ДИС 05.11

$\frac{1}{1+x^2}$ Разложете в сума елементарни дроби

$$\frac{1}{(x-i)(x+i)} = \frac{A}{x-i} + \frac{B}{x+i} = \frac{A(x+i)+B(x-i)}{x^2+1}$$

$$A(x+i)+B(x-i) = (A+B)x + iA - iB = 1$$

$$\left|\begin{array}{l} A+B=0 \\ iA-iB=1 \end{array}\right\} \Rightarrow \left|\begin{array}{l} A=-B \\ -iB-iB=1 \Rightarrow B=-\frac{1}{2i}=\frac{i}{2} \Rightarrow A=-\frac{i}{2} \end{array}\right.$$

Заместваме $\frac{-i}{2(x-i)} + \frac{i}{2(x+i)}$

$$1 = A(x-x_1)(x-x_2) + B(1+x)(x_2-x_2) + C(1+x)(x_2-x_1)$$

$$\left|\begin{array}{ll} 1 = B(1+x_1)(x_1-x_2) & \text{при } x = x_1 \\ 1 = C(1+x_2)(x_2-x_1) & \text{при } x = x_2 \\ 1 = A(-1-x_1)(-1-x_2) & \text{при } x = -1 \end{array}\right.$$

$$\frac{x^2+1}{(x^2-2)(x-1)^2} = \frac{ax+b}{x^2-2} + \frac{c}{(x-1)^2} + \frac{d}{x-1}$$

Άσκ 6 $$\frac{1}{(x^2-1)^2(x_3-3)^3} = \frac{Ax+B_1}{x^2+1} + \frac{A_2x+B_2}{(x^2+1)^2} + \frac{A_3x^2+B_3x+C_3}{x^3-3} +$$

$$+ \frac{A_4x^2+B_4x^2+C_4}{(x^3-3)^2} + \frac{A_5x^2+B_5x+C_5}{(x^3-3)^3}$$

$$\frac{2}{x^4+x^2} = \frac{2}{x^2(x^2+1)} = \frac{A(x)}{x^2} + \frac{B(x)}{x^2+1}$$

$$\frac{A_1}{x} + \frac{A_2}{x^2} + \frac{B_1x+C_1}{x^2+1}$$ Άσκ 4

$$\frac{A_1}{x} + \frac{A_2}{x^2} + \frac{B_2}{x+i} + \frac{B_1}{x-i}$$

$$\frac{A_1(x-i)(x+i)x + A_2(x-i)(x+i) + B_1x^2(x+1) + B_2x^2(x-1)}{x^4+x^2} =$$

για $x=0$ $\quad A_2 = 2$

για $x=i$ $\quad B_1 = -2i$

για $x=-i$ $\quad B_2 = 2i$

για $x=1$ $\quad 2A_1 + 4 + (-2i)(1+i) + 2i(1-i) \Rightarrow A_1 = -3$