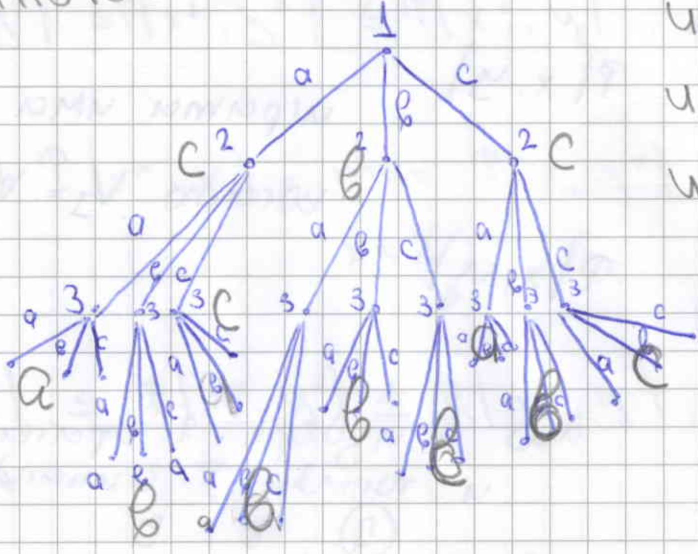
Теория на игрите

доминото:



$u_1(c) < u_1(b) < u_1(a)$

$u_2(b) < u_2(a) < u_2(c)$

$u_3(a) < u_3(c) < u_3(b)$

задача

$x_1 \in X_1 \dots x_n \in X_n$

$u_1(x_1 \dots x_n)$

$\dots$

$u_n(x_1 \dots x_n)$

$(x_1^*, x_2^*, x_3^* \dots x_n^*)$

ако вземем i-тия играч и той и отклони от стратегията си, той няма да получи повече

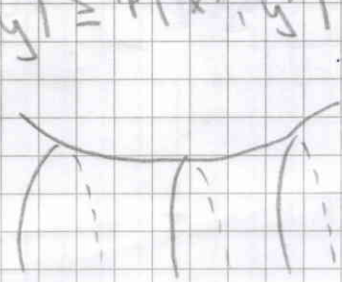
I играч $x \in X$ II играч $y \in Y$

$P(x,y) + Q(x,y) = 0$

$P(x, y^*) \le P(x^*, y^*)$ $P(x^*, y) \ge P(x^*, y^*) > P(x, y^*)$

$Q(x^*, y) \le Q(x^*, y^*)$

$P(x^*, y) \le P(x^*, y^*)$



Търсим игрова точка = равновесие по Неш

$\min\limits_{y \in Y} P(x, y)$

$V_I = \max\limits_{x \in X} \min\limits_{y \in Y} P(x, y)$

играта има цена, когато $V_I = V_{II}$

$V_2 = \min\limits_{y \in Y} \max\limits_{x \in X} P(x, y)$

задача (пример)

$$\begin{array}{c|c} I & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = P(1,3) \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ & II \end{array}$$

ако II избере 1 стратегия и тогава I винаги избира I

7 8 9

ако I избере 1 стратегия

за втория 1 4 7

нормално I – 3

II – 1

игрите с цена си държат цената

т.е, ако някой се отклони от цената ще загуби

$$\begin{array}{c c} & V_I \\ \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} & \begin{array}{c} -1 \\ -1 \end{array} \\ V_{II} \quad 1 \quad 1 & \end{array}$$

$V_I \le V_{II}$ $\quad \min P(\bar{x}, y) \le \max\limits_{x \in X} P(x, \bar{y})$

$\max\limits_{x \in X} \min\limits_{y \in Y} P(x, y) \le \max\limits_{x \in X} P(x, \bar{y})$

V_I

$$\varphi(x, y^*) \le \varphi(x^*, y^*) \le \varphi(x^*, y), \quad y \in Y$$

$$v = \min_{y} \varphi(x^*, y) = \max_{x \in X} \varphi(x, y^*)$$

$$\varphi(x, y^*) \le \varphi(x^*, y^*) \le \varphi(x^*, y), \quad y \in Y$$
$$x \in X$$

може да има две равновесия по Неш.

$$(x_1^*, y_1^*) = \varphi(x_2^*, y_2^*)$$
$$(x_1^*, y_2^*) - (x_2^*, y_2^*)$$

$$\varphi(x_1, y_1^*) \le \varphi(x_1^*, y_1^*) \le \varphi(x_1^*, y_1), \quad y \in Y$$
$$x \in X$$

$$\varphi(x, y_2^*) \le \varphi(x_2^*, y_2^*) \le \varphi(x_2^*, y)$$

$$\varphi(x_2^*, y_1^*) \le \varphi(x_1^*, y_1^*) \le \varphi(x_1^*, y_2^*) \le \varphi(x_2^*, y_2^*) \le \varphi(x_2^*, y_1^*)$$

$$\varphi(x, y_2^*) \le \varphi(x_1^*, y_2^*) \le \varphi(x_1^*, y)$$

$x \ni \left[0, \frac{1}{2}\right] \ni y$

$P(x, y) = 1 - x - y$

$q(z) = \frac{z}{2} - \frac{3}{4} z^2$

$u_1(x, y) = x(1 - x - y) - q(x)$

$u_2(x, y) = y(1 - x - y) - q(y)$

$$\begin{cases} u_1(x, y) = 1 - x - y + \cancel{x} - \frac{1}{2} + \frac{3}{2}x \\ u_2(x, y) = 1 - x - \cancel{y} + \cancel{y} - \frac{1}{2} + \frac{3}{2}y \end{cases}$$

$u_1 = \frac{1}{2} - y + \frac{3}{2}x$

$u_2 = \frac{1}{2} - x + \frac{3}{2}y$

$u_1 = x(1 - x - y) - \frac{x}{2} + \frac{3}{4}x^2$

$= x - x^2 - xy - \frac{x}{2} + \frac{3}{4}x^2$

$u_1' = 1 - 2x - y - \frac{1}{2} + \frac{3}{2}x = \frac{1}{2} - \frac{1}{2}x - y$

$$\begin{cases} y = \frac{1}{2} - \frac{1}{2}x \\ x = \frac{1}{2} - \frac{1}{2}y \end{cases}$$

$x_1^* = 0 \to u_2(0, y) \to \max$

$\to u_1\left(x, \frac{1}{2}\right) \to \max$

$y^* = \frac{1}{2} \quad x^* = 0$

$y = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}y\right)$

$y = \frac{1}{2} - \frac{1}{4} + \frac{1}{4}y$

$y = \frac{1}{4} + \frac{1}{4}y$

$\frac{3}{4}y = \frac{1}{4}$

$\boxed{y = \frac{1}{3} \quad x = \frac{1}{3}}$

$\left(\frac{1}{2}, 0\right)\left(0, \frac{1}{2}\right)\left(\frac{1}{3}, \frac{1}{3}\right)$

Теория на игрите

x	y	u_1	u_2
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{36}$	$\frac{1}{36}$
$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{36}$	$\frac{1}{48}$
0	$\frac{1}{2}$	0	$-\frac{1}{16}$
$\frac{1}{2}$	0	$-\frac{1}{16}$	0

$$\begin{array}{c} x_1 \\ x_2 \end{array} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$X = \{(x_1, x_2) \in \mathbb{R}^2,\ x_1 \geq 0,\ x_2 \geq 0,\ x_1 + x_2 = 1\}$ - множеството от смесени стратегии

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$$X = \{(x_1 \dots x_n) \in \mathbb{R}^n,\ x_i \geq 0,\ i = 1, \dots m,\ \sum_{i=1}^{m} x_i = 1\}$$

$$Y = \{(y_1 \dots y_n) \in \mathbb{R}^n,\ y_i \geq 0,\ i = 1, \dots n,\ \sum_{i=1}^{n} x_i = 1\}$$

$$P(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} x_i \; x_j \; a_{ij} = \sum_{i=1}^{m} x_i \left(\sum_{j=1}^{n} a_{ij} y_j\right) =$$

$$= \sum_{j=1}^{n} y_j \left(\sum_{i=1}^{m} a_{ij} x_i\right) =$$

$$= \sum_{i=1}^{m} x_i P(i,y) = \sum_{j=1}^{n} y \cdot P(x,j)$$

$$P(x,y) = \sum_{i=1}^{m} x_i P(i,y) = \sum_{j=}^{n} y_j = P(x,j)$$

$x \in X$ $\quad$ $y \in Y$

$$P(x,y^*) \le P(x^*,y^*) \le P(x^*,y)$$

$x^* = (x_1^* \dots x^*_m)$ $\quad$ $x_i^* > 0 \to P(i_0, y^*) = V$

$y^* = (y_1^* \dots y^*_n)$ $\quad$ $y_j^* > 0 \to P(x^*, j_0) = P(x^*, y^*)$

$P(i_0, y^*) < P(x^*, y^*) \cdot x_0^*$

$P(i_0, y^*) < P(x^*, y^*) \; x_0^*$

$P(i, y^*) \le P(x^*, y^*) \, x_i^*$ $\quad$ $i = 1, 2, \dots n, \; i_0 \ne 0$

$$\sum_{i=1}^{m} x_i^* P(i, y^*) < \underbrace{\sum_{i=1}^{m} x_i^*}_{1} P(x^*, y^*) = P(x^*, y^*)$$