Домашно № 2 по ДИС II

на Симеон Йовков Цветков ПМ 31308 I курс II гр.

① Да се направи смяна на променливите

$z'_x z''_{xy} - z'_y z''_{xx} = 0$

$x = w(u(x,y), v(x,y))$
$y = u(x,y)$
$z = v(x,y)$

$\left| \begin{array}{l} 1 = w'_u u'_x + w'_v v'_x \\ 0 = u'_x \\ z'_x = v'_x = \frac{1}{w'_v} \end{array} \right.$ $\left| \begin{array}{l} 0 = w'_u u'_y + w'_v v'_y \\ 1 = u'_y \\ z'_y = v'_y = -\frac{w'_u}{w'_v} \end{array} \right.$

$z''_{xx} = -(w'_v)^{-2}(w''_{uv} \cdot u'_x + w''_{vv} v'_x) = -(w'_v)^{-2} \cdot \frac{w''_{vv}}{w'_v} =$

$= -\frac{w''_{vv}}{w'^3_v}$

$z''_{xy} = -(w'_v)^{-2}(w''_{uv} u'_y + w''_{vv} \cdot v'_y) = -(w'_v)^{-2} \cdot (w''_{uv} - \frac{w''_{vv} w'_u}{w'_v}) =$

$= -\frac{w''_{uv} w'_v - w''_{vv} w'_u}{w'^3_v} \Rightarrow z'_x z''_{xy} - z'_y z''_{xx} = 0$

$\Rightarrow \frac{w''_{vv} w'_u - w''_{uv} w'_v}{(w'_v)^4} - \frac{w'_u \cdot w''_{vv}}{(w'_v)^4} = 0$

$\boxed{\Rightarrow w''_{uv} w'_v = 0}$

(2) За кои стойности на α функцията е:

$$f(x,y) = \begin{cases} (x^2+y^2)^\alpha \sin\frac{1}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0 & x^2+y^2 = 0 \end{cases}$$

а) непрекъсната

б) диференцуема в т. (0,0)

а) $f(x,0) = x^{2\alpha} \sin\frac{1}{x^2} = x^{2\alpha} \cdot \frac{\sin\frac{1}{x^2}}{\frac{1}{x^2}} \cdot \frac{1}{x^2} =$

$= x^{2\alpha-2} \cdot \underbrace{\frac{\sin\frac{1}{x^2}}{\frac{1}{x^2}}}_{\to 0} \xrightarrow[x\to 0]{} 0$

$f(x,0) \xrightarrow[x\to 0]{} 0$ за $\forall\alpha$

Аналогично $f(0,y) = y^{2\alpha-2} \cdot \underbrace{\frac{\sin\frac{1}{y^2}}{\frac{1}{y^2}}}_{\to 0} \xrightarrow[y\to 0]{} 0$

$f(0,y) \xrightarrow[y\to 0]{} 0$ за $\forall\alpha$

$\Rightarrow$ Непрекъсната за $\forall\alpha$

б)

③ Нека $\varphi(u, v)$ е диференцуема, а $f(x,y,z) = \varphi(xy, \frac{y}{z})$

Да се докаже, че: $-x \cdot f'_x + y \cdot f'_y + z \cdot f'_z = 0$

$$f'_x = \varphi'_u \, u'_x + \varphi'_v \cdot v'_x = \varphi'_u \cdot y$$

$$f'_y = \varphi'_u \cdot u'_y + \varphi'_v \, v'_y = \varphi'_u \cdot x + \varphi'_v \cdot \frac{1}{z}$$

$$f'_z = \varphi'_u \cdot u'_z + \varphi'_v \cdot v'_z = -\varphi'_v \frac{y}{z^2}$$

$$\Rightarrow \quad -x f'_x + y f'_y + z f'_z = \cancel{-x \varphi'_u y} + \cancel{y \varphi'_u x} +$$

$$+ \cancel{y \cdot \frac{1}{z}} \cdot \varphi'_v + \cancel{z} \left(-\varphi'_v \cdot \frac{y}{z^{\cancel{2}}}\right) = 0$$

④ $z = x^y$ Да се докаже, че

$$\frac{x}{y} \cdot z'_x + \frac{1}{\ln x} \cdot z'_y = 2z$$

$$z'_x = y x^{y-1} \qquad z'_y = x^y \ln x$$

$$\Rightarrow \frac{x}{y} z'_x + \frac{1}{\ln x} \cdot z'_y = \frac{x}{\cancel{y}} \cancel{y} \cdot x^{y-1} + \frac{1}{\cancel{\ln x}} \cdot x^y \cancel{\ln x} =$$

$$= 2x^y = 2z$$