$$\frac{u_{n+1}}{u_n} = \frac{3n-1}{3n+3} \to 1 < 1$$

Р-Д

$$n\left(\frac{3n+3}{3n-1} - 1\right) = n\left(\frac{\cancel{3n}+3-\cancel{3n}+1}{3n-1}\right) =$$

$$= \frac{4n}{3n-1} > 1 \Rightarrow сх.$$

4-сх.

4 сх.

Контролно: Диф.
смяна пр.
Лагранж
НМС, НГС
редове
ст. редове
Фурие

# ЗАДАЧИ ОТ ДЪРЖАВЕН ИЗПИТ

30.05.2013г.

четвъртък

Лекция

Зад. От Държавен Изпит 2007 !

Да се развие в степенен ред около т. $x = -1$

$$f(x) = \int_0^x \frac{t+1}{t^2+2t+2}\,dt$$

$$f'(x) = \frac{x+1}{x^2+2x+2} = \frac{(x+1)}{(x+1)^2+1} =$$

$$\frac{a}{x-1}$$

$$= (x+1)\sum_{n=0}^{\infty}(-1)^n (x+1)^{2n} =$$

$$\frac{1}{1+q} = 1 - q + q^2 - q^3 + \dots + (-q)^n + \dots =$$

$$= \sum_{n=0}^{\infty}(-q)^n$$

$$= \sum_{n=0}^{\infty}(-1)^n (x+1)^{2n+1}$$

$$f(x) = \int f'(x)\,dx + C$$

$R \Rightarrow |q| < 1$

$|x+1| < 1$

$-1 < x+1 < 1$

$-2 < x < 0$

разх. ( сх. ) разх.

$-2 \quad -1 \quad 0$

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^{2n+2}}{2n+2} + C}$$

$f(0) = 0 + C = 0$

$\Rightarrow C = 0$

$x = 0$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+2} \quad \text{сходящ} \quad a_n = \frac{1}{2n+2} \searrow 0$$

$x = -2$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot (-1)^{2n+2}}{2n+2} \quad \text{сходящ} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+2} \quad \text{сходящ}$$

развитието е валидно $[-2, 0]$ по Тh Абел

Пн. $\displaystyle\int \frac{t+1}{t^2+2t+2}\,dt =$

$$= \frac{1}{2} \ln\left(1 + (x+1)^2\right) - \frac{\ln 2}{2} = f(x)$$

степенен ред

---

Зад. $f'(x) = \dfrac{1}{\sin x + 2\cos x + 3}$

Търси се $f$ $(-\pi, 2\pi)$

---

Интеграл на Пуасон

$$J(\tau) = \int_0^{\pi} \ln(1 - 2\tau\cos x + \tau^2)\,dx$$

$|\tau| < 1$

Диф. под знака на $\int$

$$I'(\tau) = \int_0^{\pi} \frac{-2\cos x + 2\tau}{1 - 2\tau\cos x + \tau^2}$$

$t = \operatorname{tg}\dfrac{x}{2}$

$\cos x = \dfrac{1-t^2}{1+t^2}$

$$I'(\tau) = \int_0^{\pi} \frac{-2\,\frac{(1-t^2)}{(1+t^2)} + 2\tau}{1 - 2\tau\,\frac{(1-t^2)}{1+t^2} + \tau^2} \cdot \frac{2}{1+t^2}\, dt =$$

$$\operatorname{arctg} t = \frac{x}{2}$$

$$x = 2\operatorname{arctg} t$$

$$dx = \frac{2}{1+t^2}$$

$$= \int_0^{\infty} \frac{-2(1-t^2) + 2\tau(1+t^2)}{1+t^2 - 2\tau(1-t^2) + \tau^2(1+t^2)} \cdot \frac{2}{1+t^2}\, dt =$$

$$= 4\int_0^{\infty} \frac{-1 + t^2 + \tau + \tau t^2}{1+t^2-2\tau+2\tau t^2+\tau^2+\tau^2 t^2} \cdot \frac{1}{(1+t^2)}\, dt$$

$$= 4\int_0^{\infty} \frac{-(1-\tau) + t^2(\tau+1)}{(1+t^2)\left[(1-\tau)^2 + (1+\tau)^2 t^2\right]}\, dt =$$

$$= \frac{4(1+\tau)}{(1+\tau)^2} \int_0^{\infty} \frac{t^2 - \frac{(1-\tau)}{1+\tau}}{(1+t^2)\left(t^2 + \left(\frac{1-\tau}{1+\tau}\right)^2\right)}\, dt$$

$$\frac{1-\tau}{1+\tau} = a$$

$$= \frac{4}{1+\tau} \int_0^{\infty} \frac{t^2 - a}{(1+t^2)(t^2+a^2)}\, dt$$

$$\frac{t^2-a}{(1+t^2)(t^2+a^2)} = \frac{u-a}{(1+u)(u+a^2)} =$$

$$u = t^2 \qquad = \frac{A}{1+u} + \frac{B}{u+a^2}$$

$$u = -1$$

$$u = -a^2$$

$$A = \frac{1}{1-a}$$

$$B = \frac{a}{a-1}$$

$$J' = \frac{4(1+\tau)}{\dots}\left[\frac{1}{1-a}\int_0^\infty \frac{dt}{1+t^2} - \frac{a}{1-a}\int_0^\infty \frac{dt}{a^2+t^2}\right] =$$

$$\left.\operatorname{arctg} t\right|_0^\infty \qquad \left.\frac{1}{a}\operatorname{arctg}\frac{t}{a}\right|_0^\infty$$

$$= \frac{4(\cancel{1+\tau})}{(1+\tau)(1-a)}\left[\frac{\pi}{2} - 0 - \frac{a}{a}\left(\frac{\pi}{2} - 0\right)\right] = 0$$

! $a > 0$

$$J'(\tau) = 0 \Rightarrow J(\tau) = \mathrm{const}$$

$$J(\tau) = J(0) = 0$$

---

Зад 2004

$$f(x) = \frac{x^4-1}{4}\operatorname{arctg} x - \frac{x^3}{12} + \frac{x}{4}$$

$$f'(x) = \frac{4x^3}{4}\operatorname{arctg} x + \frac{x^4-1}{4}\cdot\frac{1}{x^2+1} - $$

$$-\frac{3x^2}{12} + \frac{1}{4}$$

упр

4

Зад

$$\sum_{n=0}^{\infty} \frac{5^n (x+2)^n}{8\sqrt{n+1}}$$

~~Даламбер~~ Коши-Адамар

$$R = \frac{1}{\lim \sqrt[n]{a_n}} = \frac{1}{\lim \frac{5}{\sqrt[n]{8}\sqrt[n]{\sqrt{n+1}}}} \qquad n \to \infty$$

$$\sqrt[n]{8} \to 1$$

$$\sqrt[n]{\sqrt{n+1}} = \sqrt{\sqrt[n]{n+1}} \to \sqrt{1}$$

$$\boxed{\Rightarrow R = \frac{1}{5}}$$

разх. сх разх.

$$-2-\frac{1}{5} \qquad -2+\frac{1}{5}$$

$$x = -2 - \frac{1}{5}$$

$$\sum \frac{5^n\left(-\frac{1}{5}\right)^n}{8\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{8\sqrt{n+1}} \quad \text{сходящ}$$

$$\frac{1}{8\sqrt{n+1}} \searrow 0$$

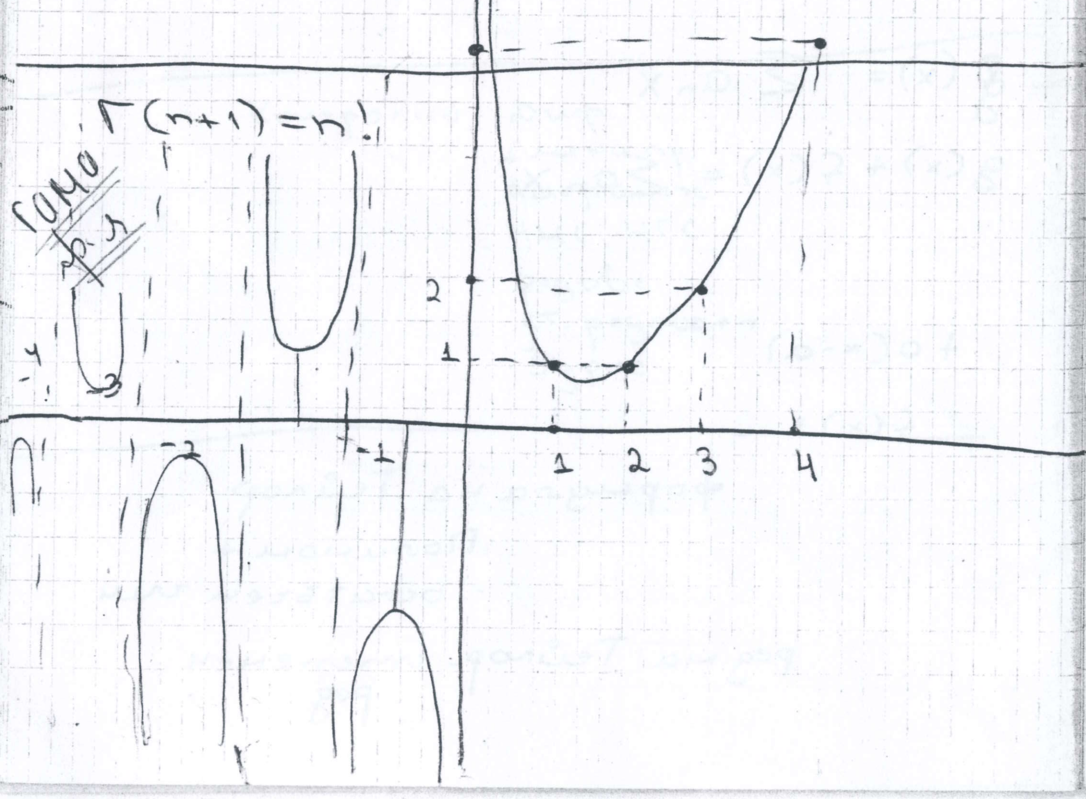
$x = -2 + \frac{1}{5}$

$$\sum \frac{5^n \frac{1}{5^n}}{8\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{1}{8\sqrt{n+1}}$$

$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ - разходящ

$\left[-2-\frac{1}{5}, 2+\frac{1}{5}\right)$

$\Gamma(n+1) = n!$




Бета ф-я

$B(a,b) = \frac{\Gamma(a) \cdot \Gamma(b)}{\Gamma(a+b)}$ Дирихле

Ойлерови интеграли

$x > 0$

$\Gamma(x+1) = x\Gamma(x)$

$\Gamma(x) > 0$

$(\ln \Gamma(x))'' > 0$

$\Gamma(1) = 1$

Условия на Бор. Молеруп

Ако ги изпълнява => тя е $\Gamma$ функцията

Нека $\phi$

$\phi(x+1) = x\,\phi(x)$

$\phi(x) \cdot \phi(x+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2x-1}}\,\phi(2x)$

$\phi(x) \cdot \phi(1-x) = \frac{\pi}{\sin \pi x}$

за $\gamma$ ф-ята изпълнени

2016

Зад. $\displaystyle\int \frac{1-\cos x}{(5+3\sin x)(5+4\cos x)}\,dx$

сметки ⊥

Зад. ~~2010~~ 2006

$$\int_{\frac{1}{3}}^{\frac{2}{3}} \frac{x^4-2x-1}{x^5-x}\,dx$$

---

Зад. 2012

$$f(x)=\frac{1}{6}\ln\frac{(x-1)^2}{x^2+x+1}+\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$$

---

Упражнение

$f(x) =$

$$f(a)+\frac{(x-a)}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+\ldots+\frac{(x-a)^n}{n!}f^{(n)}(a)+\ldots$$

ред на Тейлор за $f(x)$

$\cancel{x \neq 0}$

$$\begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases} = f(x) \text{ - непрекъсната}$$

$$f^{(n)}(x)=\begin{cases} P\left(\frac{1}{x}\right)\cdot e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x=0 \end{cases}$$

по ММИ

$0+0\ldots+0 \neq$ ред Тейлор ст-т ф-я

$g(x) = \sum a_n x^n$

$g(x)+f(x)=\sum a_n x^n$

$+\,o(x-a)^{n+1}$

$f(x)=\ldots$

формула на Тейлор

Полином + остатъчен член

ред на Тейлор - степенен ред

6