МО 03.06.15

Разпределителен метод

$\min z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$; $\sum_{j=1}^{n} x_{ij} = a_i,\ i=1..m$; $\sum_{i=1}^{m} x_{ij} = b_j,\ j=1..n,\ x_{ij}\geq 0$

$\bar{x}$ - начален връх, $m+n-1$ - базисни, $\bar{x} = [\bar{x}_{ij}]_{m\times n}$

(1) $\gamma_{kl} : \underset{+}{(k,l)}\underset{-}{(k,j_1)}\underset{+}{(i_1,j_1)} \dots \underset{+}{(i_s,j_s)}\underset{-}{(i_s,l)} \Rightarrow \gamma_{kl}^{+} \cup \gamma_{kl}^{-}$

(2) $\bar{c}_{kl} = c_{kl} - c_{kj_1} + c_{i_1j_1} - \dots + c_{i_sj_s} - c_{i_sl} = \sum_{\gamma_{kl}^{+}} c_{ij} - \sum_{\gamma_{kl}^{-}} c_{ij}$

$x' = [x'_{ij}]_{m\times n} \Rightarrow$ (4) $x'_{ij} = \begin{cases} x_{ij}+\theta, & (i,j)\in\gamma_{kl}^{+} \\ x_{ij}-\theta, & (i,j)\in\gamma_{kl}^{-} \\ x_{ij}, & (i,j)\notin\gamma_{kl} \end{cases}$, където

θ - най-малкото количество т.е. (3) $\theta = \min_{(i,j)\in\gamma_{kl}^{-}} \bar{x}_{ij} = x_{i_pj_p} \geq 0$

$z(\bar{x}) = \overbrace{\sum_{\gamma_{kl}^{+}} c_{ij}x_{ij}}^{S^+} + \overbrace{\sum_{\gamma_{kl}^{-}} c_{ij}x_{ij}}^{S^-} + \overbrace{\sum_{(i,j)\notin\gamma_{kl}} c_{ij}}^{S} = S^+ + S^- + S$

$z(x') = \sum_{\gamma_{kl}^{+}} c_{ij}(\bar{x}_{ij}+\theta) + \sum_{\gamma_{kl}^{-}} c_{ij}(\bar{x}_{ij}-\theta) + \sum_{(i,j)\notin\gamma_{kl}} c_{ij}x_{ij} =$

$\underbrace{S^+ + S^- + S}_{z(\bar{x})} + \theta \underbrace{\left(\sum_{\gamma_{kl}^{+}} c_{ij} - \sum_{\gamma_{kl}^{-}} c_{ij}\right)}_{\bar{c}_{kl}} \Rightarrow z(x') = z(\bar{x}) + \theta\bar{c}_{kl} \underset{\bar{c}_{kl}<0}{\leq} z(\bar{x})$

Критерий $\forall\ \bar{c}_{ij} \geq 0$

Ст0 Начален връх

Ст1 Построяване на единствен цикъл γ_{kl} за $\forall (k,l)$ празна свързваща я с другите клетки

Ст2 Пресмятане на оценките $\bar{c}_{kl}$ за $\forall$ празни клетки, като използваме (2)

Ст3 Проверка критерий за оптималност

а) $\bar{c}_{kl} \geq 0 \Rightarrow$ опт. връх $\Rightarrow$ край

б) $\exists (k,l)$ празна за която $\bar{c}_{kl} < 0 \Rightarrow$ Ст4

Ст4 Използвайки (3) и (4) минаване в съседен опорен връх $x' \Rightarrow$ Ст1

Зад

	B_1	B_2	B_3	B_4	a_i
A_1	100^{3}	^{5}	^{7}	^{11}	~~100~~
A_2	50^{1}	80^{4}	^{6}	^{3}	~~130~~ ~~80~~
A_3	^{6}	40^{8}	80^{12}	50^{7}	~~170~~ ~~130~~ ~~50~~
b_j	~~150~~ 50	~~120~~ 40	~~80~~	~~50~~	400 / 400

$\bar{c}_{12} = 5-3+1-4 = -1 < 0$ не е опт

$\theta = \min(80, 100) = 80$

т.е.

	B_1	B_2	B_3	B_4	a_i
A_1	20^{3}	80^{5}	^{7}	^{11}	
A_2	130^{1}	^{4}	^{6}	^{3}	
A_3		40	80	50	
b_j					

$\Rightarrow$

$\bar{c}_{13} = 7-5+8-12 = -2 < 0 \Rightarrow$

$\theta = \min(80, 80) = 80$

	B_1	B_2	B_3	B_4	a_i
A_1	20^{3}	0^{5}	80^{7}	^{11}	
A_2	130^{1}	^{4}	^{6}	^{3}	
A_3	^{6}	120^{8}	^{12}	50^{7}	

$\bar{c}_{14} = 11-5+8-7 > 0$

$\bar{c}_{22} = 4-1+3-5 > 0$

$\bar{c}_{24} = 3-1+3-5+8-7 > 0$

$\bar{c}_{31} = 6-3+5-8 = 0$

$\bar{c}_{33} = 12-8+5-7 > 0$

$\min z = 3.20 + 7.80 + 8.120 + 7.50 + 130 = 2060$

$$X_1^* = \begin{bmatrix} 20 & 0 & 80 & 0 \\ 130 & 0 & 0 & 0 \\ 0 & 120 & 0 & 50 \end{bmatrix}$$

$\theta = \min(20, 120) = 20$

$$X_2^* = \begin{bmatrix} 0 & 20 & 80 & 0 \\ 130 & 0 & 0 & 0 \\ 20 & 100 & 0 & 50 \end{bmatrix}$$

$X_\lambda^* = \lambda X_1^* + (1-\lambda) X_2^*, \ \lambda \in [0, 1]$

Задачи с нарушен баланс

1) $\sum a_i < \sum b_j$, тогава

$A_1 .. A_m : A_{m+1}$; $B_1 .. B_n$

$a_{m+1} = \sum b_j - \sum a_i$, $c_{m+1,j} = 0$

$\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1..m$

$\sum_{i=1}^{m} x_{ij} \le b_j$, $j = 1..n$

2) $\sum a_i > \sum b_j$, тогава

$A_1 - A_m$, $B_1 .. B_{n+1}$

$b_{n+1} = \sum a_i - \sum b_j$, $c_{i,n+1} = 0$

$\sum_{j=1}^{n} x_{ij} \le a_i$, $i = 1..m$

$\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1..n$

Зад

	B_1	B_2	B_3	a_i
A_1	^{1}	^{2}	^{4}	30
A_2	^{3}	^{4}	^{3}	25
b_j	45	10	15	55 / 70

$\min z = x_{11} + 2x_{12} + 4x_{13} + 3x_{21} + 4x_{22} + 3x_{23}$

$x_{11} + x_{12} + x_{13} = 30$

$x_{21} + x_{22} + x_{23} = 25$

$x_{11} + x_{21} \le 45$

$x_{12} + x_{22} \le 10$ $\Rightarrow$

$x_{13} + x_{23} \le 15$

	B_1	B_2	B_3	a_i
A_1	30^1	^{2}	^{4}	30
A_2	15^3	10^4	^{3}	25 ~~10~~
A_3	^{0}	0^0	15^0	15
	~~45~~ 15	10	15	70 / 70

$\bar{c}_{12} = 2-1+3-4=0$

$\bar{c}_{13} = 4-1+3-4+0-0=2>0$

$\bar{c}_{23} = 3-4+0-0<0$ $\Rightarrow$

$\theta = \min(10,15)=10$

	B_1	B_2	B_3	a_i
A_1	30^1	^{2}	^{4}	
A_2	15^3	^{4}	10^3	
A_3	^{0}	10^0	15^0	
b_j				

$\bar{c}_{12} = 2-0+0-3+3-1>0$

$\bar{c}_{13} = 4-1+3-3>0$

$\bar{c}_{22} = 4-0+0-3>0$

$\bar{c}_{31} = 0-0+3-3=0$

$X_1^* = \begin{bmatrix} 30 & 0 & 0 \\ 15 & 0 & 10 \end{bmatrix}$ $\min z = 30 + 3\cdot15 + 3\cdot10 = 105$

? има ли оптимално решение при $x_{23} = 12$

	B_1	B_2	B_3
A_1	30^1	^{2}	^{4}
A_2	-15^3	^{4}	$+10^3$
A_3	$+\cdot^0$	10^0	-5^0

$\theta = \min(5,15)=5$ $\Rightarrow$

$X_2^* = \begin{bmatrix} 30 & 0 & 0 \\ 10 & 0 & 15 \end{bmatrix}$

$X_\lambda^* = \lambda X_1^* + (1-\lambda)X_2^*, \ \lambda\in[0,1]$

$\lambda: 10\lambda + (1-\lambda)15 = 12 \Rightarrow \lambda = 3/5$

$\Rightarrow \frac{3}{5}X_1^* + \frac{2}{5}X_2^* = \begin{bmatrix} 30 & 0 & 0 \\ 13 & 0 & 12 \end{bmatrix}$

? $x_{11} = 22$

	B_1	B_2	a_i
A_1	^{5}	^{2}	55
A_2	^{7}	^{4}	30
A_3	^{4}	^{3}	25
b_j	55	35	90 \ 110

$$\min z = 5x_{11} + 2x_{12} + 7x_{21} + 4x_{22} + 4x_{31} + 3x_{32}$$

$$x_{11} + x_{12} \le 55$$

$$x_{21} + x_{22} \le 30$$

$$x_{31} + x_{32} \le 25 \quad \sim$$

$$x_{11} + x_{21} + x_{31} = 55$$

$$x_{12} + x_{22} + x_{32} = 35$$

	B_1	B_2	B_3	a_i
	55^5	^{2}	^{0}	~~55~~
	0^7	30^4	^{0}	~~30~~
	^{4}	5^3	20^0	~~25~~ 20
	~~55~~	~~35~~ 5	~~20~~	110 \ 110

$$\bar{c}_{12} = 2-5+7-4=0$$

$$\bar{c}_{13} = 0-5+7-4+3-0 = 1 > 0$$

$$\bar{c}_{23} = 0-4+3-0 = -1 < 0$$

$$\theta = \min(20, 30) = 20 \Rightarrow$$

	B_1	B_2	B_3	a_i
A_1	55^5	^{2}	^{0}	
A_2	0^7	10^4	20^0	
A_3	^{4}	25^3	^{0}	

$$\bar{c}_{11} > 0$$

$$\bar{c}_{13} > 0$$

$$\bar{c}_{31} = 4-7+4-3 < 0$$

$$\theta = \min(0, 25) = 0 \Rightarrow$$

	B_1	B_2	B_3
A_1	55^5	^{2}	^{0}
A_2	^{7}	10^4	20^0
A_3	0^4	25^3	^{0}

$\sim$

	B_1	B_2	B_3
A_1	30^5	25^2	^{0}
A_2	^{7}	10^4	20^0
A_3	25^4	^{3}	

$$\bar{c}_{13} > 0$$

$$\bar{c}_{21} > 0$$

$$\bar{c}_{32} > 0$$

$$\bar{c}_{33} > 0$$

$$X_1^* = \begin{bmatrix} 30 & 25 & 0 \\ 0 & 10 & 20 \\ 25 & 0 & 0 \end{bmatrix} \sim X_2^* = \begin{bmatrix} 30 & 25 \\ 0 & 10 \\ 25 & 0 \end{bmatrix}$$

$$\bar{c}_{12} = 2-5+4-3 < 0$$

$$\theta = \min(25, 55) = 25$$

$$\min z = 150+50+40+100 = 340$$

упр

	B_1	B_2	B_3	B_4	a_i
A_1	^{5}	^{4}	^{2}	^{6}	50
A_2	^{1}	^{2}	^{4}	^{7}	60
	20	50	70	50	190 \ 110

а) $a = 5$

б) a? намереното оптимално решение остава оптимално