ЕДУ 18.11.14

$u_{tt} = a^2 u_{xx} \qquad x \in (0, L), \quad t > 0$

$$\begin{cases} u(x,0) = x = \varphi & \varphi'(0) = 1 \neq 0 \\ u_t(x,0) = 1 = \psi & \varphi'(L) = 1 \neq 0 \end{cases}$$

$u(x,t) = X(x)T(t) \neq 0$

$$\frac{\ddot{T}(t)}{a^2 T(t)} = \frac{X''(x)}{X(x)} = -\lambda, \quad \begin{cases} X'' + \lambda X = 0 \\ X'(0) = X'(L) = 0 \end{cases}$$

$\lambda_0 = 0, \quad X_0 = 1$

$\lambda_k = \left(\frac{k\pi}{L}\right)^2, \quad X_k = \cos\frac{k\pi x}{L}$

$\ddot{T}_k + a^2 \frac{k^2\pi^2}{L^2} T_k = 0$

$T_k(t) = A_k \cos\frac{k a\pi t}{L} + B_k \sin\frac{k\pi a t}{L}, \quad \ddot{T}_0 = 0$

$T_0 = A_0 t + B_0$

$u(x,t) = X_0 T_0 + \sum_1^\infty X_k(x) T_k(t)$

$u(x,t) = A_0 t + B_0 + \sum_1^\infty \cos\frac{k\pi x}{L}\left(A_k \cos\frac{k\pi a t}{L} + B_k \sin\frac{k\pi a t}{L}\right)$

$u_t = A_0 + \sum_1^\infty \cos\frac{k\pi x}{L}\left(A_k\left(-\frac{k\pi a}{L}\right)\sin\frac{k\pi a t}{L} + B_k \frac{k\pi a}{L}\cos\frac{k\pi a t}{L}\right)$

$u_t|_{t=0} = 1 = A_0 + \sum_1^\infty \cos\frac{k\pi x}{L} + \frac{k\pi a}{L} B_k$

$\left\{1, \cos\frac{k\pi x}{L}\right\} \Rightarrow A_0 = 1, \quad B_k = 0$

$u(x,t) = t + B_0 + \sum_1^\infty A_k \cos\frac{k\pi x}{L} \cdot \cos\frac{k\pi a t}{L}$

$$u(x,0) = B_0 + \sum_1^{\infty} A_k \cos\frac{k\pi x}{L} = x$$

$$x = C_0 + \sum C_k \cos\frac{k\pi x}{L}$$

$$C_0 = \frac{1}{L}\int_0^L x\,dx$$

$$C_k = \frac{2}{L}\int_0^L x\cos\frac{k\pi x}{L}\,dx$$

$$B_0 = \frac{1}{L}\int_0^L x\,dx = \frac{x^2}{2L}\Big|_0^L = \frac{L}{2}$$

$$A_k = \frac{2}{L}\int_0^L x\cos\frac{k\pi x}{L}\,dx = \frac{L}{k\pi}\,\frac{2}{L}\int_0^L x\,d\sin\frac{k\pi x}{L} =$$

$$\frac{2L}{k\pi L}\,x\sin\frac{k\pi x}{L}\Big|_0^L - \frac{L}{k\pi}\,\frac{2}{L}\int_0^L \sin\frac{k\pi x}{L}\,dx$$

$$A_k = \frac{L^2}{k\pi^2}\,\frac{2}{L}\cos\frac{k\pi x}{L}\Big|_0^L \Rightarrow A_{2k+1} = \frac{-4}{L\pi^2}\,\frac{1}{(2k\pi)^2}$$

$$u(x,t) = t + \frac{L}{2} - \frac{4}{\pi^2 L^2}\sum_1^{\infty}\frac{1}{(2k+1)^2}\cos\frac{(2k\pi)\pi x}{L}\;\cos^2\frac{(k\pi)\pi a t}{L}$$