Славка Цанкова Ф. № 31307

Четвърто домашно

Фурие

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ a & x = 1 \\ 0 & 1 \le x \le 2 \end{cases}$$

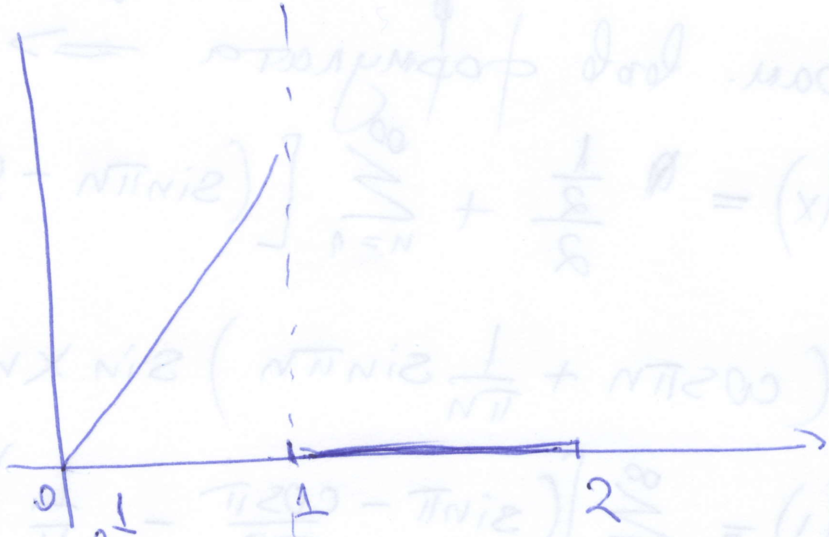
$2l = 2 \Rightarrow l = 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \int_0^1 x\,dx$$

$$a_0 = \frac{x^2}{2}\Big|_0^1 =$$

$$= \frac{1}{2} - 0 = \frac{1}{2}$$



$$a_n = \int_0^1 x.\cos(n\pi x)\,dx = \frac{1}{\pi n}\int_0^1 x.\cos(\pi n x)\,d\,x\pi n =$$

$$= \frac{1}{\pi n}\int_0^1 x\,d\,n\pi.\sin(x\pi n) = \frac{1}{\cancel{\pi n}}\left(x.\cancel{n\pi}\sin(x.\pi n)\Big|_0^1 - \int_0^1 \sin(x.\pi n)\,dx\right) =$$

$$= x.\sin(x.\pi n)\Big|_0^1 - \frac{1}{\pi^2 n^2}\int_0^1 \sin(x\pi n)\,dx\pi n =$$

$$= x.\sin(x\pi n)\Big|_0^1 - \frac{1}{\pi^2 n^2}\cos(x\pi n)\Big|_0^1 =$$

2.

$$= \sin(\pi n) - \frac{\cos(\pi n)}{n^2\pi^2} - \frac{1}{n\pi}$$

$$b_n = \int_0^1 x \cdot \sin(n\pi x)\,dx = \frac{1}{\pi n}\int_0^1 x \cdot \sin(n\pi x)\,dx\,\pi n =$$

$$= \frac{1}{\pi n}\int_0^1 x\,d\cos(n\pi x) \cdot \pi n = \frac{1}{\pi n}\left(\cos(n\pi x) \cdot \pi n \cdot x\Big|_0^1 - \ldots\right.$$

$$\left. - \int_0^1 \cos(\pi n x) \cdot \pi n\,dx\right) = x \cdot \cos(n\pi x)\Big|_0^1 - \frac{1}{\pi n}\int_0^1 \cos(\pi n x)\,d\,x(\pi n) =$$

$$= \cos\pi n + \frac{1}{n\pi}\sin(n\pi x)\Big|_0^1 = \cos\pi n + \frac{1}{n\pi}\sin n\pi$$

Зам. във формулата $\Longrightarrow$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left[\left(\sin\pi n - \frac{\cos\pi n}{\pi^2 n^2} - \frac{1}{\pi n}\right)\cos\pi x + \ldots\right.$$

$$\left. + \left(\cos\pi n + \frac{1}{\pi n}\sin\pi n\right)\sin x n\right]$$

$$f(1) = \sum_{n=1}^{\infty}\left[\left(\sin\pi - \frac{\cos\pi}{\pi^2} - \frac{1}{\pi}\right)\cos\pi a + \left(\cos\pi + \frac{1}{\pi}\sin\pi\right)\sin\pi a\right] =$$

$$= \sum_{n=1}^{\infty}\left[\left(\frac{1}{\pi^2} - \frac{1}{\pi}\right)\cos\pi a + \left(-1 + \frac{1}{\pi} \cdot 0\right)\sin\pi a\right] =$$

$$= \sum_{n=1}^{\infty}\left[\left(\frac{1-\pi}{\pi^2}\right)\cos\pi a - \sin\pi a\right]$$

$$f(1) = a$$