05.VII.2013г. Консултация - Галя

$\boxed{1}$ пръстен R относно ~~(+, *)~~ $(\mathbb{C}, +, *)$

а) 0) $R_1 = \left\{ \frac{a}{p^k} \,\middle|\, a \in \mathbb{Z}, k \in \mathbb{N}, p \text{ не дели } a \right\}$ $a \in \mathbb{Z}$ $p \in \mathbb{N}$ $k \in \mathbb{N}$

Нека $k_1 \le k_2$

$$\frac{a_1}{p^{k_1}} + \frac{a_2}{p^{k_2}} = \frac{p^{k_2}a_1 + a_2 p^{k_1}}{p^{k_1+k_2}} \qquad \frac{a}{p^k} \in R_1 \qquad \frac{p^{k_2}a_1 + a_2 p^{k_1}}{p^{k_1+k_2}} \in R_1 \qquad \frac{\cancel{p^{k_1}}(a_2 + a_1 p^{k_2-k_1})}{p^{k_2}\,\cancel{p^{k_1+k_2}}}$$

1) $$\left(\frac{a_1}{p^{k_1}} + \frac{a_2}{p^{k_2}}\right) + \frac{a_3}{p^{k_3}} = \frac{p^{k_2}a_1 + a_2 p^{k_1}}{p^{k_1+k_2}} + \frac{a_3}{p^{k_3}} = \frac{p^{k_3}(p^{k_2}a_1 + a_2 p^{k_1}) + a_3 p^{k_1+k_2}}{p^{k_1+k_2+k_3}}$$

Без ограничение на общността: $k_1 \le k_2 \le k_3$

$$\cancel{\frac{a_1 p^{k_2+k_3} + a_2 p^{k_1+k_3} + a_3 p^{k_1+k_2}}{p^{k_1+k_2+k_3}} = \frac{}{p}}$$

$$\frac{p^{k_2}a_1 + a_2 p^{k_1}}{p^{k_1+k_2}} = \frac{\cancel{p^{k_1}}(a_2 + a_1 p^{k_2-k_1})}{\cancel{p^{k_1}}\,p^{k_2}}$$

$k_2 \le k_3$

$$\frac{a_2 + a_1 p^{k_2-k_1}}{p^{k_2}} + \frac{a_3}{p^{k_3}} = \frac{a_2 p^{k_3} + a_1 p^{k_2+k_3-k_1} + a_3 p^{k_2}}{p^{k_2+k_3}} =$$

$$\frac{\cancel{p^{k_2}}(a_3 + a_2 p^{k_3-k_2} + a_1 p^{k_3-k_1})}{\cancel{p^{k_2}}\,p^{k_3}} \qquad (a_3 + a_2 p^{k_3-k_2} + a_1 p^{k_3-k_1}) \in \mathbb{Z} \qquad p^{k_3} \in \mathbb{N}$$

2) $\exists$ неутрален елемент $\in R_1$ $\left(\frac{x_1}{p^{x_2}} = \frac{0}{1}\right)$

$$\frac{a_1}{p^{k_1}} + \frac{x_1}{p^{x_2}} = \frac{a_1}{p^{k_1}} \qquad \frac{x_1}{p^{x_2}} = \frac{0}{p^1}, \frac{0}{p^2}, \ldots$$

$$\frac{a_1 p^{x_2} + p^{k_1}x_1}{p^{k_1+x_2}} = \frac{a_1}{p^{k_1}} \qquad x_2 = 0 \quad p^{x_2} = 1 \quad p^{k_1}x_1 = 0$$

$$\cancel{a_1 p^{x_2} p^{k_1}} + p^{k_1}x_1 p^{k_1} = \cancel{a_1 p^{k_1+x_2}} \qquad p^{k_1} \ne 0 \quad x_1 = 0$$

$x_1 = 0$

3) $\exists$ обратен елемент $\in R_1$:

$$\frac{a_1}{p^{k_1}} + \frac{x_1}{p^{x_2}} = \frac{0}{1}$$

$$\frac{a_1 p^{x_2} + x_1 p^{k_1}}{p^{k_1+x_2}} = \frac{0}{1} \qquad p^{k_1+x_2} = 1$$

$$k_1 = -x_2$$
$$x_2 = -k_1$$

$$a_1 p^{x_2} + x_1 p^{k_1} = 0$$

$$a_1 p^{x_2} = -x_1 p^{k_1}$$

$$a_1 p^{-k_1} = -x_1 p^{k_1}$$

$$\cancel{a_1 = -x_1}$$

$$x_1 p^{k_1} = -a_1 p^{-k_1}$$

$$x_1 = -\frac{a_1}{p^{k_1}}$$

$\Rightarrow R_1$ не е пръстен

б) $R_2 = \{ \frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0, (a,b) = 1, \cancel{b \text{ не дели } a} \}$

0) $$\frac{a_1}{b_1} + \frac{a_2}{b_2} = \frac{a_1 b_2 + a_2 b_1}{b_1 b_2} = \frac{\cancel{b_2}(a_1 + a_2 \frac{b_1}{b_2})}{b_1 \cancel{b_2}} \in \mathbb{Z}$$

$$b_1 b_2 \in \mathbb{Z} \qquad a_1 + a_2 \frac{b_1}{b_2} \in \mathbb{Z}$$

1) $$\left(\frac{a_1}{b_1} + \frac{a_2}{b_2}\right) + \frac{a_3}{b_3} = \frac{a_1 b_2 + a_2 b_1}{b_1 b_2} + \frac{a_3}{b_3} = \frac{a_1 b_2 b_3 + a_2 b_1 b_3 + a_3 b_1 b_2}{b_1 b_2 b_3}$$

$$\frac{a_1}{b_1} + \left(\frac{a_2}{b_2} + \frac{a_3}{b_3}\right) = \frac{a_1}{b_1} + \frac{a_2 b_3 + a_3 b_2}{b_2 b_3} = \frac{a_1 b_2 b_3 + a_2 b_3 b_1 + a_3 b_1 b_2}{b_1 b_2 b_3}$$

2) $$\frac{a_1}{b_1} + \frac{x_1}{x_2} = \frac{a_1}{b_1}$$

$$\frac{a_1 x_2 + x_1 b_1}{b_1 x_2} = \frac{a_1}{b_1} \qquad \Rightarrow R_2 \text{ не е пръстен}$$

$$a_1 x_2 b_1 + x_1 b_1^2 = a_1 b_1 x_2$$

$$\cancel{b_1}(a_1 x_2 + x_1 b_1) = a_1 x_2 \cancel{b_1}$$

$$a_1 x_2 + x_1 b_1 = a_1 x_2 \qquad x_1 = 0$$
$$\forall x_2$$

в) $R_3 = \{ x + y\sqrt[3]{2} \mid x, y \in \mathbb{Q} \}$

0) $x_1 + y_1\sqrt[3]{2} + x_2 + y_2\sqrt[3]{2} = x_1 + x_2 + \sqrt[3]{2}(y_1 + y_2)$

$x_1 + x_2 \in \mathbb{Q}$

$y_1 + y_2 \in \mathbb{Q}$

1) $(x_1 + y_1\sqrt[3]{2} + x_2 + y_2\sqrt[3]{2}) + (x_3 + y_3\sqrt[3]{2}) = \{x_1 + x_2 + (y_1 + y_2)\sqrt[3]{2} +$

$+ x_3 + y_3\sqrt[3]{2} = x_1 + x_2 + x_3 + (y_1 + y_2 + y_3)\sqrt[3]{2}$

$x_1 + y_1\sqrt[3]{2} + (x_2 + y_2\sqrt[3]{2} + x_3 + y_3\sqrt[2]{3}) =$

$= x_1 + y_1\sqrt[3]{2} + x_2 + x_3 + (y_2 + y_3)\sqrt[3]{2} = x_1 + x_2 + x_3 + (y_1 + y_2 + y_3)\sqrt[3]{2}$

2) $x_1 + y_1\sqrt[3]{2} + a_1 + a_2\sqrt[3]{2} = x_1 + y_1\sqrt[3]{2}$

$x_1 + a_1 = x_1$ $\quad a_1 = 0$

$y_1\sqrt[3]{2} + a_2\sqrt[3]{2} = y_1\sqrt[3]{2}$ $\quad a_2 = 0$

∃ неутрален елем.
$(a_1, a_2) = (0, 0)$

3) ∃ обратен елемент $(a_1, a_2) = (-x_1, -y_1)$

$x_1 + y_1\sqrt[3]{2} + a_1 + a_2\sqrt[3]{2} = 0 + 0$

$x_1 + a_1 = 0$ $\quad (y_1 + a_2)\sqrt[3]{2} = 0\sqrt[3]{2}$

$a_1 = -x_1$ $\quad y_1 + a_2 = 0$

$a_2 = -y_1$

4) $x_1 + y_1\sqrt[3]{2} + x_2 + y_1\sqrt[3]{2} = x_2 + y_1\sqrt[3]{2} + x_1 + y_1\sqrt[3]{2}$

5) $(x_1 + y_1\sqrt[3]{2})(x_2 + y_2\sqrt[3]{2}) =$

$= x_1x_2 + x_1y_2\sqrt[3]{2} + x_2y_1\sqrt[3]{2} + y_1y_2\sqrt[3]{2^2} =$

~~$x_1x_2 \in \mathbb{Q}$~~

$= x_1x_2 + \sqrt[3]{2}(x_1y_2 + x_2y_1 + y_1y_2\sqrt[3]{2})$

$x_1 . y_2 \in \mathbb{Q}$

$x_2 . y_1 \in \mathbb{Q}$ $\quad \Rightarrow$ не е пръстен R_3

$\sqrt[3]{2}y_1 . y_2 \notin \mathbb{Q}$

2) $R_4 = \{ x + y\sqrt[3]{2} + z\sqrt[3]{4} \mid x, y, z \in \mathbb{Q} \}$

0) $x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4} + x_2 + y_2\sqrt[3]{2} + z_2\sqrt[3]{4} = \underbrace{x_1 + x_2}_{\in\mathbb{Q}} + \underbrace{(y_1+y_2)}_{\in\mathbb{Q}}\sqrt[3]{2} + \underbrace{(z_1+z_2)}_{\in\mathbb{Q}}\sqrt[3]{4}$

1) $(x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4} + x_2 + y_2\sqrt[3]{2} + z_2\sqrt[3]{4}) + x_3 + y_3\sqrt[3]{2} + z_3\sqrt[3]{4} =$

$= x_1 + x_2 + (y_1+y_2)\sqrt[3]{2} + z_1\sqrt[3]{4} + x_3 + y_3\sqrt[3]{2} + z_3\sqrt[3]{4} =$

$= \underbrace{x_1 + x_2 + x_3}_{\in\mathbb{Q}} + \sqrt[3]{2}\underbrace{(y_1+y_2+y_3)}_{\in\mathbb{Q}} + \sqrt[3]{4}\underbrace{(z_1+z_2+z_3)}_{\in\mathbb{Q}}$

2) $\exists$ неутрален елемент $(a_1, a_2, a_3) = (0,0,0)$

$x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4} + a_1 + a_2\sqrt[3]{2} + a_3\sqrt[3]{4} = x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4}$

$(x_1 + a_1)\, 1 = x_1$ $\quad a_1 = 0 \quad x_1 + a_1 = x_1$

$(y_1 + a_2)\sqrt[3]{2} = y_1\sqrt[3]{2}$ $\quad a_2 = 0 \quad y_1 + a_2 = y_1$

$(z_1 + a_3)\sqrt[3]{4} = z_1\sqrt[3]{4}$ $\quad a_3 = 0 \quad z_1 + a_3 = z_1$

3) $\exists$ обратен елемент $(a_1, a_2, a_3) = (-x_1, -y_1, -z_1)$

$x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4} + a_1 + a_2\sqrt[3]{2} + a_3\sqrt[3]{4} = 0 + 0\sqrt[3]{2} + 0\sqrt[3]{4}$

$1(x_1 + a_1) = 0$ $\quad x_1 + a_1 = 0 \quad a_1 = -x_1$

$\sqrt[3]{2}(y_1 + a_2) = 0\sqrt[3]{2}$ $\quad y_1 + a_2 = 0 \quad a_2 = -y_1$

$\sqrt[3]{4}(z_1 + a_3) = 0\sqrt[3]{4}$ $\quad z_1 + a_3 = 0 \quad a_3 = -z_1$

4) $x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4} + x_2 + y_2\sqrt[3]{2} + z_2\sqrt[3]{4} = x_1 + x_2 + \sqrt[3]{2}(y_1+y_2) + \sqrt[3]{4}(z_1+z_2)$

$x_2 + y_2\sqrt[3]{2} + z_2\sqrt[3]{4} + x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4} = x_1 + x_2 + \sqrt[3]{2}(y_1+y_2) + \sqrt[3]{4}(z_1+z_2)$

5) $(x_1 + y_1\sqrt[3]{2} + z_1\sqrt[3]{4})(x_2 + y_2\sqrt[3]{2} + z_2\sqrt[3]{4}) =$

$= x_1x_2 + x_1y_2\sqrt[3]{2} + x_1z_2\sqrt[3]{4} + x_2y_1\sqrt[3]{3} + y_1y_2\sqrt[3]{2} + y_1z_2\underbrace{\sqrt[3]{8}}_{2} +$

$+ x_2z_2\sqrt[3]{4} + y_2z_1\underbrace{\sqrt[3]{8}}_{2} + z_1z_2\underbrace{\sqrt[3]{16}}_{2\sqrt[3]{2}} =$

$= \underbrace{x_1x_2}_{\in\mathbb{Q}} + \sqrt[3]{2}\underbrace{(x_1y_2 + x_2y_1 + y_1y_2)}_{\in\mathbb{Q}} + \sqrt[3]{4}\underbrace{(x_1z_2 + x_2z_2)}_{\in\mathbb{Q}} + \sqrt[3]{8}\underbrace{(y_1z_2 + y_2z_1)}_{\in\mathbb{Q}}$

$+ \sqrt[3]{16}\underbrace{(z_1z_2)}_{\in\mathbb{Q}\sqrt[3]{2}}$ $\checkmark$

6) $[(x_1+y_1\sqrt[3]{2}+z_1\sqrt[3]{4})(x_2+y_2\sqrt[3]{2}+z_2\sqrt[3]{4})](x_3+y_3\sqrt[3]{2}+z_3\sqrt[3]{4}) =$

$= [x_1x_2+x_1y_2\sqrt[3]{2}+x_1z_2\sqrt[3]{4}+y_1x_2\sqrt[3]{2}+y_1y_2\sqrt[3]{4}+y_1z_2\cdot 2+$

$+x_2z_1\sqrt[3]{4}+2z_1y_2 + z_1z_2\cdot 2\sqrt[3]{2}](x_3+y_3\sqrt[3]{2}+z_3\sqrt[3]{4}) =$

$= [x_1x_2 + \sqrt[3]{2}(x_1y_2+x_2y_1+2z_1z_2)+2(y_1z_2+z_1y_2)+$

$+\sqrt[3]{4}(x_1z_2+y_1y_2+x_2z_1)](x_3+y_3\sqrt[3]{2}+z_3\sqrt[3]{4}) =$

$= x_1x_2x_3+\sqrt[3]{2}x_3(x_1y_2+x_2y_1+2z_1z_2)+2x_3(y_1z_2+z_1y_2)+$

$+\sqrt[3]{4}x_3(x_1z_2+y_1y_2+x_2z_1)+x_1x_2y_3\sqrt[3]{2}+\sqrt[3]{4}y_3(x_1y_2+x_2y_1+2z_1z_2)$

$+2y_3\sqrt[3]{2}(y_1z_2+z_1y_2)+y_3 2(x_1z_2+y_1y_2+x_2z_1)+$

$+x_1x_2z_3\sqrt[3]{4}+2z_3(x_1y_2+x_2y_1+2z_1z_2)+2\sqrt[3]{4}x_3z_3(y_1z_2+z_1y_2)$ *

* . . . .

– аналогично нататък се прилагат формулите и се получава че R е пръстен

11 $G=\{(a,b,c) \mid a,b,c\in\mathbb{R}\}$

$(a_1,b_1,c_1)\circ(a_2,b_2,c_2)=(a_1+a_2,\ b_1+a_1c_2+b_2,\ c_1+c_2)$

а) ~~G е група~~ ? $(G,\circ)$ е група

б) ? $H=\{(0,b,0)\mid b\in\mathbb{R}\} \trianglelefteq G$

? $H\cong(\mathbb{R},+)$

$G/H\cong(\mathbb{R}^2,+)$

а) 0) $(a_1,b_1,c_1)\circ(a_2,b_2,c_2)=(a_1+a_2,\ b_1+a_1c_2+b_2,\ c_1+c_2)$

$a_1+a_2\in\mathbb{R}$

$b_1+a_1c_2\in\mathbb{R}$

$c_1+c_2\in\mathbb{R}$

1) $((a_1,b_1,c_1)\circ(a_2,b_2,c_2))\circ(a_3,b_3,c_3)=(a_1+a_2,\ b_1+a_1c_2+b_2,\ c_1+c_2)\circ$

$(a_3,b_3,c_3)=(\underline{a_1+a_2+a_3},\ \underline{b_1}+a_1c_2+b_2+(a_1+a_2)c_3+b_3,\ \underline{c_1+c_2+c_3})$

$(a_1,b_1,c_1)\circ((a_2,b_2,c_2)\circ(a_3,b_3,c_3))=(a_1,b_1,c_1)\circ$

$\circ(a_2+a_3,\ b_2+a_2c_3+b_3,\ c_2+c_3)=(\underline{a_1+a_2+a_3},\ \underline{b_1}+a_1(c_2+c_3),\ \underline{c_1+c_2+c_3})$

$\cancel{b_1+a_1c_2+b_2}$

1) $[(a_1,b_1,c_1)\circ(a_2,b_2,c_2)]\circ(a_3,b_3,c_3)=(a_1+a_2,b_1+a_1c_2+b_2,c_1+c_2)\circ(a_3,b_3,c_3)=$

$=(\underline{a_1+a_2+a_3},\ \underline{b_1}+\underline{a_1c_2}+\underline{b_2}+\underline{(a_1+a_2)c_3}+\underline{b_3},\ \underline{c_1+c_2+c_3})$

$(a_1,b_1,c_1)\circ[(a_2,b_2,c_2)\circ(a_3,b_3,c_3)]=(a_1,b_1,c_1)\circ(a_2+a_3,b_2+a_2c_3+b_3,c_2+c_3)=$

$=(\underline{a_1+a_2+a_3},\ \underline{b_1}+\underline{a_1(c_2+c_3)}+\underline{b_2}+\underline{a_2c_3}+\underline{b_3},\ \underline{c_1+c_2+c_3})$

2) $\exists$ неутрален елемент $(x_1,x_2,x_3)=(0,0,0)$

$(a_1,b_1,c_1)\circ(x_1,x_2,x_3)=(a_1,b_1,c_1)$

$(a_1+x_1,\ b_1+a_1x_3+x_2,\ c_1+x_3)=(a_1,b_1,c_1)$

$a_1+x_1=a_1 \qquad x_1=0$

$b_1+a_1x_3+x_2=b_1 \qquad a_1x_3+x_2=0 \qquad x_2=0$

$c_1+x_3=c_1 \qquad x_3=0$

3) $\exists$ обратен елемент $(x_1,x_2,x_3)=(-a_1,-b_1+a_1c_1,-c_1)$

$(a_1,b_1,c_1)\circ(x_1,x_2,x_3)=(0,0,0)$

$(a_1+x_1,\ b_1+a_1x_3+x_2,\ c_1+x_3)=(0,0,0)$

$a_1+x_1=0 \qquad x_1=-a_1$

$b_1+a_1x_3+x_2=0 \qquad x_2=-b_1+a_1c_1$

$c_1+x_3=0 \qquad x_3=-c_1$

$\Rightarrow$ G е група $(G,\circ)$ относно $\circ$

δ) $H \subset G$

$g^{-1}hg = (-a_1, -b_1+a_1c_1, -c_1) \circ (0, b_2, 0) \circ (a_1, b_1, c_1) =$

$= (-a_1, -b_1+a_1c_1+b_2+(-a_1).0, -c_1) \circ (a_1, b_1, c_1) =$

$= (0, \underbrace{\underset{\in R}{-b_1}+\underset{\in R}{a_1c_1}+\underset{\in R}{b_2}\underset{\in R}{-a_1c_1}+\underset{\in R}{b_1}}_{\in R}, 0) \Rightarrow H \trianglelefteq G$

~~$\varphi(H,\circ) \to (R,+)$~~

~~$\varphi((0,b,0)) \to b$~~

~~$\mathrm{Ker}\,\varphi = \{(0,b,0) : \varphi((0,b,0)) = b-b=0 \mid b \in R\}$~~

$\mathrm{Im}\,\varphi =$

$\varphi((0,b,0)) = b$

$\varphi((0,b_1,0)) \circ \varphi((0,b_2,0)) = \cancel{(0, b_1+b_2, 0) = b_1+b_2}$ $b_1 \circ b_2 = b_1+b_2$

$\varphi((0,b_1,0) \circ (0,b_2,0)) = \varphi(0, b_1+b_2, 0) = b_1+b_2$ ✓

$b_1 = \varphi((0,b_1,0))$ ~~$\neq \varphi((0,b_2,0)) = x_2$~~ $\neq b_2 = \varphi((0,b_2,0))$

$\Rightarrow \varphi((0,b_1,0)) \neq \varphi((0,b_2,0))$.

$b = \varphi((0,b,0))$

$\Rightarrow H \cong (R,+)$

$G/H \cong (\mathbb{R}^2, +)$

$\varphi((a,b,c)) = \cancel{a+c=0}\ (a,c)$

$\varphi((a_1,b_1,c_1)) \circ \varphi((a_2,b_2,c_2)) = (a_1,c_1) \circ (a_2,c_2) = (a_1,c_1) + (a_2,c_2)$

$\varphi((a_1,b_1,c_1) \circ (a_2,b_2,c_2)) = \varphi((a_1+a_2, b_1+a_1c_2+b_2, c_1+c_2)) = (a_1+a_2, c_1+c_2) =$
$= (a_1,c_1) + (a_2,c_2)$

$Ker\,\varphi = \{ (a,b,c) : \varphi((a,b,c)) = (a,c) = 0 \mid a,b,c \in \mathbb{R} \} =$

$= \{ (a,b,c) : (a,c) = 0 \mid a,b,c \in \mathbb{R} \} = H$

$H \trianglelefteq G$ е изп. $\Rightarrow G/H = Im\,\varphi \overset{?}{=} \mathbb{R}^2$

$\varphi((a,b,c)) = (a,c) \in \mathbb{R}^2 \Rightarrow Im\,\varphi = \mathbb{R}^2$

$\boxed{2}$ 2.
$$\begin{cases} 3x+1 \equiv 0 \pmod{35} \\ 7x \equiv 11 \pmod{20} \end{cases}$$

$35 \to 36 \to 3.12$
$20 \to 21 \to 7.3$
$20|40|60|80|100|$
$120|140|160$

$$\begin{cases} 3x \equiv -1 \pmod{35} \\ 7x \equiv 11 \pmod{20} \end{cases}$$

$3x = 35y - 1$

$\cancel{x = \frac{35}{3}y - \frac{1}{3} = \frac{35.4}{3} - \frac{1}{3} = \frac{139}{3}}$

$\cancel{7.\frac{35}{3}y - \frac{7}{3} \equiv 11 \pmod{20}}$ /.3

$\cancel{7.35y - 7 \equiv 13 \pmod{20}}$

$\cancel{7.35y \equiv 0 \pmod{20}}$

$\cancel{7.15y \equiv 0 \pmod{20}}$

$\cancel{105y \equiv 0 \pmod{20}}$

$\cancel{5y \equiv 0 \pmod{20}}$

$\cancel{y = 4}$

$x = 30 \pmod{35} + 13 \pmod{20}$

$x = 43 \pmod{140}$

8 $x = 140t + 43$

$\cancel{3y \pmod{}}$

$3 \pmod{35} = -1 \pmod{35}$

$$\begin{cases} x \equiv -12 \pmod{35} \\ x \equiv 13 \pmod{20} \end{cases}$$

$20y + 13 = \cancel{35y - 12}$ x

$\cancel{15y = 1}$

y

$\cancel{20y + 13 \equiv 13 \pmod{20}}$

$\cancel{20y \equiv 0 \pmod{20}}$

$\cancel{7y \equiv 0 \pmod{20}}$ /.(3)

$\cancel{y \equiv 0 \pmod{20}}$

$20y + 13 \equiv -12 \pmod{35}$

$20y \equiv -25 \pmod{35}$ /.4

$y \equiv -30 \pmod{35} = 5 \pmod{35}$

$\cancel{20}y = 35t - 25$

$5y = 7t - 5$

$5y \equiv -5 \pmod{7}$

$\cancel{y} \equiv -1 \pmod{7}$

$y = \frac{7t-1}{5}$

$20y \equiv -25 \pmod{35}$

$5y \equiv -5 \pmod{7}$ $\quad -15$

$y \equiv -1 \pmod{7}$

$y \equiv 6 \pmod{7}$

$x = 20y + 13 =$

$x = 20 + 13 = -7 \pmod{140}$

$x \equiv 133 \pmod{140}$

$\boxed{1}$ 2.
$$\begin{cases} 5x + 1 \equiv 0 \pmod{24} \\ 4x \equiv 19 \pmod{21} \end{cases}$$

$5x \equiv -1 \pmod{24} \,/ \cdot 5$

$x \equiv -5 \pmod{24} = \cancel{x} = 24y - 5$

$\cancel{4(-5) \pmod{24} \equiv 19 \pmod{21}}$

$\cancel{-20 \pmod{24} \equiv 19 \pmod{21}}$

$\cancel{4 \cdot (24y - 5) \equiv 19 \pmod{21}}$

$\cancel{96y - 5 \pmod{21} - 5}$

$96y - 20 \equiv 19 \pmod{21}$

$96y \equiv -3 \pmod{21} \equiv 18 \pmod{21}$

$12y \equiv 18 \pmod{21}$

$4y \equiv 6 \pmod{7} \,/ \cdot 2$

$y \equiv 5 \pmod{7} =$

$\cancel{120}$ $x = 24y - 5 =$

$= 120 \pmod{7} - 5 \pmod{24} =$

$= 115 \pmod{168}$

$39 = 21 \cdot 2 - 3$

$96 = 4 \cdot 21 + 12$

$12y = 21t + 18$

$4y = 7t + 6$

12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 |

21 42 63 84 105 120

4 | 8

7, 14, 21, 28, 35, 42

$24 \cdot 7$

168