \bar{s} < s^{i}\end{gathered}}$$

$\gamma(0) \in (m, M)$

$0 < m < M$

$$\boxed{u(t) = \gamma(t) \cdot y(t)}$$

отвора, с който пускаме нашето влияние

$$u(t) = \frac{k \mu s(t) x(t)}{s^{i} - \bar{s}}$$

не се мери

искаме само измерими и да стабилизираме процеса автономна с-ма - добавяме $\dot{\gamma}(t) = \ldots$

Заместваме и в с-мата

$$\begin{cases} \dot{s}(t) = \underline{\underline{y(t)}} \left[\gamma(t) \cdot (s' - s(t)) - \frac{k}{\lambda} \right] \\ \dot{x}(t) = y(t) \left[\frac{1}{\lambda} - \alpha \underline{\gamma(t)}\, x(t) \right] \\ \dot{p}(t) = y(t) \left[\frac{h}{\lambda} - \beta \gamma(t)\, p(t) \right] \\ \dot{\gamma}(t) = y(t) \cdot c\, (\bar{s} - s(t))(M - \gamma(t))(\gamma(t) - m) \end{cases}$$

има единствено реш.

$\gamma \neq 0$, защото там има реш, а то е единствено

$\gamma > 0$

$s, x, p \neq 0$ - съответства реш.

$\Rightarrow y(t) > 0$

$\mu(0) = 0$

$\mu(s) > 0 \rightarrow s > 0$

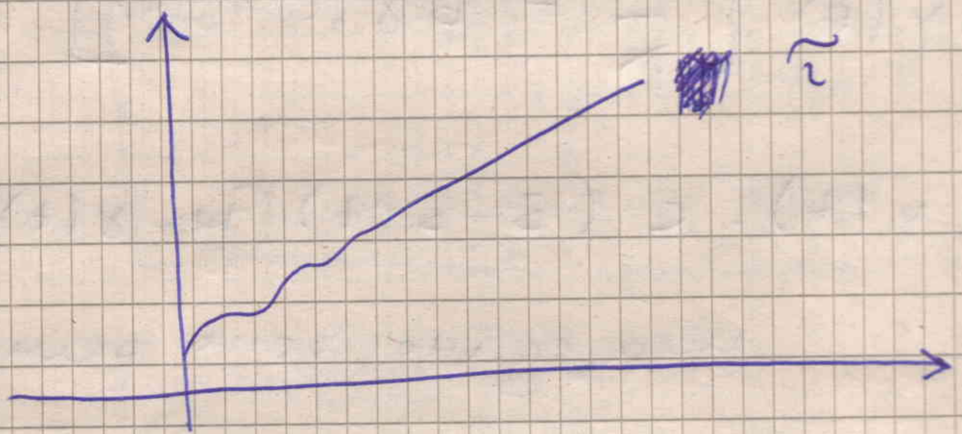
Сменяме скалата на времето

$$z(t) = \tau := \int_0^t y(s)\,ds$$

$$\tau > 0$$

$$z'(t) = y(t) > 0$$

$\Rightarrow z$ има обратна



$$\tau = z\left(z^{-1}(\tau)\right)$$

$$1 = z'\left(z^{-1}(\tilde{\tau})\right) \cdot \frac{d}{d\tilde{\tau}} z^{-1}(\tilde{\tau})$$

Взимаме s и правим смяна

$$\tilde{s}(\tilde{\tau}) = s\left(z^{-1}(\tilde{\tau})\right)$$

$$\frac{d}{d\tilde{\tau}} \tilde{s}(\tau) = \frac{d}{dt} s\left(z^{-1}(\tilde{\tau})\right) \cdot \frac{d}{d\tilde{\tau}} z^{-1}(\tilde{\tau})$$

$$= y(z^{-1}(\tilde{\tau})) \cdot \left[\gamma(z^{-1}(\tilde{\tau})) \cdot \left(S^{i} - S(z^{-1}(\tilde{\tau})) - \frac{k}{\lambda} \right] \cdot \frac{1}{y(z^{-1}(\tilde{\tau}))}$$

$$\Rightarrow \frac{d}{d\tilde{\tau}} z^{-1}(\tilde{\tau}) = \frac{1}{z'(z^{-1}(\tilde{\tau}))} =$$

$$= \frac{1}{y(z^{-1}(\tilde{t}))}$$

$$\Rightarrow \frac{d}{d\tilde{\tau}} \tilde{S}(\tilde{\tau}) = \left[\gamma(z^{-1}(\tilde{\tau})(S^{i} - \tilde{S}(\tilde{\tau})) - \frac{k}{\lambda} \right]$$

$\gamma \to \tilde{\gamma}$ изчезнало е $y(t)$

:))

$\tilde{x} \to x$ и т.н.

$$\tilde{x}(t) = x(z^{-1}(\tilde{\tau}))$$

$$\tilde{p}(t) = p(z^{-1}(\tilde{\tau}))$$

$$\tilde{\gamma}(\tilde{\tau}) = \gamma(z^{-1}(\tilde{\tau}))$$

$$\frac{d}{d\tilde{\tau}} \tilde{S}(\tilde{\tau}) = \tilde{S}'(\tilde{\tau}) = \tilde{\gamma}(\tilde{\tau})(S^{i} - \tilde{S}(\tilde{\tau}) - \frac{k}{\lambda}$$

$$\frac{d}{d\tilde\tau}\tilde s(\tilde\tau) = \tilde s'(\tilde\tau) = \tilde\gamma(\tilde\tau)\,(s' - \tilde s(\tilde\tau) - \frac{k}{\lambda}$$

$$\frac{d}{d\tilde\tau}\tilde x(\tilde\tau) = \frac{1}{\lambda} - \alpha\,\tilde\gamma(\tilde\tau)\,\tilde x(\tilde\tau)$$

$$\frac{d}{d\tilde\tau}\tilde p(\tilde\tau) = \tilde p'(\tilde\tau) = \frac{h}{\lambda} - \beta\,\tilde\gamma(\tilde\tau)\,\tilde p(\tilde\tau)$$

$$\frac{d}{d\tilde\tau}\tilde\gamma(\tilde\tau) = \tilde\gamma'(\tilde\tau) = c\,(\bar s - \tilde s(\tilde\tau))(M - \tilde\gamma(\tilde\tau))(\tilde\gamma(\tilde\tau) - m) \quad (*)$$

$$v(\tilde\tau) = s' - \tilde s(\tilde\tau) \quad (**)$$

$$v'(\tilde\tau) = -\tilde s'(\tilde\tau) = \frac{k}{\lambda} - \tilde\gamma(\tilde\tau)\,v(\tilde\tau)$$

$$\tilde\gamma'(\tilde\tau) = c\,(M - \tilde\gamma(\tilde\tau))(\tilde\gamma(\tilde\tau) - m)(v(\tilde\tau) - \bar v)$$

$$\bar s - \tilde s(\tilde\tau) = \bar s - s' + \overbrace{s' - \tilde s(\tilde\tau)}^{v(\tilde\tau)} = v(\tilde\tau) - \bar v$$

$$\bar v = \bar s - s'$$

→ получаване двумерна система

Принцип La Salle $\tilde{\gamma}$

$$\cancel{F(v,\tilde{\gamma}) = \int_{\bar{v}} \frac{\xi - \bar{v}}{\xi} d\xi + \int \eta\ \gamma}$$

!

$$F(v,\tilde{\gamma}) = \int_{\bar{v}}^{v} \frac{\xi - \bar{v}}{\xi} d\xi + \int_{\bar{\gamma}}^{\gamma} \frac{\eta - \bar{\gamma}}{c(H-\eta)(\eta - m)} d\eta$$

$$F'(v,\tilde{\gamma}) =$$

анулира ⊛

$\bar{\gamma} - \bar{S}$

$\bar{\gamma}$ - прави 0 производната по S

$$\bar{\gamma} = \frac{k}{\lambda(S' - \bar{S})} = \frac{k}{\lambda \bar{V}}$$

$$\bar{V} = S' - \bar{S}$$

$$\dot{F}^{*}(v,\tilde{\gamma})$$

градиент . дясната страна (**)

$$\dot{F}(v,\tilde{\gamma}) = \left(\frac{v-\bar{v}}{v}, \frac{\gamma - \bar{\gamma}}{c(H-\gamma)(\gamma - m)}\right) \cdot \begin{pmatrix} \gamma \\ \cdot \end{pmatrix}$$

$$\dot F(v,\gamma) = \frac{v-\bar v}{v}\cdot(\bar v\bar\gamma - v\gamma) +$$

$$+ \frac{\gamma-\bar\gamma}{c\cancel{(\mu-\gamma)}\cancel{(\gamma-m)}}\cdot \cancel{c(\mu-\gamma)(\gamma-m)}(v-\bar v)$$

$$\dot F(v,\gamma) = \frac{v-\bar v}{v}\left[\bar\gamma\bar v - \cancel{\gamma v} + \cancel{\gamma v} - \bar\gamma v\right] =$$

$$= -\frac{\bar\gamma(v-\bar v)^2}{v} \qquad \text{o.k. :))}$$

От Принципа на La Salle:

комп. на $s = \bar s$

$\overset{=0}{\dot S}$ само при $v = \bar v$

$$v'(\tilde\tau) = \bar v\bar\gamma - \tilde\gamma(\tau)v(\tau)$$

$v \to \bar v$

$\gamma \to \bar\gamma$:)

при $\gamma = \bar\gamma$

$$\tilde x'(\tilde\tau) = \frac{1}{\gamma} - 2\bar\gamma\tilde x(\tilde\tau)$$

$$\tilde p'(\tilde\tau) = \frac{b}{\gamma} - \beta\bar\gamma\tilde\beta(\tilde\tau)$$

$$\begin{cases} \tilde{x}'(\tilde\tau) = \dfrac{1}{\lambda} - \alpha\bar\gamma\,\tilde{x}(\tilde\tau) \\ \tilde{p}'(\tilde\tau) = \dfrac{1}{\lambda} - \beta\bar\gamma\, p'(\tilde\tau) \end{cases}$$

$$\cancel{x(\tilde\tau) = e^{\int_{t_0}^{t} \alpha\bar\gamma\, ds} \cdot x_0 + \int_0^{x_0} e}$$

$$\tilde{x}'(\tilde\tau) = -\alpha\bar\gamma\,\tilde{x}(\tilde\tau) + \frac{1}{\lambda}$$

$$\tilde{x}'(\tilde\tau) = \alpha\bar\gamma\left(\frac{1}{\lambda\alpha\bar\gamma} - \tilde{x}(\tilde\tau)\right) =$$

$$= -\alpha\bar\gamma\left(\tilde{x}(\tilde\tau) - c\right)$$

$$\frac{d}{d\tilde\tau}\left(\tilde{x}(\tilde\tau) - c\right)$$

$$\tilde{x}(\tilde\tau) - c = (x_0 - c)\cdot e^{-\alpha\bar\gamma}$$

$$x \xrightarrow[t \to \infty]{} c$$

$$c = \frac{1}{\lambda\alpha\bar\gamma}$$

$\cancel{=}$ с въпроси :))

? Как се учи за изпит ?

A_0 $\quad$ A_1 $\quad$ дни

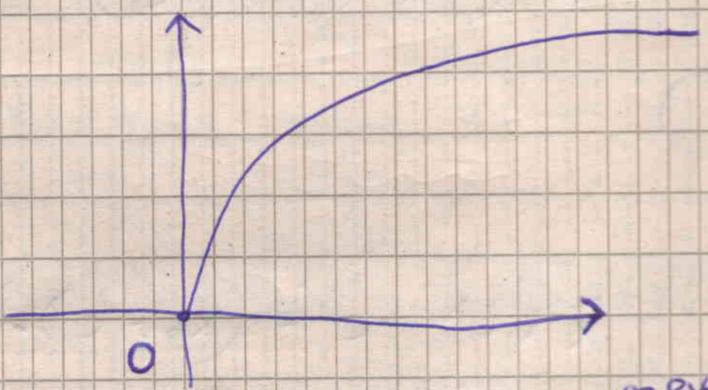
$$A_0, \; A_1, \; \dots, \; A_m = \bar{A}$$

A_1 - познанията 1 ден

$$A_1 = A_0 - \underbrace{\alpha A_0}_{\text{забравили сме}} + \beta Y_1^{1/2}$$

- времето, в което сме учили

$$f(y) = y^2 \quad \alpha \in (0,1)$$



$$A_2 = A_1 - \alpha A_1 + \beta Y_2^{1/2}$$

- ефективност на ученето

$\vdots$

$$A_n = A_{n-1} - \alpha A_{n-1} + \beta Y_n^{1/2}$$

$$Y > 0$$

искаме да min времето

$$\sum_{i=1}^{n} y_i \to \min$$

$$y_1 = \left(\frac{A_1 + \alpha A_0 - A_0}{\beta}\right)^2$$

$$y_2 = \left(\frac{A_2 + \alpha A_1 - A_1}{\beta}\right)^2$$

$$\vdots$$

$$y_n = \left(\frac{A_n + \alpha A_{n-1} - A_{n-1}}{\beta}\right)^2$$

$$A_1 + A_0(\alpha - 1)$$

$$y_1 + y_2 + \dots + y_n = \frac{1}{\beta^2}\left[\left(A_1 + \alpha A_0 - A_0\right)^2 + \dots + \left(A_n + \alpha A_{n-1} - A_{n-1}\right)^2\right]$$

$k = 0, 1, \dots, n-1$

$$A_{k+1} \overset{?}{=} \gamma A_k + \beta y_{k+1}^{1/2} \qquad \gamma = 1 - \alpha$$

знаем A_0 и A_n

делим γ^{k+1}

$$\frac{A_{k+1}}{\gamma^{k+1}} = \frac{A_k}{\gamma^k} + \frac{\beta}{\gamma^{k+1}} y_{k+1}^{1/2}$$ и събиране

$$C B_k = \frac{A_k}{\gamma_k}$$

$$\cancel{\Rightarrow B_{k+1} = B_k}$$

$$\Rightarrow C_{k+1} = C_k + \frac{B}{\gamma^{k+1}} \cdot y_{k+1}^{1/2}$$

сумираме

$$k = 0, 1, \dots, n-1$$

$$\underset{\substack{\uparrow \\ \text{знаем}}}{d} = C_m - C_0 = B \sum_{k=1}^{m} \frac{y_k^{1/2}}{\gamma^k}$$

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$a = (a_1, \dots, a_n)$
$b = (b_1, \dots, b_n)$

$$\langle a, b \rangle \leq \|a\| \cdot \|b\|$$

$$\left(\sum_{i=1}^{m} y_i\right)^{\frac{1}{2}} \geq \sum_{i=1}^{m} \frac{y_i^{1/2}}{\gamma^i} = F \quad , \gamma = \left(\sum_{i=1}^{n} \frac{1}{\gamma^{2i}}\right)^{\frac{1}{2}}$$

$\cancel{i = 1, \dots, n}$

$\langle a, b \rangle = \|a\| \cdot \|b\|$

$a_i = \tilde{\tau}\, b_i$

$b = \left(\frac{1}{\gamma}, \frac{1}{\gamma^2}, \dots, \frac{1}{\gamma_n} \right)$

$a = \left(y_1^{1/2}, \dots, y_n^{1/2} \right)$

$y_i^{1/2} = \frac{\tilde{\tau}}{\gamma_i} \Rightarrow y_i = \frac{\tilde{\tau}^2}{\gamma^2_i}$

$\sum_{i=1}^{n} \frac{\tilde{\tau}}{\gamma^2_i} = A$

$\tilde{\tau} = \frac{A}{\sum_{i=1}^{n} \frac{1}{\gamma^2_i}} \dots$

$A_{i+1} = \gamma A_i + \beta y_i^{1/2} \quad \gamma \in (0;1)$

$u(c, l)$ - ф-я на полезност

c - потребление на стоки

l - мързел :D

$u'_c > 0$

$u'_l > 0$

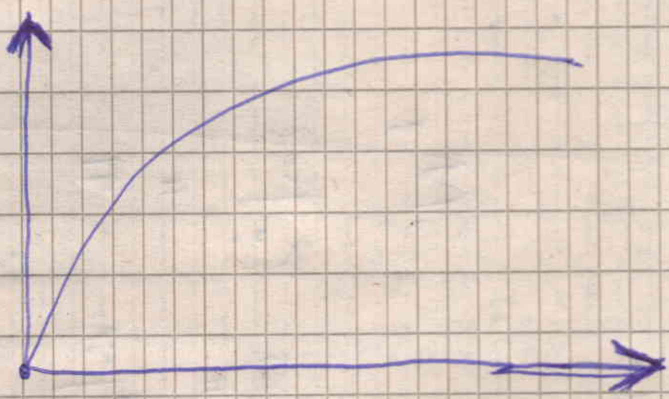
$u(c, l) \longrightarrow \max$ $\qquad$ $u(0, l) = 0$

$u(c, 0) = 0$

$$\lim_{c \to 0^+} u'_c (c, l) = \infty$$

$$\lim_{l \to 0^+} u'_l (c, l) = \infty$$

силно вдлъбната



под графиката - изпъкнало

$f: S \longrightarrow R \ , \ S \subseteq \mathbb{R}^n$

! S - изпъкнало ⊙

$x_1 \in S \ , \ x_2 \in S \ , \ \lambda \in [0, 1]$

$x_1 \neq x_2 \qquad \lambda \neq \{0, 1\}$

$$f(\lambda x_1 + (1-\lambda) x_2) > \lambda f(x_1) + (1-\lambda) f(x_2)$$

$\{(x, \tau) \in \mathbb{R}^{n+1} : \tau \leq f(x)\}$ подграфика — изпъкнало $\odot$ при f - вдлъбнатата

$(x_1, \tau_1) \in \odot \quad f(x_1) \geq \tau_1$

$(x_2, \tau_2) \quad f(x_2) \geq \tau_2$

$\Rightarrow \cancel{f(\alpha x_1 + (1-\alpha) x_2) \geq \alpha f(x_1) + (1-\alpha) f(x_2)}$

$\alpha(x_1, \tau_1) + (1-\alpha)(x_2, \tau_2) =$

$= (\alpha x_1 + (1-\alpha) x_2, \alpha \tau_1 + (1-\alpha)\tau_2)$

изпъкнало $\odot$

$f(\alpha x_1 + (1-\alpha) x_2) > \alpha f(x_1) + (1-\alpha) f(x_2)$

$\geq \alpha \tau_1 + (1-\alpha)\tau_2$

:))

$\{x \in \mathbb{R}^n : f(x) \geq \tau\}$

$x_1 \quad f(x_1) \geq \tau_1$

$x_2 \quad f(x_2) \geq \tau_2$

$f(\alpha x_1 + (1-\alpha) x_2) > \alpha f(x_1) + (1-\alpha) f(x_2)$

$\geq \alpha r + (1-\alpha) r$

$= r$

k - капитала

n - времена работа

$$\begin{cases} u(c,\ell) \to \max \\ 0 \le c \le \overset{\text{заплата}}{w}(1-\ell) + rk \\ \overset{\text{цяло време}}{1} = n + \ell \\ k \to k + \mu k = (1+\mu)k \\ 0 \le k \le k_0 \\ 0 \le \ell \le 1 \end{cases}$$

цена капитала

$k = k_0$ влагаме целия капитал за повече потребление

$\ell \neq 0$

$\ell \neq 1$

$c > 0$

$\Leftrightarrow$

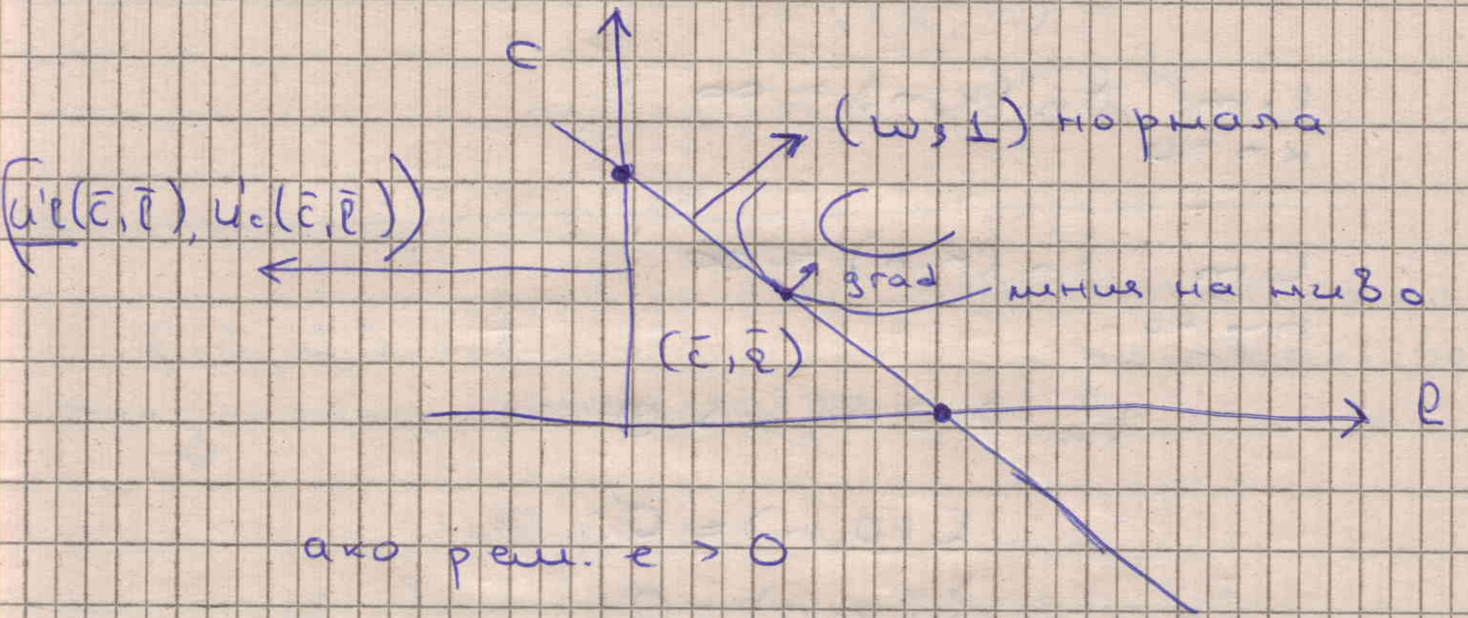
$$\begin{cases} u(c,\ell) \to \max \\ c \le w(1-\ell) + rk_0 \end{cases}$$

Б.О. $\begin{cases} c = w(1-\ell) + rk_0 \\ u(c,\ell) \to \max \end{cases}$

$$L(c,\ell,\lambda) = u(c,\ell) + \lambda(w(1-\ell) + rk_0 \overset{-c}{)}$$

$$\begin{cases} 0 = L_c = u'_c(\bar{c},\bar{\ell}) - \lambda \\ 0 = L_\ell = u'_\ell(\bar{c},\bar{\ell}) - \lambda w \\ c + w\ell = w + rk_0 \end{cases}$$

$u'_c(\bar{c}, \bar{\ell}) = \lambda$
$u'_\ell(\bar{c}, \bar{\ell}) = \lambda w$



ако реш. $\ell > 0$

$\{x \in \mathbb{R}^n : f(x) \geq \alpha\}$ - изпъкнало
α – произволно

$\{(c, \ell) : u(c, \ell) = u(\bar{c}, \bar{\ell})\}$
изпъкнало

$$\begin{cases} u'_c(\bar{c}, \bar{\ell}) = \lambda \\ u'_\ell(\bar{c}, \bar{\ell}) = \lambda w \end{cases} \Rightarrow w\, u'_c(\bar{c}, \bar{\ell}) = u'_\ell(\bar{c}, \bar{\ell})$$

$$w\, u'_c(\bar{c}, \bar{\ell}) = u'_\ell(\bar{c}, \bar{\ell})$$

капитал, труд

За фирма

$$f(k,n) - wn - rk \longrightarrow \max$$

заплати — цена на капитала

$$\lim_{k\to+0} f'_k(k,n) = \infty$$

$$\lim_{n\to+0} f'_n(k,n) = \infty$$

$$f'_k > 0 \quad f'_n > 0$$

$$f(0,n) = 0$$

$$f(k,0) = 0$$

$$f'_k(\bar{k},\bar{n}) = r$$

$$f'_n(\bar{k},\bar{n}) = w$$

и добавяме $\quad w u'_c(\bar{c},\bar{l}) = u'_l(\bar{c},\bar{l})$

$$f(\lambda k, \lambda n) = \lambda f(k,n) \quad \lambda > 0$$

хомогенно

напр. $f(k,n) = k^{\frac{1}{3}} n^{\frac{2}{3}}$

$$f(\lambda\bar{k}, \lambda\bar{n}) = \lambda f(\bar{k},\bar{n})$$

диф. по λ

$$f(\bar{k},\bar{n}) = f'_k(\lambda\bar{k},\lambda\bar{n})\bar{k} + f'_n(\lambda\bar{k},\lambda\bar{n})\bar{n}$$

$\lambda = 1$

$f'_k(\bar{k}, \bar{n}) \cdot \bar{k} + f'_n(\bar{k}, \bar{n}) \cdot \bar{n} =$

$= f(\bar{k}, \bar{n}) =$

$= r\bar{k} + \omega \bar{n} = f(\bar{k}, \bar{n})$

$f(\bar{k}, \bar{n}) = \bar{c}$ равенство на производство

предлагане консумация

$\bar{n} + \bar{\ell} = \underline{1}$

$\bar{k} = k_0$

1 $\omega u'_c(\bar{c}, \bar{\ell}) = u'_\ell(\bar{c}, \bar{\ell})$

2 $f'_k(\bar{k}, \bar{n}) = r$

3 $f'_n = (\bar{k}, \bar{n}) = \omega$

4 $f(\bar{k}, \bar{n}) = \bar{c}$

5 $\bar{n} + \bar{\ell} = 1$

6 $\bar{k} = k_0$

$\circledast$

3 1

$f'_n(k_0, 1-\bar{\ell})\, u'_c(f(k_0, 1-\ell)) = u'_\ell(f(k_0, 1-\ell), \ell)$

намираме ℓ

---

k_0 - капитал, наличен в обществото

Пример:

$$u(c, l) = 2c^{\frac{1}{2}} + l$$
$$f(k, n) = k^{\frac{1}{3}} n^{\frac{2}{3}}$$

$$f'_k(\bar{k}, \bar{n}) = n^{\frac{2}{3}} \cdot \frac{1}{3} k^{-\frac{2}{3}}$$

$$f'_n(\bar{k}, \bar{n}) = \frac{2}{3} k^{\frac{1}{3}} n^{-\frac{1}{3}}$$

$$u'_c = c^{-\frac{1}{2}} \qquad u'_l = 1$$

$$f(k_0, 1-l) = k_0^{\frac{1}{3}} \cdot (1-l)^{\frac{2}{3}}$$

$$\frac{2}{3} k_0^{\frac{1}{3}} \cdot (1-l)^{-\frac{1}{3}} \cdot c^{-\frac{1}{2}} \; ($$

$$\frac{2}{3} k_0^{\frac{1}{3}} \cdot (1-l)^{-\frac{1}{3}} \cdot k_0^{-\frac{1}{6}} (1-l)^{-\frac{1}{3}} = 1$$

$$k_0^{\frac{1}{6}} (1-l)^{-\frac{2}{3}} = \frac{3}{2}$$

$$\frac{3}{2(1-l)^{\frac{2}{3}}} = k_0^{\frac{1}{6}}$$

$$\frac{3}{2k_0^{\frac{1}{6}}} = (1-l)^{\frac{2}{3}}$$

$$\left(\frac{3}{2k_0^{\frac{1}{6}}}\right)^{\frac{3}{2}} = 1 - l$$

$$l = 1 - \left(\frac{3}{2k_0^{\frac{1}{6}}}\right)^{\frac{3}{2}}$$

Кога има реш.?

$$u(c,\ell) \longrightarrow \max$$
$$\angle$$
$$c = f(k_0, 1-\ell)$$
$$0 \le \ell \le 1$$

Допустими стойности да $\exists\ c, \ell$

непразно огр. зат. ⊙

ако $u(c,\ell) \to$ непрек. :))

1. има решение

$$u(f(k_0, 1-\ell), \ell) \longrightarrow \max$$
$$\ell \in (0,1)$$

Диференц.

$$-u'_c(f(k_0, 1-\bar{\ell}), \bar{\ell}) \cdot f'_n(k_0, 1-\ell)$$
$$+ u'_\ell(f(k_0, 1-\bar{\ell}), \bar{\ell}) = 0$$

? единствено

Доп., че има 2 решения ℓ_1 и ℓ_2

$$c_1 = f(k_0, 1-\ell_1)$$
$$c_2 = f(k_0, 1-\ell_2)$$

$$\Rightarrow u(c_1, \ell_1) = u(c_2, \ell_2) = \max u(c,\ell)$$

M

$\parallel$

$$\frac{c_1+c_2}{2} = \frac{1}{2}c_1 + \frac{1}{2}c_2 = \frac{1}{2}f(k_0, 1-\ell_1)$$
$$+ \frac{1}{2}f(k_0, 1-\ell_2) \quad \angle$$

силно вдлъбната

$$f\left(k_0, \frac{1}{2}(1-l_1)+\frac{1}{2}(1-l_2)\right) =$$

$$= f\left(k_0, 1-\left(\frac{1}{2}l_1+\frac{1}{2}l_2\right)\right)$$

$\Rightarrow \frac{c_1+c_2}{2}$ доп.

$$u\left(\frac{c_1+c_2}{2}, \frac{l_1+l_2}{2}\right) > \frac{1}{2}u(c_1, l_1) +$$

$$+ \frac{1}{2}u(c_2, l_2) = M$$

max

$\overset{\circ}{x}$

$\Rightarrow$ реш. е единствено

Държава → данъци

$u(c, l) \to \max$
c

държавата
но е
↑обложила

приход $\tilde{\tau}_w w n + \underbrace{r k_0}_{\text{капитал}} \; \tilde{\tau}_k > 0$

$\tilde{\tau}_w > 0$

д приход държава

$$\begin{cases} u(c, l) \to \max \\ c \le (1-\tau_w) w (1-l) + (1-\tau_k) r k_0 \end{cases}$$

$$f(k, n) - wn - rk \to \max$$

$$c + g = f(\bar{k}, \bar{n})$$

$$1 = \bar{n} + \bar{l}$$

$$\bar{k} = k_0$$

$$(1-\tau_w) w u'_c(\bar{c}, \bar{l}) = u'_l(\bar{c}, \bar{l})$$

$$(1-\tau_w) f'_n(k_0, 1-\bar{l}) u'_c\big(f(k_0, 1-\bar{l}) -$$

$$- \tau_w f'_n(k_0, 1-\bar{l})(1-\bar{l}) - \tau_k (f'_k(k_0, 1-\bar{l}) k_0, \bar{l}\big)$$

$= u'_l$ (те же аргумент)

21.01.2014г.

едномерна система
1 тема + пример (с д-во)
изтъкнала
без биопроцеси с добавяне (адаптивно) управление
Принцип La Salle - без д-во
биопроцеси с 2 популации (без)

Казино 2 игри

	I	II
1 игра	5% - 1000лв.	5% - 5000лв.
	95% - 0лв.	95% - -200лв.

$$E(R_1) = \sum_{i=1}^{k} p_i a_i$$

I игра
очакване
<u>случайна</u>
<u>величина</u>

p_i - вероятност
a_i - стойност

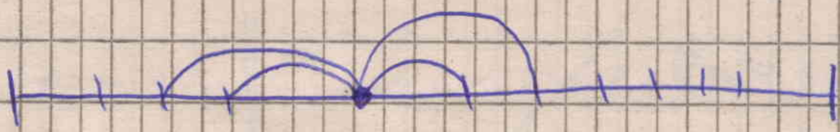
$$E(R_1) = \frac{5}{100} \cdot 1000 + \frac{95}{1000} \cdot 0 = 50$$

$$E(R_2) = 5000 \cdot \frac{5}{100} - \frac{95}{100} \cdot 200 =$$

$$= 250 - 190 = \cancel{440}\ 60$$

<u>Риск</u> - σ средно квадратично отклонение
дисперсия

$$\sigma^2 = E\left[(R - E(R))^2\right]$$



E(R) - изпъкнала комбинация

$$\sigma^2 = E_1\left[\left(R_1 - E(R_1)\right)^2\right]$$

$$E(R_1) = 50$$

$$\sigma_1^2 = \frac{5}{100}(1000-50)^2 + \frac{95}{100}(0-50)^2$$

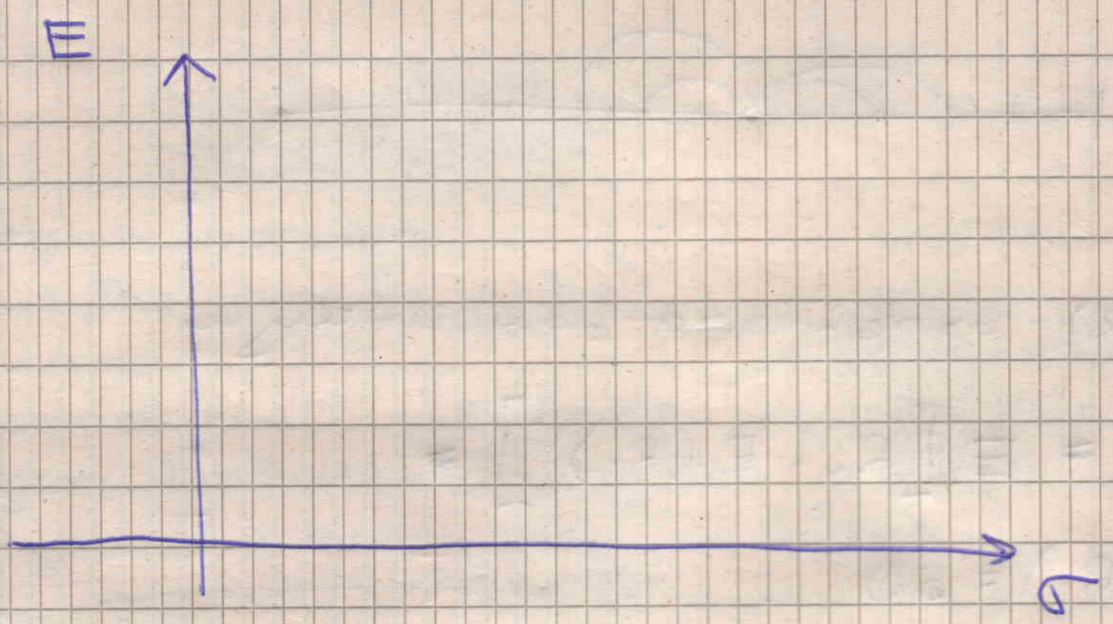
95 · 25 = 475, 190, = 2370

$$= \frac{5}{10\cancel{0}}(950)^2 + \frac{95}{1\cancel{00}} \cdot 25\cancel{00} =$$

$$= \frac{475 + 2370}{10} = \frac{2845}{10} = \cancel{284},5$$

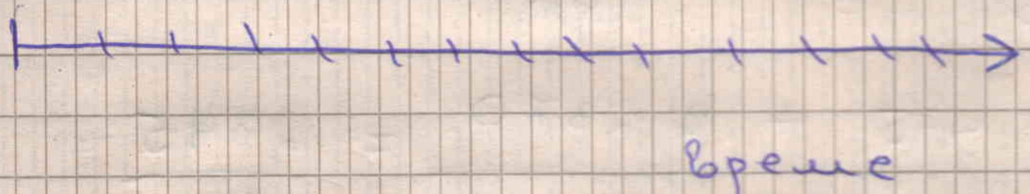
45125 + 2375

$$\sigma_1 = 212,48$$



Доходност на една акция

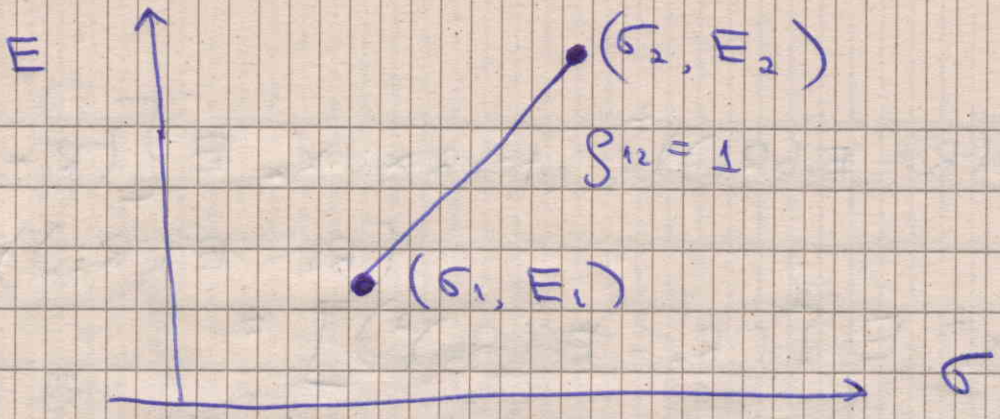
$$R = \frac{\text{продажна / покупна цена (в момента)} + \text{дивидент} - \text{покупна цена}}{\text{покупна цена}}$$



$$R \longrightarrow (\underbrace{\sigma_R}_{\text{риск}},\ \underbrace{E(R)}_{\text{очаквана доходност}})$$

средна ст-т → средно

$$\frac{R_1 + R_2 + \dots + R_k}{k}$$



само 2 акции

Портфейл от 2 акции - случайна величина

$$R = x_1 R_1 + x_2 R_2$$

$$x_1 + x_2 = 1 \qquad x_1 \geq 0$$
$$x_2 \geq 0$$

$x_1 = \frac{1}{3}$ $\quad \frac{1}{3}$ от парите в I акция

$\frac{2}{3} \to$ във II акция

$E(R) = x_1 E(R_1) + x_2 E(R_2)$

очакване

$$\sigma_R^2 = E\left[\left(x_1 R_1 + x_2 R_2 - x_1 E(R_1) - x_2 E(R_2)\right)^2\right]$$

$$= E\left[\left(x_1 (R_1 - E(R_1)) + x_2 (R_2 - E(R_2))\right)^2\right] =$$

$$= E\left\{x_1^2 (R_1 - E(R_1))^2 + 2 x_1 x_2 (R_1 - E(R_1))(R_2 - E(R_2)) + x_2^2 (R_2 - E(R_2))^2\right.$$

$$= x_1^2 E(R_1 - E(R_1))^2 + 2x_1 x_2 E\left[(R_1 - E(R_1))(R_2 - E(R_2))\right] +$$

$$+ x_2^2 E\left[(R_2 - E(R_2)^2)\right] =$$

$$= x_1^2 \sigma_1^2 + 2x_1 x_2 \sigma_1 \sigma_2 \rho_{12} + x_2^2 \sigma_2^2$$

$$\rho_{12} = \frac{E\{(R_1 - E(R_1)(R_2 - E(R_2))\}}{\sigma_1 \sigma_2}$$

<u>коефициент на корелация</u>

$\rho \in [-1, 1]$ (доказва се)

Ако $\rho_{1,2} = 1$

$\rightarrow \sigma_R = x_1 \sigma_1 + x_2 \sigma_2$

$$(\sigma_R, E(R)) = (x\sigma_1 + (1-x)\sigma_2, \; xE_1 + (1-x)E_2)$$

$x \in [0, 1]$

отсечка :))

$\rho_{1,2} = -1$

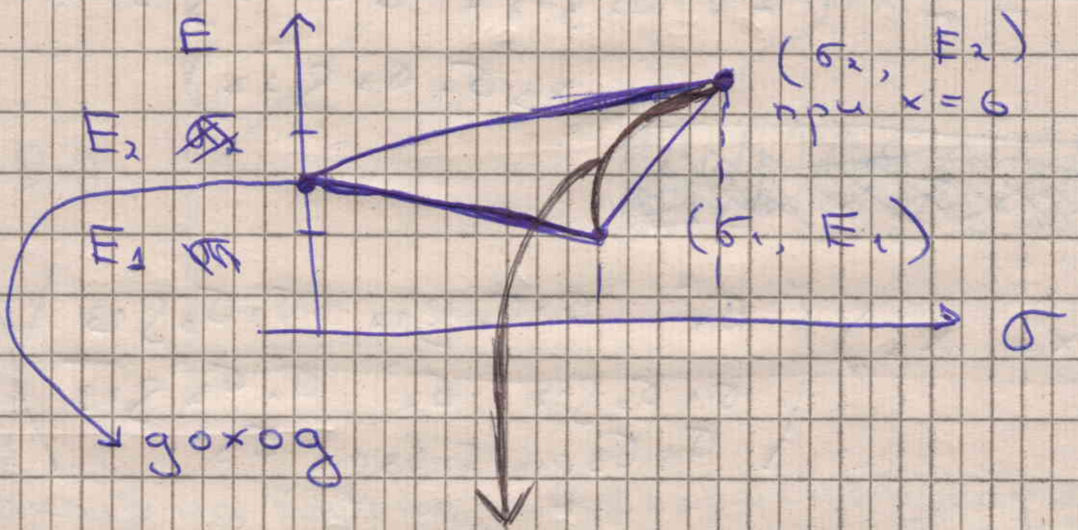
$$\sigma_R = |x\sigma_1 - |1-x|\sigma_2|$$

$$x\sigma_1 - (1-x)\sigma_2 = 0$$

$$x\sigma_1 - \sigma_2 + x\sigma_2 = 0$$

$$\overline{x} = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

! винаги може 0-ев риск



$-1 < \rho_{12} < 1$ строго

с k величини

$$R = \sum_{i=1}^{k} x_i R_i$$

$$x_i \geq 0$$

$$\sum_{i=1}^{k} x_i = 1$$

$2x_1x_2\rho_{12}\sigma_1\sigma_2 + 2x_1x_3\rho_{13}\sigma_1\sigma_3 + 2x_2x_3\rho_{23}\sigma_2\sigma_3$

$$E(R) = \sum_{i=1}^{k} x_i E(R_i)$$

$2x_1x_2(R_1 - E(R_1))(R_2 - E(R_2)) * \ldots$

$x_1(R_1 - E(R_1)) + x_2(R_2 - E(R_2)) + x_3(R_3 - E(R_3))$

$x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + x_3^2 \sigma_3^2 + 2$

с 3 величини:

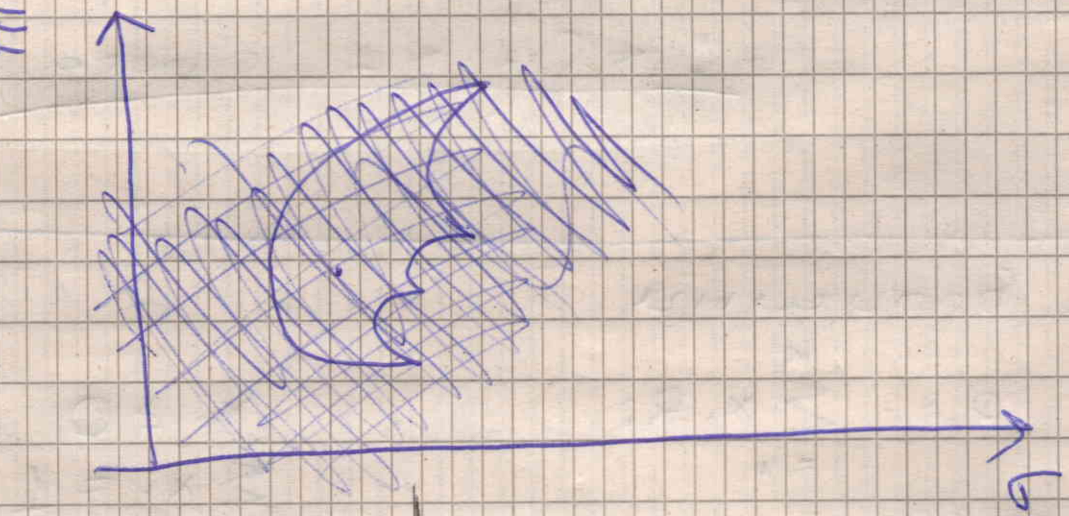
$$\sigma_R^2 = E\left[\left(x_1R_1 + x_2R_2 + x_3R_3 - x_1E(R_1) - x_2E(R_2) - x_3E(R_3)\right)^2\right]$$

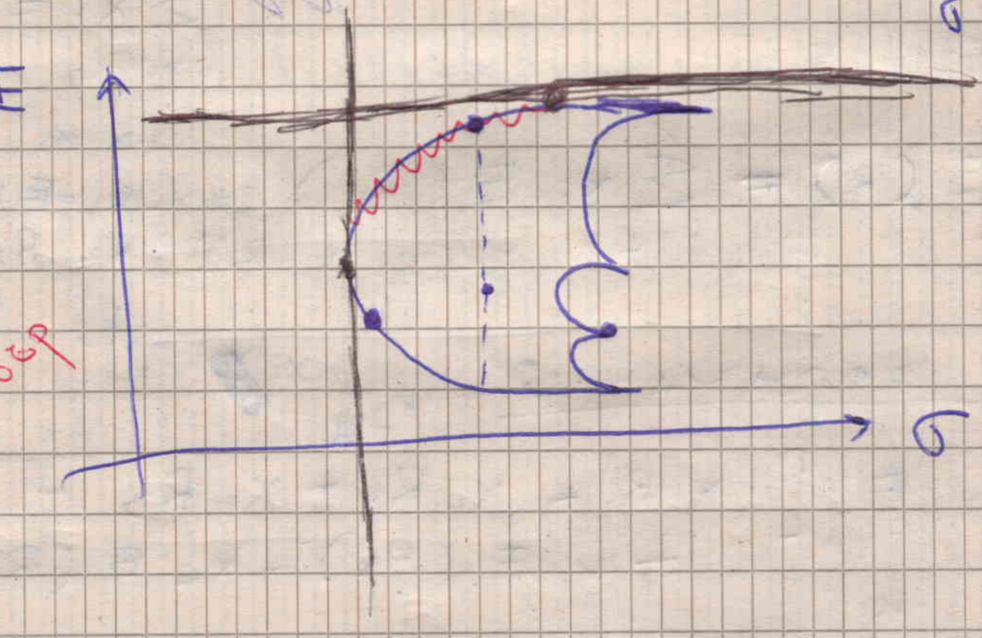
$\cancel{\sigma_R^2 = \sum_{i=1}^{k} x_i^2 \sigma_i^2 + 2}$

$$\sigma_R^2 = \left(x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + x_3^2\sigma_3^2\right) + 2x_1\left(x_2\sigma_1\sigma_2\rho_{12} + x_3\sigma_1\sigma_3\rho_{13}\right) + x_2x_3\sigma_2\sigma_3\rho_{23}\Big)$$

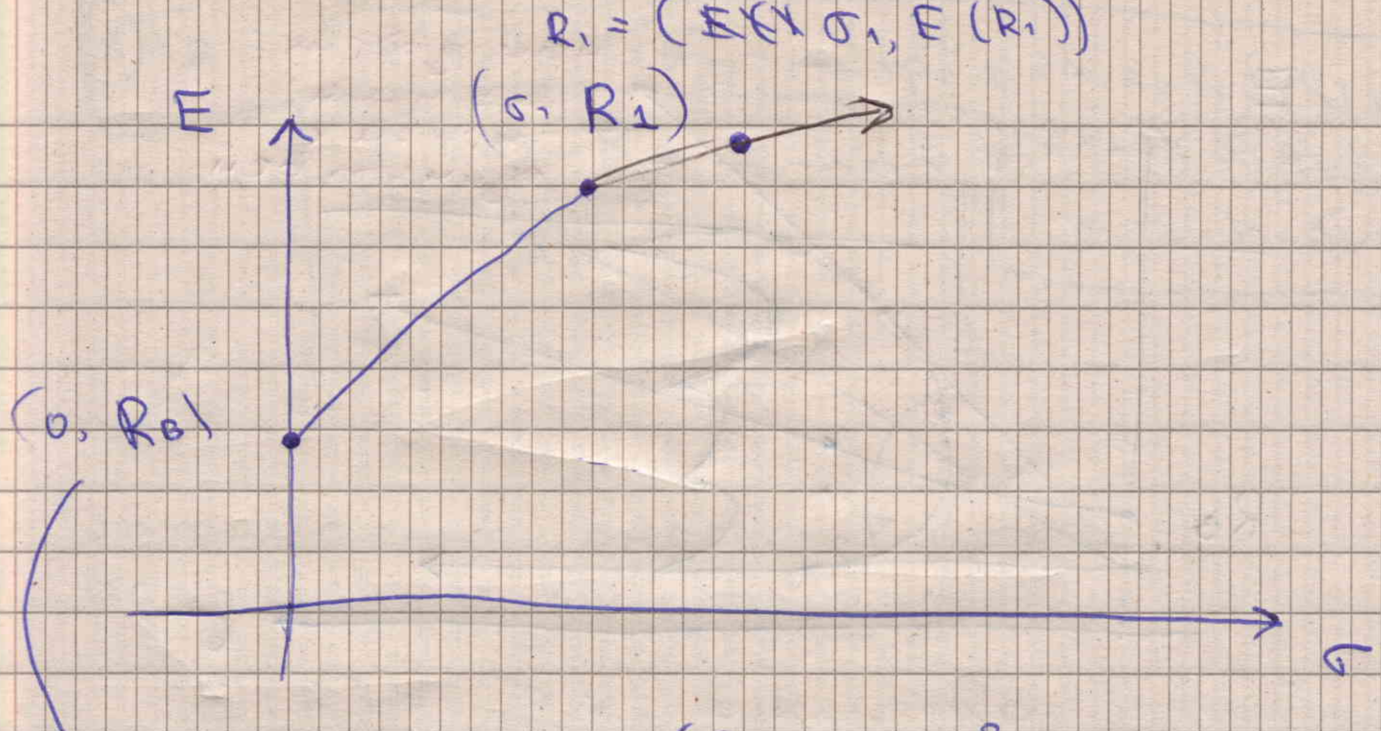
[redacted]

$$= (x_1, x_2, x_3)\begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} & \sigma_1\sigma_3\rho_{13} \\ \sigma_1\sigma_2\rho_{12} & \sigma_2^2 & \sigma_2\sigma_3\rho_{23} \\ \sigma_1\sigma_3\rho_{13} & \sigma_2\sigma_3\rho_{23} & \sigma_3^2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$





хубави портфейли
МПТ няма
най-добър



актив на банката (безрисков, гарантирано)

част от парите в банка, част - рисков

$$\sigma_2 = 0$$

$$\sigma_R = x_1 \sigma_1$$

$$(\sigma_1, R_1 - R_B)$$

$$\begin{cases} x = 0 + \sigma_1 . \lambda \\ y = R_B + (R_1 - R_B) . \lambda \end{cases}$$

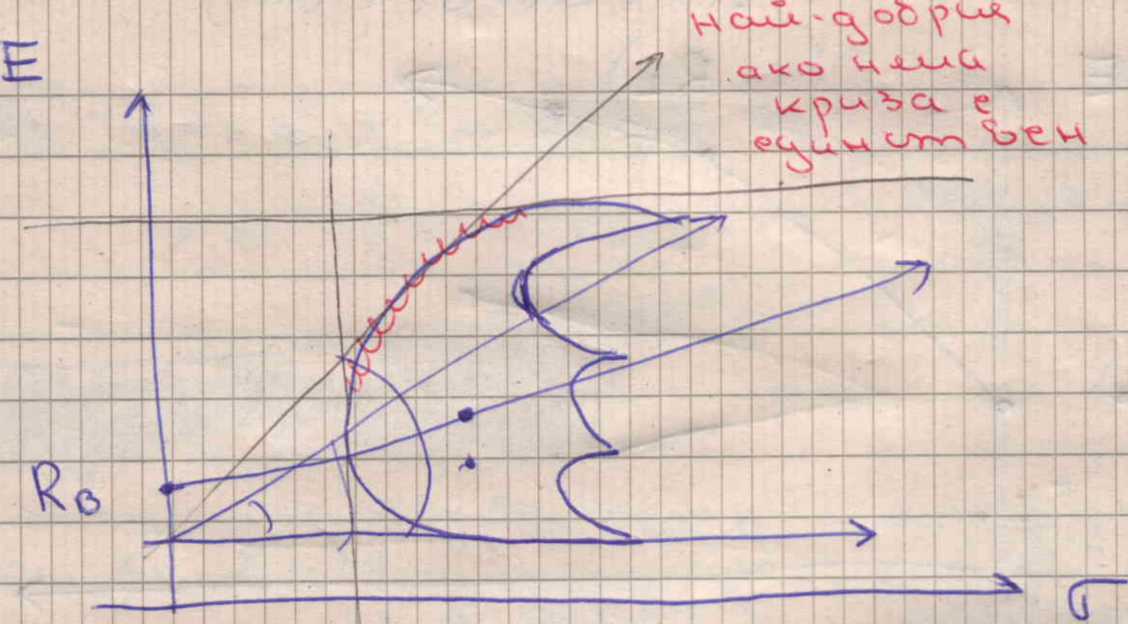
$$E(R) = R_B + \frac{E(R_1) - R_B}{\sigma_1} \sigma_R$$

Разгл. лъча

$$E(R) = x E(R_1) + (1-x) R_B$$

Нека $x > 1$

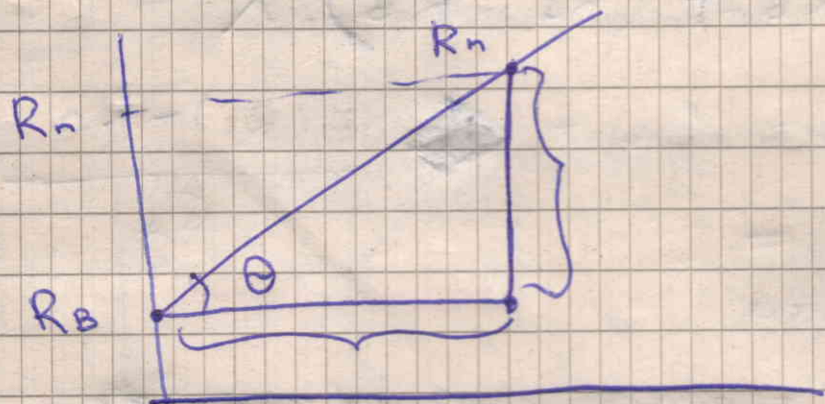
$$= R_B + x(E(R_1) - R_B)$$



Кой портфейл е най-добър?

Нека активът на банката е 3

Търсим портфейл с най-голям ъгъл.



$$tg\,\theta = \frac{E(R_{\cancel{m}}) - R_B}{\sigma_{\cancel{R}}} \longrightarrow max$$

$$x_i \geq 0 \quad \sum_{i=1}^{m} x_i = 1$$

$$\boxed{\sigma_R = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + x_3^2 + \sigma_3^2 + 2x_1x_2\sigma_1\sigma_2\rho_{12} + 2x_1x_3\sigma_1\sigma_3\rho_{13} + 2x_2x_3\sigma_2\sigma_3\rho_{23}}}$$

$$\left|\begin{array}{l} \dfrac{E(R) - R_B}{\sigma_R} \to \max \\ x_i \geq 0 \qquad \sum_{i=1}^{k} x_i = 1 \end{array}\right.$$

?. оптималният портфейл при разрешени къси продажби

$$E(R) = \sum_{i=1}^{n} x_i E(R_i) - R_B =$$

$$= \sum_{i=1}^{n} x_i (E(R_i) - R_B)$$

за $\underline{\underline{n = 3}}$

Гледаме само целевата ф-я
Диф. по $\forall x_i$:

$$\Rightarrow E(R) = x_1(E(R_1) - R_B) + x_2(E(R_2) - R_B) + x_3(E(R_3 - R_B)$$

Диф.:

$$\frac{E(R_1) - R_B}{\sigma_R} + \frac{1}{2}\left(\frac{E(R) - R_B}{\sigma_R^3}\right) \cdot 2\left[\cancel{2}x_1\sigma_1^2 + x_2\sigma_1\sigma_2\rho_{12} + x_3\sigma_1\sigma_2\rho_{13}\right]$$

$$E(R_1) - R_B = \frac{E(R) - R_B}{\sigma_R^2} \left(x_1 \sigma_1^2 + x_2 \sigma_1 \sigma_2 \rho_{12} + x_3 \sigma_1 \sigma_3 \rho_{13} \right)$$

$\to \lambda$

$$\lambda x_1 = z_1$$
$$\lambda x_2 = z_2$$
$$\lambda x_3 = z_3$$

I ред матрица

$$E(R_1) - R_B = z_1 \sigma_1^2 + z_2 \sigma_1 \sigma_2 \rho_{12} + z_3 \sigma_1 \sigma_3 \rho_{13}$$

Едно и също диф. по x_1, x_2, x_3

Диф. по x_2

$$E(R_2) - R_B = z_1 \sigma_1 \sigma_2 \rho_{12} + z_2 \sigma_2^2 + z_3 \sigma_2 \sigma_3 \rho_{23}$$

$$E(R_3) - R_B = z_1 \sigma_1 \sigma_3 \rho_{13} + z_2 \sigma_2 \sigma_3 \rho_{23} + z_3 \sigma_3^2$$

има z_1, z_2, z_3

$$z_1 + z_2 + z_3 = \lambda x_1 + \lambda x_2 + \lambda x_3 = \lambda (x_1 + x_2 + x_3) = \lambda$$

$\underbrace{x_1 + x_2 + x_3}_{1}$

$$x_i = \frac{z_i}{\lambda}$$

зад. 1

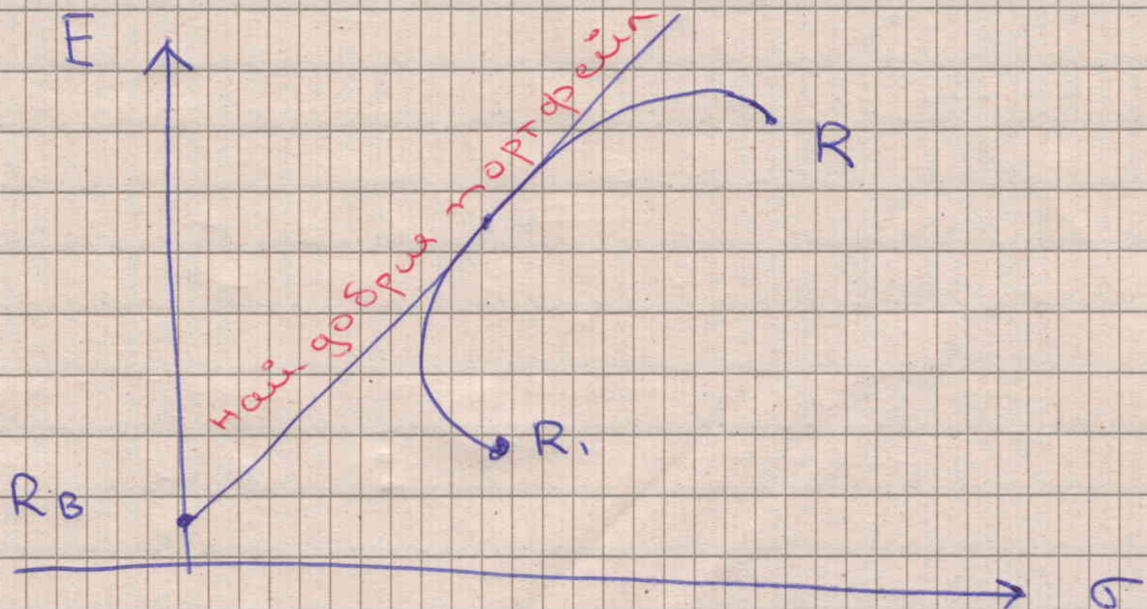
$E(R_1) = 14\%$ $\sigma_1 = 6\%$

$E(R_2) = 8\%$ $\sigma_2 = 3\%$

$E(R_3) = 20\%$ $\sigma_3 = 15\%$

Ако $x_i < 0$ - продадем и купим друго

$x_1 R_1 + x_2 R_2$ 2 актива
индекс



? при какви условия еднот x е 0 (не инвестираме в чужбина)

за 2 актива

$$\begin{cases} E(R_1) - R_B = z_1 \sigma_1^2 + z_2 \sigma_1 \sigma_2 \rho_{12} \\ E(R_2) - R_B = z_1 \sigma_1 \sigma_2 \rho_{12} + z_2 \sigma_2^2 \end{cases}$$

$z_1 \sigma_1^2 + z_2 \sigma_1 \sigma_2 \rho_{12}$

$z_1 \sigma_1 \sigma_2 \rho_{12} + z_2 \sigma_2^2$

$$\rho_{12} = \frac{E(R_2) - R_B}{\cancel{k}\ \sigma_1 \sigma_2}$$

$$E(R_2) - R_3 = \sigma_1^2$$

Модел на ?Маклувиг?