ТВ лекции 25.03.15

$$F(x) = P(X \le x)$$

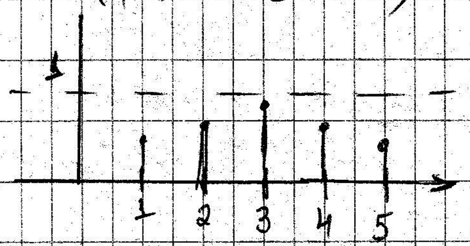
Функция на разпределение на всяко случайна ве...

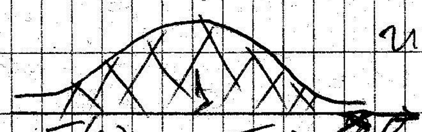
Броими случайни величини $P(X=k)=p_k$, $k=\overline{0,\infty}$

Така се задава дискретна случайна величина, като вероятността е p_k

Непрекъснати случайна величина

$f(x) \ge 0$, $\int_{-\infty}^{\infty} f(x)dx = 1$





плътност

$f(x) = F'(x)$, $F(x) = \int_{-\infty} f(t)dt$

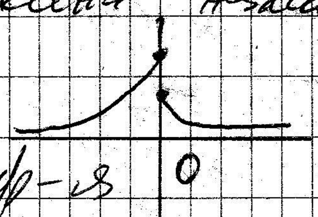
Математическо очакване

$$EX = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{-\infty}^{\infty} x\, dF(x)$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

Смес на две случайни величини - няма производна в нулата

Може да изберем коя да е точка в 0-та м/у едната и другата ф-я



Експоненциално разпределени величина

$f(x) = \lambda e^{-\lambda x}$, $x \ge 0$, $\int f(x)dx = \int \lambda e^{-\lambda x} dx =$

$= \int e^{-\lambda x} d\lambda x = 1$

$x \geq 0$

$$EX = \int_0^\infty x \lambda e^{-\lambda x} dx = -\int_0^\infty x \, d e^{-\lambda x} =$$

$$= -\underbrace{e^{-\lambda x}\Big|_0^\infty}_{0} + \int_0^\infty e^{-\lambda x} dx = -\frac{1}{\lambda} e^{-\lambda x}\Big|_0^\infty = \frac{1}{\lambda}$$

Нареждането е a^x, x^n, $\ln x$, която най-бързо расте a^x, $a > 1$

$F(x) = 1 - e^{-\lambda x}$ - експоненциално разпределение

Специална гама ф-я $\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx$

$\Gamma(p) = (p-1)\Gamma(p-1) \Rightarrow$ може да се стигне до $\Gamma(p) = (p-1)!$

$$EX^2 = DX = \int_0^\infty x^2 \lambda e^{-\lambda x} dx = \frac{1}{\lambda^2} \int_0^\infty e^{-\lambda x} (\lambda x)^2 d\lambda x =$$

$$\frac{1}{\lambda^2} \Gamma(3) = \frac{2!}{\lambda^2} \Rightarrow DX = \frac{2}{\lambda^2} - (EX)^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

Разгл. $\Gamma(p)$. Ако p е рационално $\Rightarrow \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$$f(x) = C x^{\alpha-1} e^{-\beta x}, x \geq 0, \quad C \int_0^\infty x^{\alpha-1} e^{-\beta x} dx = 1$$

$$C \frac{1}{\beta^\alpha} \int_0^\infty (\beta x)^{\alpha-1} e^{-\beta x} d\beta x = \frac{C}{\beta^\alpha} \Gamma(\alpha) = 1 \Rightarrow$$

$$f x = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}, \quad x \geq 0 \qquad X \sim \Gamma(\alpha, \beta)$$

α - параметър на формата

$$EX = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty x^\alpha e^{-\beta x} dx = \frac{\beta^\alpha}{\Gamma(\alpha) \beta \beta^\alpha} \int_0^\infty (\beta x)^\alpha e^{-\beta x} d\beta x =$$

$$\frac{\Gamma(\alpha+1)}{\beta \Gamma(\alpha)} = \frac{\alpha \cancel{\Gamma(\alpha)}}{\beta \cancel{\Gamma(\alpha)}} = \frac{\alpha}{\beta}$$

hazard $F \Rightarrow h(x) = \dfrac{f(x)}{\bar{F}(x)} = \dfrac{\lambda e^{-\lambda x}}{e^{-\lambda x}} = \lambda$ - const

$F(x) = 1 - e^{-\lambda x},\ x \geq 0,\ \bar{F}(x) = 1 - F(x) = e^{-\lambda x}$

Св-вото липса на памет

$\bar{F}(x+y) = \bar{F}(x)\bar{F}(y)$

Нормалното разпределение. Плътността

$$X \sim f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

$EX = a$, $DX = \sigma^2$, $\sqrt{DX} = \sigma$ което трябва да е > 0

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-a)^2}{2\sigma^2}} dt = P(X \leq x)$$

$$Z = \frac{X-a}{\sigma}, \quad F_Z(z) = P(Z \leq z) = P\left(\frac{X-a}{\sigma} \leq z\right) =$$

$$P(X \leq \sigma z + a) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\sigma z + a} e^{-\frac{(t-a)^2}{2\sigma^2}} dt =$$

(полагаме $\frac{t-a}{\sigma} = v$) $\Rightarrow$

$$= \frac{\sigma \cdot 1}{\sigma \cdot \sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{v^2}{2}} dv \ ; \quad f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \ -\infty < x < \infty$$

$X \sim N(a, \sigma)$, $Z \sim N(0, 1)$

Стандартно нормално разпределение

$$EX = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{(x-a)^2}{2\sigma^2}} dx$$ (прибавяме и вадим a)

$$\Rightarrow \frac{1\sigma^2}{\sigma\sqrt{2\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-\frac{(x-a)^2}{2\sigma^2}} d\frac{(x-a)^2}{2\sigma^2}}_{0} + a \underbrace{\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-a)^2}{2\sigma^2}} dx}_{1} = a$$

$DX = E(X - EX)^2 = E(X-a)^2 =$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(x-a)^2 e^{-\frac{(x-a)^2}{2\sigma^2}}\,d(x-a)=$$

$$=\frac{1\cdot\sigma^2}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(x-a)e^{-\frac{(x-a)^2}{2\sigma^2}}\,d\frac{(x-a)^2}{2\sigma^2}=-\frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(x-a)\,de^{-\frac{(x-a)^2}{2\sigma^2}}=$$

$$-\frac{\sigma}{\sqrt{2\pi}}\underbrace{(x-a)e^{-\frac{(x-a)^2}{2\sigma^2}}\Big|_{-\infty}^{\infty}}_{0}+\frac{\sigma\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{(x-a)^2}{2\sigma^2}}\,d\frac{x-a}{\sigma}=\sigma^2$$