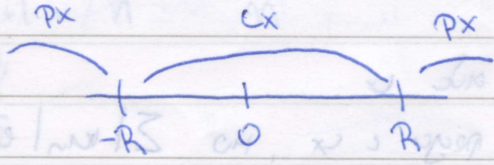
29.05.2013г. ДИС - упражнение

$\sum_{n=0}^{\infty} a_n x^n$ - степенни редове

$\sum a_n (ax+b)^n$

1. $R \geq 0$



R радиус на сходимост

2. $f(x) = \sum_{n=0}^{\infty} a_n x^n$ $\quad f'(x) = \sum_{n=0}^{\infty} n . a_n x^{n-1}$

$$\int f(x) dx = \sum_{n=0} a_n \frac{x^{n+1}}{n+1} + C$$

двата реда имат един и същи радиус

3. област на сходимост

4. в областта на сх. редът $f(x) = \sum a_n x^n$
представлява непрекъснатата f
Th на Абел

Коши

$\sum u_n$ < 1 сх абс. сх.

1. $\sqrt[n]{|u_n|} \to l$ > 1 рх

= 1 $\sqrt[n]{u_n} \geq 1$ без граници ако е изп. рх
ако не няма резултат

② $\left|\frac{u_{n+1}}{u_n}\right| \longrightarrow \ell$ <1 абс. сх.

>1 рх

$\left|\frac{u_{n+1}}{u_n}\right| \geq 1$ рх.

③ $n\left(\left|\frac{u_n}{u_{n+1}}\right| - 1\right) \xrightarrow{\quad}$ $\lambda > 1$ абс сх

$\lambda < 1$ реда е сх, но $\sum|u_n|$ е рх

$\lambda \geq 0$

$\lambda > 0$ $\quad |u_n| \longrightarrow 0$

Лайбниц.

④ $\sum\limits_{n=0}^{\infty} (-1)^n u_n$, $\quad u_n > u_{n+1}$

$u_n > 0$ то реда е сх.

$u_n \to 0$

Пример

$\sum\limits_{n=1}^{\infty} \frac{x^n}{2n+1}$

$\sqrt[n]{a_n} \to a$

$a_n \to a > 0$

$\sqrt[n]{a} \to 1 \quad a > 0$

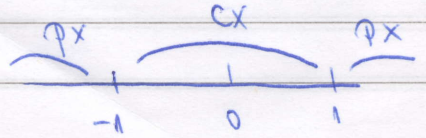
$\sqrt[n]{n} \to 1$

$$\sqrt[n]{\left|\frac{x^n}{2n+1}\right|} = \left|\frac{x}{\sqrt[n]{2n+1}}\right| =$$

$$= \sqrt[n]{2n+1} = \underset{\overset{\downarrow}{1}}{\sqrt[n]{n}}\ \underset{\overset{\downarrow}{1}}{\sqrt[n]{2+\frac{1}{n}}} \Rightarrow \longrightarrow 1$$

$\Rightarrow \longrightarrow |x|$ ако $|x| < 1$ сх.

$|x| > 1$ рх



радиуса е 1

ако търсим областта

$$\sum \frac{1}{2n+1}$$

$$\frac{1}{2n+1} = \underset{\underset{\frac{1}{2}}{\downarrow}}{\frac{n}{2n+1}} \cdot \frac{1}{n} \sim \frac{1}{n} \longrightarrow \text{рх.}$$

$$\sum \frac{(-1)^n}{2n+1}$$ по Лайбниц.

$\frac{1}{2n+1}$ – намаляваща $\to 0$

$\rightsquigarrow$

задача

$$\sum_{n=0} \binom{2n}{n} x^n$$

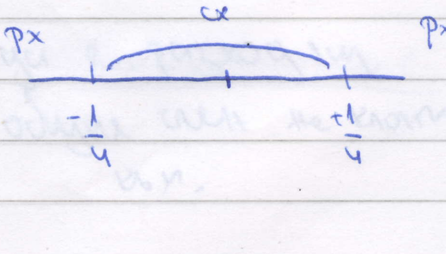
$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{\binom{2n+2}{n+1} x^{n+1}}{\binom{2n}{n} x^n} = (2n+2)(2.$$

$$= \frac{\overbrace{(2n+2)(2n+1)\,2n \;\ldots\; (2n+2-n-\cancel{1}+\cancel{1})}^{n+2}\, x^{n+1} \quad \cancel{n!}}{(n+1)\,\cancel{n!} \quad \underbrace{2n\,(2n-1) \;\ldots\; (2n-n+1)}_{n+1} \quad x^n} = \frac{|x|\,(\overset{2}{\cancel{2n+2}})\,(2n+1)}{(n+1)\,\cancel{(n+1)}} =$$

$$= |x|\,\frac{2\,(2n+1)}{n+1} \longrightarrow 4|x|$$

$|x| < \frac{1}{4}$ сх

$|x| > \frac{1}{4}$ рх



при $x = \frac{1}{4}$ ~~4~~ заместване в $4|x|^2 \frac{|2n+1|}{n+1} = \frac{4n+2}{4n+4} < 1$

нищо не знаем

$$n\left(\frac{u_n}{u_{n+1}} - 1\right) = n\left(\frac{4n+4}{4n+2} - \frac{4n+2}{4n+2}\right) = n\left(\frac{2}{4n+2}\right) = \frac{1n}{2n+1} \to \frac{1}{2}$$

$\Rightarrow$ реда е рх

при $x = -\frac{1}{4}$

$$\sum \underbrace{\binom{2n}{n}\frac{(-1)^n}{4^n}}_{u_n}$$

от Th $\lambda > 0$

$\Rightarrow |u_n| \to 0$

и понеже границата на u_n е $\frac{1}{2}$

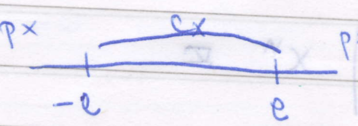
ако $\sum (-1)^n u_n$

$u_n > u_{n+1} > 0$

$u_n \to 0 \Rightarrow$ сх

$$\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n = \left|\frac{u_{n+1}}{u_n}\right| =$$

$$= \left|\frac{\cancel{(n+1)}\cancel{n!} \; \cancel{x^{n+1}} \; \cancel{n^n}}{(n+1)^{n+1} \; \cancel{n!} \; \cancel{x^n}}\right| = \frac{|x|}{\left(1+\frac{1}{n}\right)^n} \longrightarrow \frac{|x|}{e}$$



$\left(1+\frac{1}{n}\right)^n \longrightarrow e$

монот. ↑ редица

$\left(1+\frac{1}{n}\right)^n < e$ монотонно ↓

$\left(1+\frac{1}{n}\right)^{n+1} \longrightarrow e$

~~монотонна ред.~~

$\Rightarrow$ границата е в края.

$$\left(1+\frac{1}{n}\right)^n < e < \left(1+\frac{1}{n}\right)^{n+1}$$

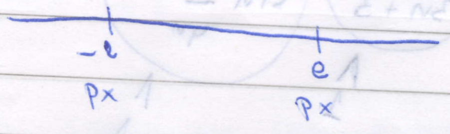
$x = e$

$$\frac{u_{n+1}}{u_n} = \frac{e}{\left(1+\frac{1}{n}\right)^n} > 1$$

$\Rightarrow$ от 2 рх.

$u_{n+1} > u_n > u_1 > 0$

$\Rightarrow u_n \not\to 0$



$$\sum_{n=1}^{\infty} \underbrace{\frac{n!}{n^n} e^n}_{\not\to 0} \Rightarrow \sum \underbrace{\frac{n!}{n^n} |-e|^n}_{\not\to 0}$$

$\Rightarrow$ реда е разходящ. ако общия член не клони към.

задача

$$\sum_{n=1}^{\infty}\left(\frac{n}{e}\right)^{n}\frac{1}{n!}\left(-2x\right)^{n}$$

за упражнение

задача

$$\frac{1}{3}\sum_{n=2}^{\infty}\frac{2.5\dots(3n-4)}{\cancel{3}.6\dots3n}\sin\left(\frac{1}{4n}\right)x^{n}$$

$$\frac{\left|\frac{2.5\dots(3n-1)}{6\dots3n+3}\right|\sin\left|\frac{1}{4^{n+1}}\right|x^{n+1}}{\quad}\qquad\frac{6\dots3n}{\quad}$$

2.5....

$$|x|\frac{\cancel{2.5\dots(3n-4)}(3n-1)\;\sin\frac{1}{4^{n+1}}\;\cancel{6\dots3n}\;\frac{1}{4^n}\cdot\cancel{4^n}}{\cancel{6.4\dots3n}\underset{\downarrow 1}{(3n+3)}\;:\frac{1}{4^{n+1}}\cdot\cancel{4^{n+1}}\;\cancel{2\dots(3n-4)}\;\sin\frac{1}{4^n}}=$$

$$=\cancel{|x|\cdot(3n-4)}\qquad\frac{|x|}{4}\leftarrow$$

$x = 4$

$$4\cdot\underset{\overset{\wedge}{1}}{\frac{3n-1}{3n+3}}\;\underset{\overset{\wedge}{1}}{\frac{\sin\frac{1}{4^{n+1}}}{\sin\frac{1}{4^n}}}$$

$$\frac{2.5\dots(3n-4)}{6\dots3n}\;\frac{\sin\frac{1}{4^n}}{\frac{1}{4^n}}\sim\frac{2.5\dots3n-4}{6\dots3n}\qquad\frac{3n-1}{3n+3}<1$$

$n\left|\frac{3n+3}{3n-1}-1\right| = \frac{4n}{3n-1} \longrightarrow \frac{4}{3}$ абс. сх. от ③

в -4 ник Сх. $\Rightarrow$ е абсолютно сх