

03.04.2013

ЗУС - упражнение

010002

Неявна функция

$$x^2 + y^2 - 1 = 0 \quad F(x, y) = 0$$

$$y = \sqrt{1-x^2}$$

$$y = -\sqrt{1-x^2}$$

$y(x)$

$$x^2 + \left| \sqrt{1-x^2} \right| - 1 \equiv 0$$

$$F(x, y(x)) = 0$$

$$\varepsilon(x) = \begin{cases} \text{поз.} \\ \text{отриц.} \end{cases}$$

$$y = \varepsilon(x) \sqrt{1-x^2}$$

F, F'_x, F'_y непрекъснати

$$F(x_0, y_0) = 0$$

$$F'_y(x_0, y_0) \neq 0$$

$\Rightarrow$ съществува ун. макс. амплит. на T x_0 , в която F -функцията която е определена от това уравнение

Пример. Взимаме $T = \left| \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right|$ в нея е изпълнено условието,

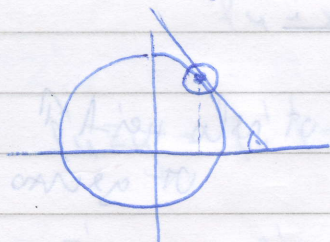
$$a = F'_y = 2y \neq 0$$

можем да намерим достатъчно

малка окръжност около тази точка, в която да има само една функция,

$\Rightarrow$ функцията има единствено

$$F(x, y(x)) \equiv 0$$



В нашия пример

$$F'_x + F'_y y' = 0$$

$$y' \left| \frac{1}{\sqrt{2}} \right| = - \frac{2 \cdot \frac{1}{\sqrt{2}}}{2 \cdot \frac{1}{\sqrt{2}}} = -1$$

$$\Rightarrow y'(x_0) = - \frac{F'_x(x_0, y_0)}{F'_y(x_0, y_0)}$$

$y' \neq$ между 0 и отрицателен
 $e = -1$

задача

$$F(x, y) = x^3 + y^3 - 3xy = 0$$

$$x^3 + y^3 - 3xy = 0$$

$$3x^2 - 3y = 0$$

$$y = \frac{3x^2}{3} = x^2$$

$$x^3 + x^6 - 3x^3 = 0$$

$$x^6 - 2x^3 = 0$$

$$x^3(x^3 - 2) = 0$$

$$x^3 = 0$$

$$x = 0$$

$$y = 0$$

$$x^3 = 2$$

$$x = \sqrt[3]{2}$$

$$y = \sqrt[3]{4}$$

$$F'_y = 3y^2 - 3x$$

ако заместим с (0,0)

$$F'_y = 0$$

$$F'_x = 0$$

$$F = 0$$

ако тези три неутрализи

изпълнени

F се нарича особена

$$F'_y(\sqrt[3]{2}, \sqrt[3]{4}) \neq 0 = \sqrt[3]{2}$$

в т. А имаме хоризонтална

асимптота

т. В е изокривна, ако не сме

F относно y и в B имаме

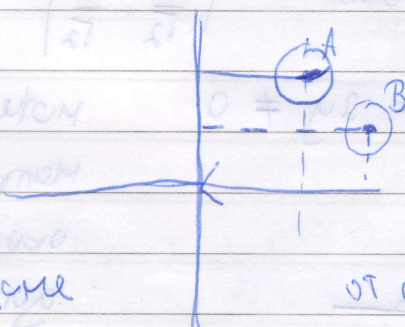
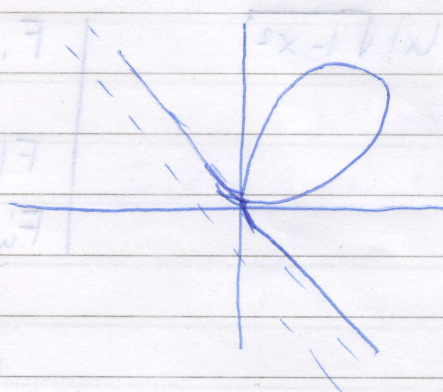
вертикална производна

$$y'(x) = \frac{3x^2 - 3y}{3y^2 - 3x} = -\frac{x^2 - y}{y^2 - x}$$

за асимптотата

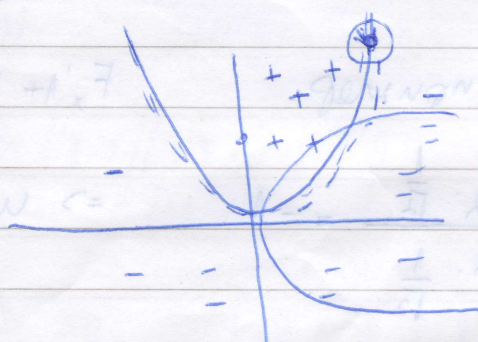
$$y(x) \rightarrow \infty$$

$$x \rightarrow -\infty$$



от посока на A ↑

от посока ↓



задача

при каждом условии от
 $f|z, y-xz| = 0$ можно задать определенную функцию
 и эту функцию удовлетворяет условию

$$f' z'_x + z \cdot f' z'_y = 0$$

$$(x_0, y_0, z_0) \text{ такая что } f|z_0, y_0 - x_0 z_0| = 0$$

$$F|x, y, z| = 0$$

$$F' F'_x, F'_y, F'_z \neq \text{непрер.}$$

$$F|x_0, y_0, z_0| = 0$$

$$F'_z|x_0, y_0, z_0| \neq 0$$

$$|f'_u + f'_v|-x| \neq 0$$

произв. по u +

произв. по v

х. произв.

по z

по z

справд.

аналогично

$$f|z_0, y_0 - x_0 z_0| = 0$$

$$f'_u + f'_v|-x| \neq 0$$

$$f|z(x, y), y - xz(x, y)| = 0$$

$$f'_u z'_x + f'_v|-z - xz'_x| = 0$$

$$z'_x = \frac{z f'_v}{f'_u - x f'_v}$$

$$f'_u z'_y + f'_v|-x z'_y| = 0$$

$$z'_y = \frac{-f'_v}{f'_u - x f'_v}$$

$$z'_x + z \cdot z'_y = 0$$

задача

задача $x^3 + y^3 - 3xy = 0$

$f(u, v)$

$f\left(x + \frac{z}{y}, y + \frac{z}{x}\right) = 0$

$z(x, y) : x \cdot z'_x + y \cdot z'_y = z - xy$

$x^6 - 2x^3 = 0 \Rightarrow x^3 = 2$

$x^3(x^3 - 2) = 0 \Rightarrow x^3 = 2$

$T(x_0, y_0, z_0) : \rho(x_0) = 0$
 $x^3 = 2 \quad x = \sqrt[3]{2} \quad y = \sqrt[3]{4}$

$F_y = 3y^2 - 0 = \sqrt[3]{0.5} \cdot x - \sqrt[3]{0.5} \cdot 0.5$
 $0 = \sqrt[3]{0.5} \cdot x - \sqrt[3]{0.5} \cdot 0.5$
 $x = 0.5$
 $F = 0$ — минимум

$F_y(\sqrt[3]{2}, \sqrt[3]{4}) \neq 0 = \sqrt[3]{2}$

В т. А — минимум

т. В — максимум, так как F относительно y и x в B имеет вертикальные касательные

$y'(x) : \frac{3x^2 - 3y}{3y^2 - 3x} = -\frac{(x^3 - y)}{(y^3 - x)}$

за асимптоты

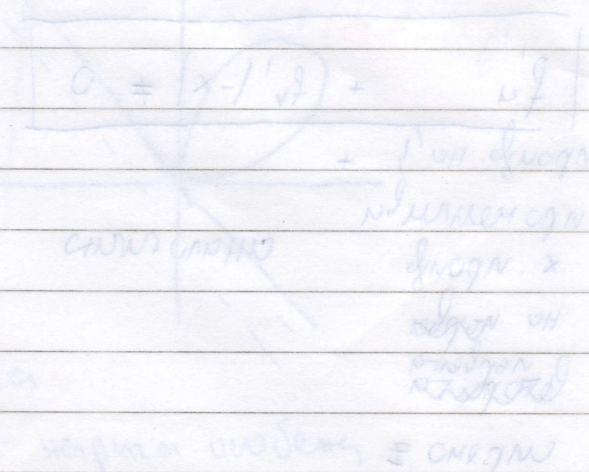
$y(x) \rightarrow \infty$
 $x \rightarrow -\infty$

задача

то задача решена

$0 = \sqrt[3]{0.5} \cdot 0.5 + \sqrt[3]{0.5} \cdot 0.5$

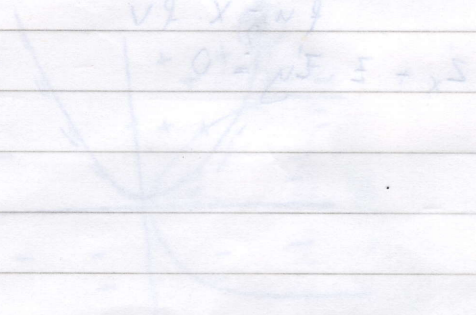
$0 = \sqrt[3]{0.5} \cdot x - \sqrt[3]{0.5} \cdot 0.5$



$0 = \sqrt[3]{0.5} \cdot x - \sqrt[3]{0.5} \cdot 0.5$
 $x = 0.5$

$0 = \sqrt[3]{0.5} \cdot x - \sqrt[3]{0.5} \cdot 0.5$

$\sqrt[3]{0.5} \cdot x - \sqrt[3]{0.5} \cdot 0.5 = 0$



задача

$$ax + z = f \mid by + z$$

и укажите $z''_{xx} \cdot z''_{yy} = z''_{xy}$

$$1 - f' \cdot 1 \neq 0 \quad z = z(x, y)$$

$$a + z'_x \cdot f' \cdot |z'_x| = 0$$

$$\begin{aligned} a + z'_x \cdot f' \\ z'_x |a - f'| = 0 \\ z'_x = \frac{a}{f' - 1} \end{aligned}$$

$$\begin{aligned} z'_y &= f' |b + z'_y| \\ z'_y - f' z'_y &= f' b \\ z'_y &= \frac{f' b}{1 - f'} \end{aligned}$$

$$z''_{xx} = - \frac{a f'' z'_x}{(f' - 1)^2} = - \frac{a^2 f''}{(f' - 1)^3}$$

$$z''_{xy} = - \frac{a \cdot f'' \cdot z'_y}{(f' - 1)^2} = \frac{a b f'' f'}{(f' - 1)^3} = - \frac{a \cdot f'' (b + z'_y)}{(f' - 1)^2}$$

$$\begin{aligned} z''_{yy} &= \frac{f'' b |b + z'_y| |1 - f'| + b f' \cdot f'' |b + z'_y|}{(1 - f')^2} = b f'' |b + z'_y| \left(\frac{1 - f' + f'}{(1 - f')^2} \right) = \\ &= b f'' \frac{|b + z'_y|}{|1 - f'|^2} = \frac{b^2 f'' \cdot \left| 1 + \frac{f'}{1 - f'} \right|}{|1 - f'|^2} = \frac{b^2 f''}{(1 - f')^3} \end{aligned}$$

$$\frac{z'_y}{b} + \frac{z'_x}{a} = \frac{1}{f-1} + \frac{f'}{f-f'} = -1$$

$$\frac{z''_{yx}}{b} + \frac{z''_{xx}}{a} = 0$$

$$\frac{z''_{yy}}{b} + \frac{z''_{xy}}{a} = 0$$

$$a \cdot z''_{xy} = -b \cdot z''_{xx}$$

$$b \cdot z''_{xy} = -a \cdot z''_{yy}$$

$$(a+b)z''_{xy} = -(a+b)(z''_{xx} + z''_{yy})$$

cb

$$(\epsilon + \mu \delta / \delta) = \epsilon = x \delta$$

$$\epsilon'' = \mu \delta' \epsilon''$$

$$(\mu \cdot x) / \epsilon = \epsilon \quad 0 \neq 1 \cdot \delta - 1$$

$$0 = (\delta' / \epsilon) \cdot \delta - \delta' \epsilon + \delta$$

$$0 = (\delta' - \delta \cdot \delta' / \epsilon) \cdot \delta - \delta' \epsilon + \delta$$

$$\begin{aligned} \delta' / \epsilon + \delta / \delta &= \mu \delta \\ \delta' &= \mu \delta' \cdot \delta - \mu \delta \\ \delta' / \delta &= \mu \delta' \\ \delta' &= \mu \delta' \end{aligned}$$

$$\delta'' = -\frac{\delta' \delta''}{\delta' - \delta} = \frac{\delta' \delta''}{\delta' - \delta}$$

$$\frac{(\mu \delta' + \delta)'' \cdot \delta}{\delta' - \delta} = \frac{\mu \delta'' + \delta''}{\delta' - \delta} = \frac{\mu \delta'' + \delta''}{\delta' - \delta}$$

$$= \frac{(\delta' + \delta - 1)}{\delta' - \delta} \cdot \frac{(\mu \delta' + \delta)'' \cdot \delta}{\delta' - \delta} = \frac{(\mu \delta' + \delta)'' \cdot \delta}{\delta' - \delta}$$

$$\frac{\delta' - \delta}{\delta' - \delta} = \frac{(\delta' + 1)'' \cdot \delta}{\delta' - \delta} = \frac{(\mu \delta' + \delta)'' \cdot \delta}{\delta' - \delta}$$

4.04.2013г.

ДУС - упражнение

задача

$$f(yz) - kz \neq 0$$

$$z(x, y)$$

$$x^2 z''_{xx} + 2xz'_x = y^2 z''_{yy} + 2yz'_y$$

$$0 = \frac{1}{5} + \frac{1}{5}, \frac{1}{5} - \frac{1}{5} = 0$$

$$f' \cdot y - x \neq 0$$

$$f(yz(x, y)) - xz(x, y) = 0$$

$$f' \cdot yz'_x - z - x \cdot z'_x = 0$$

$$yz'_x - z - xz'_x = 0$$

$$z'_x = \frac{z}{yf' - x}$$

$$z''_{xx} = \frac{z'_x (yf' - x) - z (yf'' - 1)}{(yf' - x)^2}$$

$$f' \cdot (1 \cdot z + y \cdot z'_y) - x \cdot z'_y = 0$$

$$z'_y = - \frac{zf'}{yf' - x}$$

$$z''_{yy} = - \frac{[z'_y f' + z \cdot f'' (z + y \cdot z'_y)] (yf' - x) - z f' (f' + y f'' (z + y z'_y))}{(yf' - x)^2}$$

$$x^2 z'_x \Big|_x = x^2 \cdot z''_{xx} + 2x z'_x$$

$$y^2 z'_y \Big|_y = \underline{\hspace{2cm}}$$

элементарно - жд

1.04.2018

Задание

задача | за упражнение

$f(u, v)$

$$u'v + uv' = x'v + xv' = x'v + v'x$$

$$0 = f'v + f v' = x'v + v'x$$

$$f(x, y) = 0$$

$$f(x-yz, y+z) = 0$$

$z(x, y)$

ya и ya.

$$|y-z| z''_{xx} = z''_{xy} + z'^2_x - p \cdot f$$

$$0 = (p'x) \cdot x - (p'x) \cdot x = 0$$

$$0 = x'v + v'x - x'v = 0$$

$$0 = x'v + v'x - x'v = 0$$

$$\frac{1 - (x'v)'v - (x'v)'v}{(x'v - x)'v} = x''v$$

$$\frac{x'v}{x - x'v} = x'v$$

$$0 = x'v + v'x - (x'v)'v = 0$$

$$\frac{x'v}{x - x'v} = x'v$$

$$\frac{(x'v + v'x)'v + (x'v)'v}{(x'v - x)'v} = x''v$$

$$x'v + v'x = x'v + v'x$$

$$| \text{---} | = p'v'v$$