

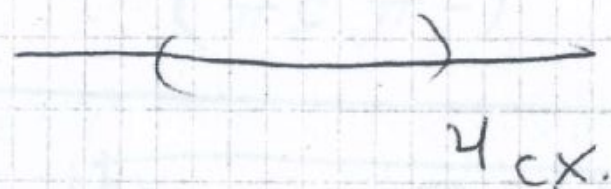
$$\frac{u_{n+1}}{u_n} = \frac{3n-1}{3n+3} \rightarrow 1 < 1$$

P-2

$$n \left(\frac{3n+3}{3n-1} - 1 \right) = n \left(\frac{3n+3-3n+1}{3n-1} \right) =$$

$$= \frac{4n}{3n-1} > 1 \Rightarrow \text{сх.}$$

4-сх.



Контролно: Диф.
смена пр.
Лагранж
НМС, НГС
редове
ст. редове
Фурье

ЗАДАЦИ ОТ ДЪРЖАВЕН

ИЗПИТ

30.05.2013г.

четвъртък

Лекция

заг. От Държавен Изпит 2007!

Да се развие в степенен ред
около т. $x = -1$

$$f(x) = \int_0^x \frac{t+1}{t^2+2t+2} dt$$

$$f'(x) = \frac{x+1}{x^2+2x+2} = \frac{(x+1)}{(x+1)^2+1} =$$

$$\frac{q}{x-1}$$

$$= (x+1) \sum_{n=0}^{\infty} (-1)^n (x+1)^{2n} =$$

$$\frac{1}{1+q} = 1 - q + q^2 - q^3 + \dots + (-q)^n + \dots =$$

$$= \sum_{n=0}^{\infty} (-q)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n (x+1)^{2n+1}$$

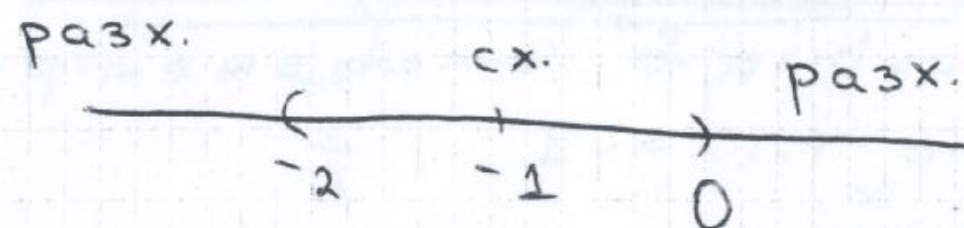
$$f(x) = \int f'(x) dx + C$$

$$R: |q| < 1$$

$$|x+1| < 1$$

$$-1 < x+1 < 1$$

$$-2 < x < 0$$



$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^{2n+2}}{2n+2} + C$$

$$f(0) = 0 + C = 0 \Rightarrow C = 0$$

$$x = 0$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+2}$$

сходящ

$$a_n = \frac{1}{2n+2}$$

↓ 0

$$x = -2$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot (-1)^{2n+2}}{2n+2}$$

сходящ

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+2}$$

сходящ

развита е в интервал
[-2, 0] по Тн Абел

$$\text{II.} \int \frac{t+1}{t^2+2t+2} dt =$$

$$= \frac{1}{2} \ln(1+(x+1)^2) - \frac{\ln 2}{2} = f(x)$$

степенен ред

$$\text{Заг. } f'(x) = \frac{1}{\sin x + 2\cos x + 3}$$

Търсим f $(-\pi, 2\pi)$

Интеграл на Пуассон

~~XXXXXX~~

$$J(z) = \int_0^{\pi} \ln(1 - 2z \cos x + z^2) dx$$

$$|z| < 1$$

Диф. пог знака на $\int$

$$J'(z) = \int_0^{\pi} \frac{-2z \cos x + 2z}{1 - 2z \cos x + z^2} dx$$

$$t = \tan \frac{x}{2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$I'(z) = \int_0^{\pi} \frac{-2 \frac{(1-t^2)}{(1+t^2)} + 2z}{1 - 2z \frac{(1-t^2)}{(1+t^2)} + z^2} \cdot \frac{2}{1+t^2} dt =$$

$$\arctg t = \frac{x}{2}$$

$$x = 2 \arctg t$$

$$dx = \frac{2}{1+t^2}$$

$$= \int_0^{\pi} \frac{-2(1-t^2) + 2z(1+t^2)}{1+t^2 - 2z(1-t^2) + z^2(1+t^2) + t^2} \cdot \frac{2}{1+t^2} dt =$$

$$= \int_0^{\pi} \frac{-2 + 2t^2 + 2z - 2zt^2}{1+t^2 - 2z + 2zt^2 + z^2 + z^2 t^2} \cdot \frac{2}{1+t^2} dt$$

$$= 4 \int_0^{\pi} \frac{-1 + t^2 + z - zt^2}{1+t^2 - 2z + 2zt^2 + z^2 + z^2 t^2} dt$$

$$= 4 \int_0^{\infty} \frac{-(1-z) + t^2(z+1)}{(1+t^2) \{ (1-z)^2 + (1+z)^2 t^2 \}} dt =$$

$$= \frac{4(1+z)}{(1+z)^2} \int_0^{\infty} \frac{t^2 - \frac{(1-z)}{1+z}}{(1+t^2) \left(t^2 + \left(\frac{1-z}{1+z} \right)^2 \right)} dt$$

$$\frac{1-z}{1+z} = a$$

$$= \frac{4}{1+z} \int_0^{\infty} \frac{t^2 - a}{(1+t^2)(t^2 + a^2)} dt$$

$$\frac{t^2 - a}{(1+t^2)(t^2 + a^2)} = \frac{u - a}{(1+u)(u + a^2)} =$$

$$u = t^2$$

$$= \frac{A}{1+u} + \frac{B}{u+a^2}$$

$$u = -1$$

$$u = -a^2$$

$$A = \frac{1}{1-a}$$

$$B = \frac{a}{a-1}$$

$$y' = \dots \left[\frac{1}{1-a} \int_0^{\infty} \frac{dt}{1+t^2} - \frac{1}{1-a} \int_0^{\infty} \frac{dt}{a^2+t^2} \right] = \frac{3ag}{2}$$

" $\arctg t$ " $\frac{1}{a} \arctg \frac{t}{a}$

$$= \frac{4(x+a)}{(1+x)(1-a)} \left[\frac{\pi}{2} - 0 - \frac{a}{a} \left(\frac{\pi}{2} - 0 \right) \right] = 0$$

! $a > 0$

$$y'(x) = 0 \Rightarrow y(x) = \text{const}$$

$$y(x) = y(0) = 0$$

zag 2004

$$f(x) = \frac{x^4-1}{4} \arctg x - \frac{x^3}{12} + \frac{x}{4}$$

$$f'(x) = \frac{4x^3}{4} \arctg x + \frac{x^4-1}{4} \cdot \frac{1}{x^2+1} -$$

$$- \frac{3x^2}{12} + \frac{1}{4}$$

4 up

4

$$\sum_{n=0}^{\infty} \frac{5^n (x+2)^n}{8 \sqrt[n]{n+1}}$$

~~Корр-Агмар~~ Корр-Агмар

$$R = \frac{1}{\lim_{n \rightarrow \infty} \sqrt[n]{a_n}} = \frac{1}{\lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{8} \sqrt[n]{n+1}}}$$

$$\sqrt[n]{8} \rightarrow 1$$

$$\sqrt[n]{n+1} = \sqrt[n]{n+1} \rightarrow \sqrt{1}$$

$$\Rightarrow R = \frac{1}{5}$$

разх. $\overbrace{\hspace{2cm}}^{cx}$ разх.

$-2 - \frac{1}{5}$ $-2 + \frac{1}{5}$

$$x = -2 - \frac{1}{5}$$

$$\sum_{n=0}^{\infty} \frac{5^n \left(-\frac{1}{5}\right)^n}{8 \sqrt[n]{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{8 \sqrt[n]{n+1}}$$

сходимость

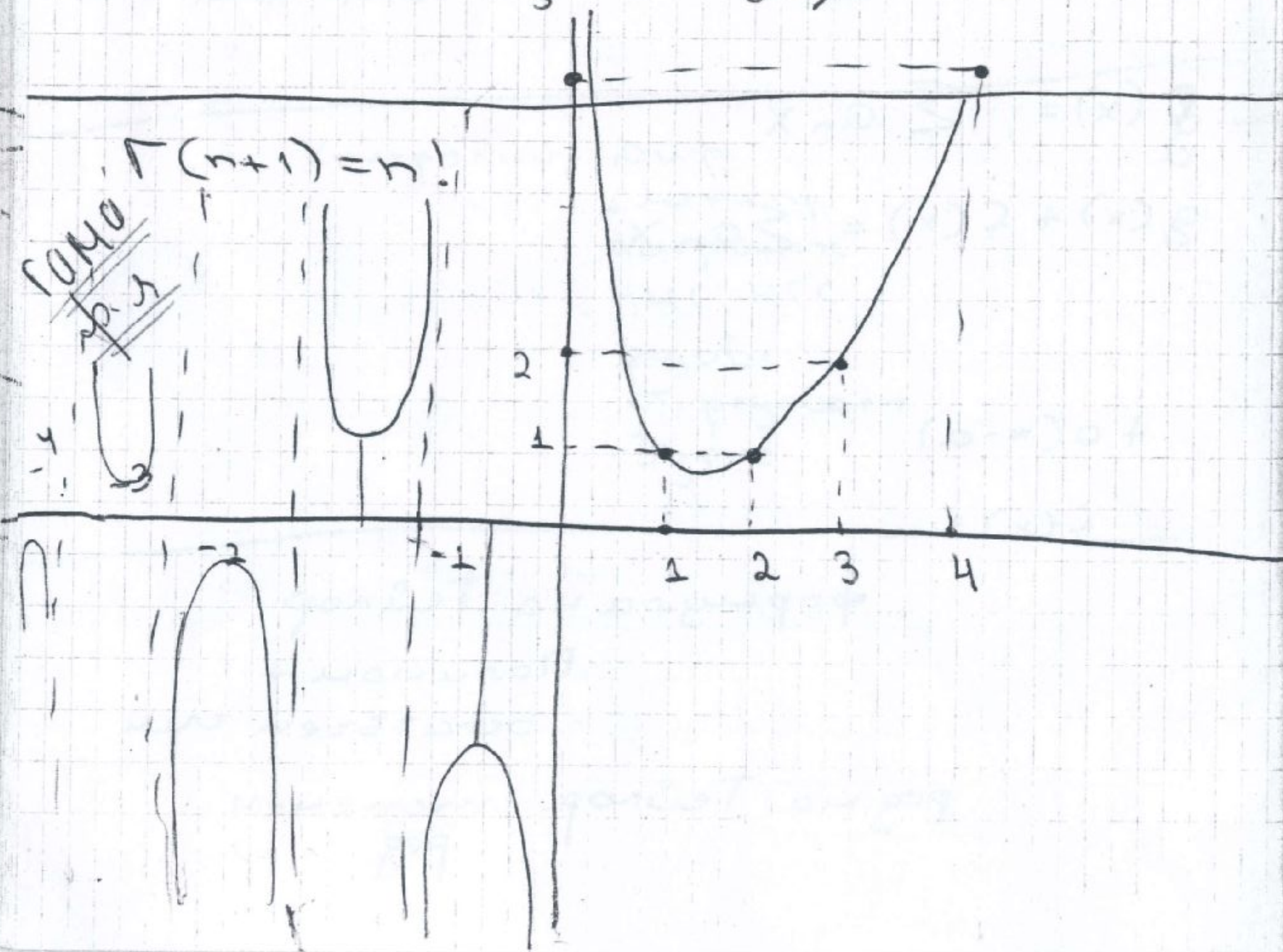
$$\frac{1}{8 \sqrt[n]{n+1}} \rightarrow 0$$

$$x = -2 + \frac{1}{5}$$

$$\sum_{n=0}^{\infty} \frac{5^n \frac{1}{5^n}}{8 \sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{1}{8 \sqrt{n+1}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}} - \text{разходещу}$$

$$\left[-2 - \frac{1}{5}, 2 + \frac{1}{5}\right)$$



Бета ф. 1

$$B(a, b) = \frac{\Gamma(a) \cdot \Gamma(b)}{\Gamma(a+b)} \quad \text{Дурихле}$$

Ойлерови интегралы

$$\Gamma(x+1) = x \Gamma(x)$$

$$\Gamma(x) > 0$$

$$(\ln \Gamma(x))'' > 0$$

$$\Gamma(1) = 1$$

$$x > 0$$

Условия на
Бор. Молерун

Ако ги изпълнява $\Rightarrow$ тя е Γ функцията

Нека ϕ

$$\phi(x+1) = x \phi(x)$$

$$\phi(x) \cdot \phi\left(x + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2x-1}} \phi(2x)$$

$$\phi(x) \cdot \phi(1-x) = \frac{\pi}{\sin \pi x}$$

за ϕ са
изпълнени

309

2016

$$\int \frac{1 - \cos x}{(5 + 3 \sin x)(5 + 4 \cos x)} dx$$

сметки :-

309

2010

2006

$$\int_{\frac{1}{3}}^{\frac{2}{3}} \frac{x^4 - 2x - 1}{x^5 - x} dx$$

309.2012

$$f(x) = \frac{1}{6} \frac{\ln(x-1)^2}{x^2+x+1} + \frac{1}{\sqrt{3}} \arctg \frac{2x+1}{\sqrt{3}}$$

Упражнение

$f(x) =$

$$f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a) + \dots$$

ред на Тейлор за $f(x)$

6

~~310~~

$$\begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases} = f(x) - \text{непрекъсната}$$

$$f^{(n)}(x) = \begin{cases} P(\frac{1}{x}) \cdot e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

по нми

$0 + 0 + \dots + 0 \neq$ ред Тейлор
ст-т ф-я

$$g(x) = \sum a_n x^n$$

$$g(x) + f(x) = \sum a_n x^n$$

$$+ o(x-a)^{n+1}$$

$$f(x) = \dots$$

формула на Тейлор

Полином +

остатък и т.н

ред на Тейлор - степенен
ред