ТВ лекции 25.04.15

Двумерни случайни величини

Пример

$Y \backslash X$	-1	1	
-1	$1/6$	$1/6$	$+\ 1/3$
0	$1/3$	0	$+\ 1/3$
1	$1/6$	$1/6$	$+\ 1/3$
	$+2/3$	$+1/3$	

$p_{ij} = P(X=i, Y=j)$

$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} p_{ij} = 1$, $p_{ij} = p_i p_j$

X	-1	1
P	$2/3$	$1/3$

Y	-1	0	1
P	$1/3$	$1/3$	$1/3$

$EX = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$

$EY = -\frac{1}{3} + 0 + \frac{1}{3} = 0$

$E[(X-EX)(Y-EY)] = Cov(X,Y)$ - ковариация

$\frac{Cov(X,Y)}{\sqrt{DX}\sqrt{DY}} = \rho$ - корелационен коефициент

$-1 \le \rho \le 1$

Ако X, Y са независими, т.е. $X \perp Y \Rightarrow \rho = 0$

$E[(X-EX)(Y-EY)] = E(XY - EXY - EYX + EXEY) =$

$= E(XY) - 2EXEY + EXEY = E(XY) - EXEY = Cov(X,Y)$

Но $EX = -1/3$, $EY = 0 \Rightarrow EXEY = 0$

$E(XY) = (-1)(-1)\frac{1}{6} + (-1)\cdot 1 \cdot 1/6 + (-1)\cdot 1 \cdot 1/6 + 1 \cdot 1 \cdot 1/6 = 0 \Rightarrow$

$\rho = 0$

Функция на разпределение $F(x,y) = P(X \le x, Y \le y)$

$\frac{\partial^2 F(x,y)}{\partial x \partial y} = f(x,y)$ - плътност на разпределение

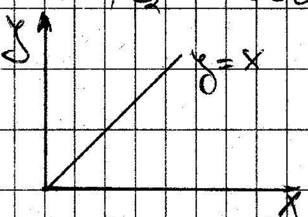
$X \perp Y \Leftrightarrow f(x,y) = f(x) \cdot f(y)$

Маргинални разпределения $\int f(x,y)\,dx = f(y)$

$\int f(x,y)\,dy = f(x)$

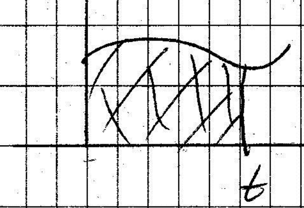
Нека $X_1 \sim exp(\lambda_1)$, $X_2 \sim exp(\lambda_2)$, $X_3 \sim exp(\lambda_3)$. Тогава
Разпределението на Marshall-Olkin

$$\left.\begin{aligned} Z_1 &= \min(X_1, X_3) \sim exp \\ Z_2 &= \min(X_2, X_3) \sim exp \end{aligned}\right\} \text{модел за провал на две независими причини}$$



$$\int f(x)\,dg(x)$$

$$F(t) = \int_0^t f(x)\,dx$$
лебег



$Q \ll P$

$P(A)$

$$\bar{F}(x) = 1 - F(x), \quad F(x) = P(X \le x, Y \le y) =$$
$$= 1 - P(X > x) - P(Y > y) + P(X > x, Y > y)$$
$$P(X > x, Y > y) = F(x,y) + P(X > x) + P(Y > y) - 1$$

Разпределение на Парето

$X^+ = \max(X, 0)$, $X^- = \max(-X, 0)$, $X = X^+ - X^-$

$EX = EX^+ + EX^-$