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Приложения на Тh за неявните ф-ии.

Множители на Лагранж.

$F(x, u, v)$

$$\begin{cases} G_1(x,u,v)=0 \\ G_2(x,u,v)=0 \end{cases}$$

F, G_1, G_2 са дефин. и имат непрек. ч. произв. в ок. на т. (x_0, u_0, v_0)

$$\begin{cases} G_1(x_0,u_0,v_0)=0 \\ G_2(x_0,u_0,v_0)=0 \end{cases} ; \quad \begin{vmatrix} \frac{\partial G_1}{\partial u} & \frac{\partial G_2}{\partial v} \\ \frac{\partial G_2}{\partial u} & \frac{\partial G_2}{\partial u} \end{vmatrix}_{(x_0,u_0,v_0)} \neq 0$$

$\Rightarrow \exists$ две ф-ии (дифер.): $\varphi(x)$ и $\psi(x)$, заместващи u и v.

$$\begin{cases} G_1(x,\varphi(x),\psi(x))=0 \\ G_2(x,\varphi(x),\psi(x))=0 \end{cases} \quad (*)$$

Тh за неявните ф-ии

$\varphi(x_0)=u_0$; $\psi(x_0)=u_0$ (от Тh-та)

Разгл. ф-ята.

Тh: К!

Разгл. ф-ията $f(x)=F(x,\varphi(x),\psi(x))$.

Нека $f(x)$ има лок. extr. в т. x_0. Тогава $\exists$ две const λ_1, λ_2 (множит. на Лагранж), т.че за $\Phi=F-\lambda_1 . G_1-\lambda_2 . G_2$

$$\frac{\partial \Phi}{\partial x}(x_0,u_0,v_0)=0 \;;\; \frac{\partial \Phi}{\partial u}(x_0,u_0,v_0)=0 \;;\; \frac{\partial \Phi}{\partial v}(x_0,u_0,v_0)=0.$$

Д-во: Търсим λ_1 и λ_2: $\frac{\partial\Phi}{\partial u}(M_0)=0$, $\frac{\partial\Phi}{\partial v}(M_0)=0$,

където $M_0=(x_0,u_0,v_0)$

$$\left|\begin{array}{l} \frac{\partial F}{\partial u}-\lambda_1\cdot\frac{\partial G_1}{\partial u}-\lambda_2\cdot\frac{\partial G_2}{\partial u}=0 \quad (\text{в } M_0) \\ \frac{\partial F}{\partial v}-\lambda_1\frac{\partial G_1}{\partial v}-\lambda_2\frac{\partial G_2}{\partial v}=0 \quad (\text{в } M_0) \end{array}\right.$$

λ_1, λ_2 - неизв.

det на с-мата: $\begin{vmatrix} \frac{\partial G_1}{\partial u} & \frac{\partial G_2}{\partial u} \\ \frac{\partial G_1}{\partial v} & \frac{\partial G_2}{\partial v} \end{vmatrix}_{(M_0)} \neq 0$

Ще покажем, че с така намерените λ_1 и λ_2
$\frac{\partial\Phi}{\partial x}(M_0)=0$.

Тъй като $f(x)$ има лок. extr. в т. (x_0), то $f'(x_0)=0$

$$(*)\quad \frac{\partial F}{\partial x}+\frac{\partial F}{\partial u}\cdot\varphi'+\frac{\partial F}{\partial v}\psi'=0 \quad (\text{в т. } x_0) \ //(x_0,u_0,v_0)$$

Диференцираме $(*)$: $\left|\begin{array}{l} \frac{\partial G_1}{\partial x}+\frac{\partial G_1}{\partial u}\cdot\varphi'+\frac{\partial G_1}{\partial v}\psi'=0 \ /\cdot(-\lambda_1) \\ \frac{\partial G_2}{\partial x}+\frac{\partial G_2}{\partial u}\varphi'+\frac{\partial G_2}{\partial v}\cdot\psi'=0 \ /\cdot(-\lambda_2) \end{array}\right. (***)$

$(*)+(***)/\cdot(-\lambda_1)+(-\lambda_2)$: $\frac{\partial F}{\partial x}-\lambda_1\frac{\partial G_1}{\partial x}-\lambda_2\frac{\partial G_2}{\partial x}+\varphi'\underbrace{\left(\frac{\partial F}{\partial u}-\lambda_1\frac{\partial G_1}{\partial u}-\lambda_2\frac{\partial G_2}{\partial u}\right)}_{=0}+\psi'\underbrace{\left(\frac{\partial F}{\partial x}-\lambda_1\frac{\partial G_1}{\partial v}-\lambda_2\frac{\partial G_2}{\partial v}\right)}_{=0}=0$

(заради избора на λ_1 и λ_2)

$$\Rightarrow \frac{\partial F}{\partial x}-\lambda_1\frac{\partial G_1}{\partial x}-\lambda_2\frac{\partial G_2}{\partial x}=0 \ \text{в т. } (x_0,u_0,v_0)$$

Това е $\frac{\partial\Phi}{\partial x}=0$