Домашно №1 по ДИС - II

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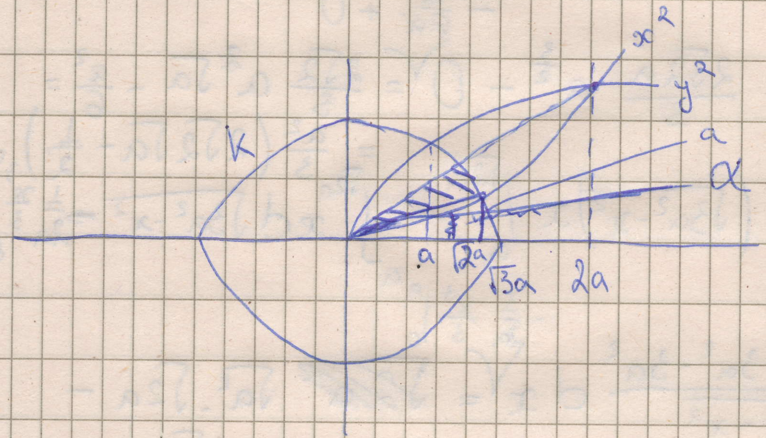
(1) Да се пресметне лицето на фигурата в I квадрант.
Окръжност: $x^2+y^2=3a^2$
Параболи: $x^2=2ay$; $y^2=2ax$

$\begin{cases} x^2+y^2=3a^2 \\ x^2=2ay \\ y^2=2ax \end{cases}$ $\quad x^2=2ay \Rightarrow y=\frac{x^2}{2a} \Big|^2 \quad y^2=\frac{x^4}{4a^2}$

$y^2=2ax \quad 2ax=\frac{x^{4}}{4a^2} \quad x^3=8a^3$

$x=2a$

не разглеждам $x=-2a$, защото желаем да разгледаме само I квадрант, а там стойностите за x и y са положителни.



$0\leq 3a^2$

$x^2+2ax=3a^2$
$x^2+2ax-3a^2=0$
ДС: $x>0$
$x_{1,2}=-a\pm\sqrt{a^2+3a^2}=-a\pm 2a \begin{cases} a\in ДС \\ -3a\notin ДС \end{cases}$

$y^2+2ay=3a^2$

$y^2+2ay-3a^2=0$

$D(: y\ge 0$

$y_{1,2}=-a\pm\sqrt{4a^2}=-a\pm 2a \begin{array}{l}\nearrow a \in Dc\\ \searrow -3a \notin Dc\end{array}$

$x^2=2a^2$

$x=\sqrt{2}a \Rightarrow \cancel{y=\frac{x^2}{2a}=\frac{2a^2}{2a}=a}$ $\quad y^2=2a(\sqrt{2}a)=2\sqrt{2}a^2 \Rightarrow y=\sqrt[4]{8}a$

$G_1 \begin{cases} 0\le x\le a \\ \frac{x^2}{2a}\le y\le\sqrt{2ax}\end{cases}$ $\qquad G_2 \begin{cases} a\le x\le\sqrt{2}a \\ \frac{x^2}{2a}\le y\le\sqrt{3a^2-x^2}\end{cases}$

$S_1=\int_0^a\left(\sqrt{2ax}-\frac{x^2}{2a}\right)dx$ $\qquad S_2=\int_a^{\sqrt{2}a}\left(\sqrt{3a^2-x^2}-\frac{x^2}{2a}\right)dx$

$S_1=\int_0^a\sqrt{2ax}\,dx-\int_0^a\frac{x^2}{2a}dx=\sqrt{2a}\int_0^a\sqrt{x}\,dx-\frac{1}{2a}\int_0^a x^2dx=$

$=\sqrt{2a}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^a-\frac{1}{2a}\,\frac{x^3}{3}\Big|_0^a=\frac{2\sqrt{2a}}{3}a^{\frac{3}{2}}-0 \underbrace{-\frac{\cancel{a^3}\,^2}{6\cancel{a}}+0}=\frac{2\sqrt{2}}{3}a^{\cancel{3}2}\sqrt{a}-\frac{a^2}{6}=$

$=\frac{a^2}{3}\left(2\sqrt{2}\sqrt{a}-\frac{1}{2}\right)$

$S_2=\int_a^{\sqrt{2}a}\sqrt{3a^2-x^2}\,dx-\int_a^{\sqrt{2}a}\frac{x^2}{2a}dx=\left(\sqrt{3a^2-x^2}\right)x\Big|_a^{\sqrt{2}a}-\int_a^{\sqrt{2}a}x\,d\sqrt{3a^2-x^2}-\frac{1}{2a}\frac{x^3}{3}\Big|_0^a=$

$=\left(\sqrt{3a^2-x^2}\right)x\Big|_a^{\sqrt{2}a}-\int_a^{\sqrt{2}a}\frac{-x^2+3a^2-3a^2}{\sqrt{3a^2-x^2}}dx \underbrace{-\frac{1}{2a}\frac{x^3}{3}\Big|_0^a}=\cancel{\sqrt{2a^2}}\sqrt{a^2}\cdot\sqrt{2}a-$

$-\sqrt{2a^2}\cdot a\ \cancel{-\frac{1}{6}a^2+0}-S_2+\cancel{\frac{1}{2a}\frac{x^3}{3}\Big|_0^a-\frac{1}{2a}\frac{x^3}{3}\Big|_0^a}+\int_a^{\sqrt{2}a}\frac{3a^2}{\sqrt{3a^2-x^2}}dx=$

$=\cancel{\sqrt{2}a^2}-\cancel{\sqrt{2}a^2}-S_2+\int_a^{\sqrt{2}a}\frac{3a^2}{\sqrt{3a^2-x^2}}dx=-S_2+\frac{3a^2}{\cancel{\sqrt{3}a}}\cdot\cancel{\sqrt{3}a}\int_a^{\sqrt{2}a}\frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{3}a}\right)^2}}d\frac{x}{\sqrt{3}a}=$

$=-S_2+3a^2\arcsin\left(\frac{x}{\sqrt{3}a}\right)\Big|_a^{\sqrt{2}a}=-S_2+3a^2\arcsin\left(\frac{\sqrt{2}}{\sqrt{3}}\right)-3a^2\arcsin\left(\frac{1}{\sqrt{3}}\right)$

$S=S_1+S_2=\frac{a^2}{3}\left(2\sqrt{2}\sqrt{a}-\frac{1}{2}\right)+\frac{3}{2}a^2\left(\arcsin\frac{\sqrt{2}}{\sqrt{3}}-\arcsin\frac{1}{\sqrt{3}}\right)$

$\boxed{2}$

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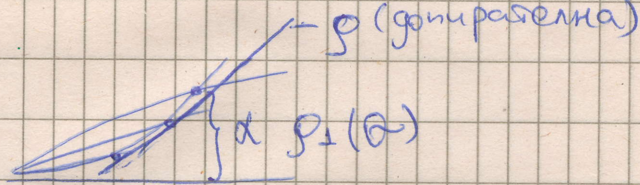
$\sin\beta = \frac{\sqrt{2}}{\sqrt{3}}$ $\quad \sin\alpha = \frac{1}{\sqrt{3}}$

$\underset{\cos^2\alpha}{\underset{\parallel}{\sin^2\beta}} + \sin^2\alpha = 1$

$\sin\alpha = \frac{1}{\sqrt{3}}$ $\quad \alpha = \arcsin\frac{1}{\sqrt{3}}$ $\quad \arcsin\frac{1}{\sqrt{3}} = \frac{\pi}{2} - \arcsin\frac{1}{\sqrt{3}}$

$S = \frac{1}{2}\left(\frac{\pi}{4} - \arcsin\frac{1}{\sqrt{3}}\right)$

$\cancel{S_2}$ $\rho = \frac{2a\sin\theta}{\cos^2\theta}$

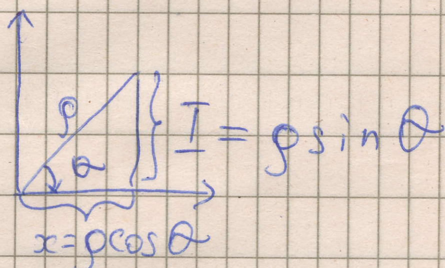


$0 \le \theta \le \alpha$

$0 \le \rho \le \rho_\perp(\theta)$

$$S_2 = \frac{1}{2}\int_0^\alpha \frac{4a^2\sin^2\theta}{\cos^4\theta}\,dt = \frac{1}{2}\,tg^3\left(\frac{\theta}{3}\right)\Big|_0^\alpha$$

$$\underline{I} = 2a^2\,\frac{tg^3\theta}{3}\Big|_0^\alpha = \frac{2a^2}{3}\,\frac{\sin^3\alpha}{\sqrt{1-\sin^2\alpha}} =$$



$$= \frac{2a^2}{3}\cdot\frac{1}{3\sqrt{3}\cdot\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^3} = \frac{1}{\cancel{9}\cancel{\sqrt{3}}_{3}}\cdot\frac{\cancel{2}a^2\cdot\cancel{3}\cancel{\sqrt{3}}^{1}}{\cancel{2}\sqrt{2}} =$$

$\cancel{\rho^2}^{1}\cos^2\theta = 2a\,\cancel{\rho}\sin\theta$

$\rho = 2a\,\frac{\sin\theta}{\cos^2\theta}$

$$= \frac{2a^2}{\cancel{2}\sqrt{2}}$$

$$S = \left(\frac{\pi}{4} - \arcsin\frac{1}{\sqrt{3}}\right)\cdot 3a^2 + \frac{2}{2\sqrt{2}}\cdot\frac{2a^2}{3}$$