16.05.2013 г. ~ ДИС ~

$f(x)$ – периодична с пер. $2l$

$-l \le x \le l$

$-\pi \le \frac{x\pi}{l} \le \pi$

Пол. $y = \frac{x\pi}{l}$, тогава $x = \frac{l \cdot y}{\pi}$ и $f(x) = f\left(\frac{ly}{\pi}\right) = g(y)$

$g(y)$ е 2π-периодична

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} g(y)\cos ky\,dy = \frac{1}{\pi}\int_{-\pi}^{\pi} f\left(\frac{ly}{\pi}\right)\cos ky\,dy = \frac{1}{\pi}\int f(x)$$

$$= \frac{1}{\pi}\int_{-l}^{l} f(x)\cos\frac{k\pi x}{l}\cdot\frac{\cancel{\pi}}{\cancel{l}}\,dx = \frac{1}{\pi}\int$$

$x = \frac{l \cdot y}{\pi}$, $d\cancel{y} = \frac{\pi}{l}dx$

$f(x)$ – $2l$-периодична

$$a_k = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{k\pi x}{l}\,dx$$

$$b_k = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{k\pi x}{l}\,dx$$

$$\boxed{\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot \cos\frac{k\pi x}{l} + b_k \cdot \sin\frac{k\pi x}{l}}$$

(К!)

Лема на Риман:

Нека ф-ията $g(t)$ е абсол. интегруема в $[a,b]$. Тогава

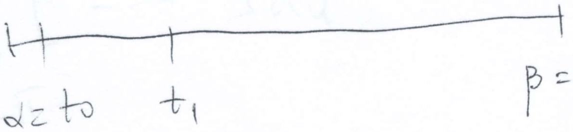
$$\lim_{p\to\infty}\int_\alpha^\beta g(t)\sin pt\,dt = 0.$$

Д-во: 1 сл.) $g(t)$ - интегруема, без особ. т-ки

$$\int_\alpha^\beta \sin pt\,dt = \frac{1}{p}\int_\alpha^\beta \sin pt\,dpt = -\frac{\cos pt}{p}\Big|_\alpha^\beta = \frac{-\cos p\beta + \cos p\alpha}{p}$$

$$\left|\frac{\cos p\alpha - \cos p\beta}{p}\right| \le \left|\frac{\cos p\alpha}{p}\right| + \left|\frac{\cos p\beta}{p}\right| = \frac{2}{p}$$

$$\Rightarrow \left|\int_\alpha^\beta \sin pt\,dt\right| \le \frac{2}{p}$$



$$\left|\int_\alpha^\beta g(t)\sin pt\,dt\right| = \left|\sum_{i=1}^n \int_{t_{i-1}}^{t_i} g(t)\sin pt\,dt\right| =$$

$m_i = \inf g(t)$ в (t_{i-1}, t_i) ; $M_i = \sup g(t)$; $\omega_i = M_i - m_i$

$$= \left|\sum_{i=1}^n \int_{t_{i-1}}^{t_i} (g(t) - m_i)\sin pt\,dt + \sum_{i=1}^n m_i \int_{t_{i-1}}^{t_i} \sin pt\,dt\right| \le$$

(*) $|g(t) - m_i||\sin pt| \le (g(t) - m_i)\cdot 1 \le M_i - m_i = \omega_i$

$$\le \sum_{i=1}^{n} \omega_i \int_{t_{i-1}}^{t_i} dt + \sum_{i=1}^{n} |m_i| \left| \int_{t_{i-1}}^{t_i} \sin pt\, dt \right| \le$$

$$\le \sum_{i=1}^{n} \omega_i \Delta t_i + \frac{2}{p} \sum_{i=1}^{n} |m_i| .$$

Зад. $\varepsilon > 0$. Нам. деление (τ) толкова (ситно / малко), че: $S_\tau - s_\tau < \frac{\varepsilon}{2}$.

$$\overset{\|}{\sum \omega_i \Delta t_i} < \frac{\varepsilon}{2}$$

Фиксир. дел-то и нам. по него m_i и изп. оценката:

$$\left| \int_{\alpha}^{\beta} g(t) \sin pt\, dt \right| \le \frac{\varepsilon}{2} + \frac{2}{p} \sum_{i=1}^{n} |m_i| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} \text{ за дост. гол. } p$$

фиксир., защото τ е фикс.

$$\Rightarrow \left| \int_{\alpha}^{\beta} g(t) \sin pt\, dt \right| < \varepsilon \text{ за дост. гол. } p, \text{ т.е. } \xrightarrow[p \to \infty]{} 0$$

2 сл.) $g(t)$ има особеност в т. β и $|g(t)|$ е интегр.

Тъй като $\int_{\alpha}^{\beta}$ е сход., то по ε се нам. δ, т.ч. $\int_{\beta-\delta}^{\beta} |g(t)|\, dt < \frac{\varepsilon}{2}$

$$\left| \int_{\beta-\delta}^{\beta} g(t) \underbrace{\sin pt}_{|\,| \le 1}\, dt \right| < \frac{\varepsilon}{2}$$

От $\left| \int_{\alpha}^{\beta-\delta} \right| < \frac{\varepsilon}{2}$ за дост. гол. p по 1 част.

Формула за n-тата парциална сума

$$S_n(x_0) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos m x_0 + b_m \sin m x_0 =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(u)\,du + \frac{1}{\pi}\sum_{m=1}^{n}\int_{-\pi}^{\pi} f(u)\left[\cos mu \cdot \cos m x_0 + \sin mu \sin m x_0\right)$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{\pi} f(u) \left[\frac{1}{2} + \sum_{m=1}^{n}\cos m(u-x_0)\right] du\right\} =$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(u)\underbrace{\frac{\sin (2n+1)\cdot\frac{u-x_0}{2}}{2\sin\frac{u-x_0}{2}}}_{2\pi\text{-периодична}}\,du =$$

$$= \frac{1}{\pi}\int_{x_0-\pi}^{x_0+\pi}\left[\ \ldots\ \right] du$$

$$u - x_0 = t$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x_0+t)\frac{\sin\frac{2n+1}{2}t}{2\sin\frac{t}{2}}\,dt = \left\{\int_0^{\pi} + \int_{-\pi}^{0}\right. \text{ — смяна } t \to -u$$

$$= \frac{1}{\pi}\left[\int_0^{\pi}\left[f(x_0+t) + f(x_0-t)\right]\cdot\underbrace{\frac{\sin\frac{2n+1}{2}\cdot t}{2\sin\frac{t}{2}}}_{\text{ядро на Дирихле}}\,dt\right.$$