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Диференциално и интегрално смятане, Домашно № 2

Задача 1: Да се направи смяна на променливите:

$$z'_x z''_{xy} - z'_y z''_{xx} = 0$$

$x = w(u(x,y), v(x,y))$
$y = u(x,y)$
$z = v(x,y)$

$$\left| \begin{aligned} 1 &= w'_u u'_x + w'_v v'_x \\ 0 &= u'_x \\ z'_x &= v'_x = \frac{1}{w'_v} \end{aligned} \right. \qquad \left| \begin{aligned} 0 &= w'_u u'_y + w'_v v'_y \\ 1 &= u'_y \\ z'_y &= v'_y = -\frac{w'_u}{w'_v} \end{aligned} \right.$$

$$z''_{xx} = -(w'_v)^{-2}\left(w''_{uv} \cdot u'_x + w''_{vv} \cdot v'_x\right) =$$

$$= -(w'_v)^{-2} \cdot \frac{w''_{vv}}{w'_v} = -\frac{w''_{vv}}{{w'_v}^3}$$

$$z''_{xy} = -(w'_v)^{-2}(w''_{uv}\cdot u'_y + w''_{vv}\cdot v'_y) =$$

$$= -(w'_v)^{-2}\left(w''_{uv} - \frac{w''_{vv}\cdot w'_u}{w'_v}\right) =$$

$$= -\frac{w''_{uv}\cdot w'_v - w''_{vv}\cdot w'_u}{{w'_v}^3}$$

$$\Rightarrow \quad z'_x z''_{xy} - z'_y\cdot z''_{xx} = 0$$

$$\Rightarrow$$

$$\cancel{\frac{w''_{vv}w'_u - w''_{uv}\cdot w'_v}{(w'_v)^4}} - \cancel{\frac{w'_u\cdot w''_{vv}}{(w'_v)^4}} = 0$$

$$\cancel{\Rightarrow w''_{uv}\cdot w'_v = 0}$$

Задача 3: Нека $\varphi(u,v)$ е диференцуема, а $f(x,y,z) = \varphi(\underset{u}{xy}, \underset{v}{\tfrac{y}{z}})$.

Да се докаже, се

$$-x \cdot f'_x + y \cdot f'_y + z \cdot f'_z = 0.$$

$$f'_x = \varphi'_u \cdot u'_x + \varphi'_v \cdot v'_x = \varphi'_u \cdot y$$

$$f'_y = \varphi'_u \cdot u'_y + \varphi'_v \cdot v'_y = \varphi'_u \cdot x + \varphi'_v \cdot \frac{1}{z}$$

$$f'_z = \varphi'_u \cdot u'_z + \varphi'_v \cdot v'_z = -\varphi'_v \cdot \frac{y}{z^2}$$

$$\Rightarrow \quad -x \cdot f'_x + y \cdot f'_y + z \cdot f'_z =$$

$$= \cancel{-x \cdot \varphi'_u \cdot y} + \cancel{y \cdot \varphi'_u \cdot x} + \cancel{y \cdot \frac{1}{z} \cdot \varphi'_v}$$

$$+ \cancel{z \cdot \left(-\varphi'_v \cdot \frac{y}{z^2}\right)} = 0$$

ЗАДАЧА 4: $z = x^y$

Да се докаже, че

$$\frac{x}{y}\cdot z'_x + \frac{1}{\ln x}\cdot z'_y = 2z$$

$$z'_x = y\cdot x^{y-1}$$

$$z'_y = x^y\cdot \ln x$$

$$\Rightarrow$$

$$\frac{x}{y}\cdot z'_x + \frac{1}{\ln x}\cdot z'_y =$$

$$= \frac{x}{\cancel{y}}\cdot \cancel{y}\cdot x^{y-1} + \frac{1}{\cancel{\ln x}}\cdot x^y\cdot \cancel{\ln x} =$$

$$= 2x^y = 2z$$

ЗАДАЧА 2: За кои стойности на α функцията

$$f(x,y) = \begin{cases} (x^2+y^2)^{\alpha} \cdot \sin\frac{1}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$$

а) е непрекъсната?

б) е диференцируема в т. $(0,0)$?

а) $f(x,0) = x^{2\alpha} \cdot \sin\frac{1}{x^2} =$

$$= \frac{x^{2\alpha} \cdot \sin\frac{1}{x^2} \cdot \frac{1}{x^2}}{\underbrace{\frac{1}{x^2}}_{\to 0}} = x^{2\alpha - 2} \cdot \underbrace{\frac{\sin\frac{1}{x^2}}{\frac{1}{x^2}}}_{\to 0} \xrightarrow[x \to 0]{} 0$$

$f(x,0) \xrightarrow[x\to 0]{} 0$ за $\forall \alpha$.

Аналогично $f(0,y) = y^{2\alpha-2} \cdot \underbrace{\frac{\sin\frac{1}{y^2}}{\frac{1}{y^2}}}_{\to 0} \xrightarrow[y \to 0]{} 0$

$f(0,y) \xrightarrow[y\to 0]{} 0$ за $\forall \alpha$.

$\Rightarrow$ Непрекъсната за всяко α.

б)