30.05.2013г. ДИС - лекция

Задачи от ДУ

1зад. 2007

$$f(x) = \int \frac{t+1}{t^2+2t+2}\,dt$$

задача

$$f'(x) = \frac{1}{\sin x + 2\cos x + 3}$$

търси се f в $(-\pi, 2\pi)$

Интеграл на Пуасон

$$I(r) = \int_0^{\pi} \ln(1 - 2r\cos x + r^2)\,dx \qquad |r|<1$$

$$I'(r) = \int_0^{\frac{\pi}{2}} \frac{-2\cos x + 2r}{1 - 2r\cos x + r^2} =$$

$$= 2\int_0^{\infty} \frac{\frac{t^2-1}{t^2+1} + r}{1 + \frac{2r(t^2-1)}{1+t^2}} \cdot \frac{2dt}{1+t^2} =$$

$$= \int_0^{\infty} \frac{t^2 - 1 + r t^2 + r}{(1+t^2)\left[1 + t^2 + 2rt^2 - 2r + r^2 + r^2t^2\right]}\,dt =$$

$$= 4\int_0^{\infty} \frac{-(1-r) + t^2(1+r)}{(1+t^2)\left[(1-r)^2 + (1+r)^2 t^2\right]}\,dt = \frac{4(1+r)}{(1+r)^2}\int_0^{\infty} \frac{t^2 - \frac{1-r}{1+r}}{(1+t^2)\left(t^2 + \left(\frac{1-r}{1+r}\right)^2\right)}\,dt =$$

$$= \frac{4}{1+r} \int_0^{\infty} \frac{t^2 - a}{(1+t^2)(t^2+a^2)}\,dt =$$

$$\frac{t^2 - a}{(1+t^2)(t^2+a^2)} = \frac{u-a}{(1+u)(u+a^2)} = \frac{A}{1+u} + \frac{B}{u+a^2} \quad \dots$$

$A = \frac{1}{1-a} \qquad B = \frac{a}{a-1}$

$$I' = 4(1+r)\left[\frac{1}{1-a}\int_0^\infty \frac{dt}{1+t^2} - \frac{a}{1-a}\int_0^\infty \frac{dt}{a^2+t^2}\right] =$$

$$\underset{\text{}}{\int_0^\infty \frac{dt}{1+t^2}} = \operatorname{arctg} t \Big|_0^\infty \qquad \int_0^\infty \frac{dt}{a^2+t^2} = \frac{1}{a}\operatorname{arctg}\frac{t}{a}\Big|_0^\infty$$

$$= \frac{4(1+r)}{1-a}\left[\frac{\pi}{2} - 0 - \frac{a}{a}\left(\frac{\pi}{2} - 0\right)\right] = 0$$

$I'(r) = 0 \Rightarrow I(r) = const \qquad I(r) = I(0) = 0$

при $r \in (-1;1) \qquad r = 0$

задача $\sum_{n=0}^{\infty} \frac{5^n (x+2)^n}{8\sqrt{n+1}}$

$\sqrt[n]{8} \to 1$

$\sqrt[n]{\sqrt{n+1}} = \sqrt{\sqrt[n]{n+1}} \to 1$

$R = \frac{1}{\lim \sqrt[n]{}}$

$x = -2 - \frac{1}{5} \qquad \sum_{n=0}^{\infty} \frac{5^n \left(-\frac{1}{5}\right)^n}{8\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{8\sqrt{n+1}} \qquad \frac{1}{8\sqrt{n+1}} \downarrow 0$

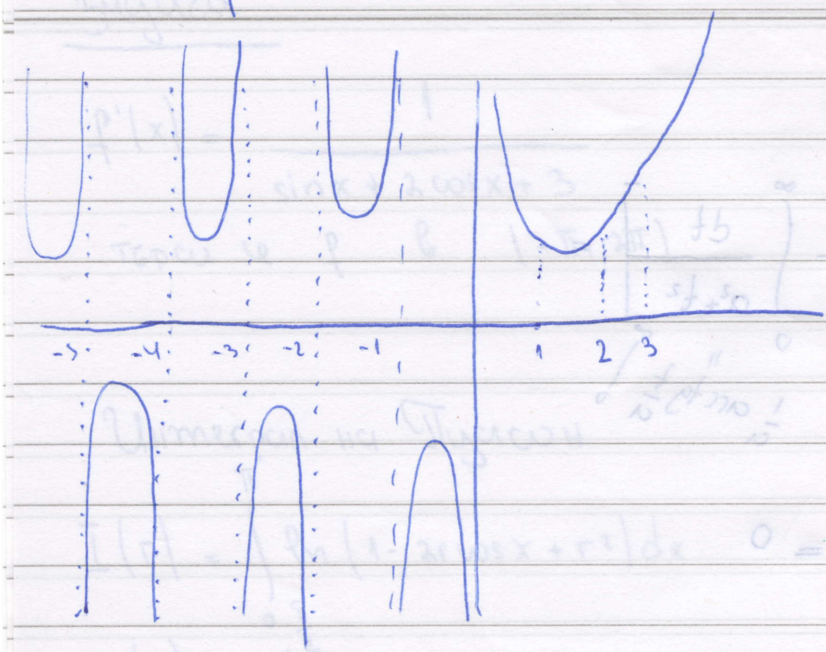
$x = -2 + \frac{1}{5} \qquad \sum \frac{5^n \frac{1}{5^n}}{8\sqrt{n+1}} = \sum \frac{1}{8} \frac{1}{\sqrt{n+1}}$

$\sum_n \frac{1}{n^{1/2}}$ - рх.

рх $\quad -2-\frac{1}{5} \quad$ сх $\quad -2+\frac{1}{5} \quad$ рх

$R = \frac{1}{5}$

Гама f



$$\Gamma(n+1) = n!$$

$$B(a,b) = \frac{\Gamma(a)\cdot\Gamma(b)}{\Gamma(a+b)}$$

Условия на Бор-Морелуп

$$\begin{cases} \Gamma(x+1) = x\Gamma(x) \\ \Gamma(x) > 0 \\ (\ln\Gamma(x))'' > 0 \\ \Gamma(1) = 1 \end{cases}$$

Нека φ

$$\varphi(x+1) = x\,\varphi(x)$$

$$\varphi(x)\,\varphi\left(x+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2x-1}}\,\varphi(2x)$$

$$\varphi(x)\,\varphi(1-x) = \frac{\pi}{\sin\pi x}$$

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$$\int \frac{(1-\cos x)\,dx}{(5+3\sin x)(5+4\cos x)}$$

$$\int_{\frac{1}{3}}^{\frac{2}{3}} \frac{x^4-2x-1}{x^5-x}\,dx$$

$$f(x) = \frac{1}{6}\ln\frac{(x-1)^2}{x^2+x+1}$$