1) 2т) Нека a = $4 + 3i$, b = $3 - 4i$. Да се пресметне a2 + b2 ; |b|; $a\bar{b}$; $\frac{a}{b}$

2) 4т) Нека a = $-\sqrt{3} + i$, b = $-\sqrt{2} - i\sqrt{2}$

а) 2т) Да се запишат с главен аргумент в тригонометричен вид числата a, b, ab, $\frac{a}{b}$

б) 2т) Да се реши уравнението $z^3 = b$ и да се пресметне $a^7 + (\bar{a})^7$

3) 4т) Да се изобрази в комплексната равнина множеството от точки z, за които :

а) $|z + 1 + i| > \sqrt{2}$

б) $|z - 4| \le |z - 2|$

в) $|\mathrm{Re}\, z| > 2$, $|\mathrm{Im}\, z| < 1$

г) $0 < \arg z < \pi$, $1 < |z| < 2$

Кои от изброените са области?

Контролно №1 по КА

1) $a = 4+3i \quad , b = 3-4i$

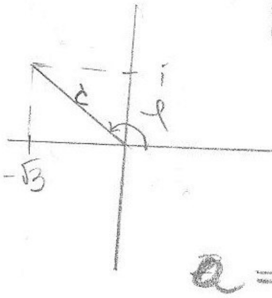
$$\left.\begin{aligned} a^2 &= (4+3i)^2 = 16+24i-9 = 7+24i \\ b^2 &= (3-4i)^2 = 9-24i-16 = -7-24i \end{aligned}\right\} \Rightarrow a^2+b^2=0$$

$$|b| = \sqrt{9+16} = \sqrt{25} = 5$$

$$a\bar{b} = (3+4i)(4+3i) = 12+25i-12 = 25i$$

$$\frac{a}{b} = \frac{4+3i}{3-4i}\,\frac{3+4i}{3+4i} = \frac{25i}{25} = i$$

2) a) $a = -\sqrt{3}+i$

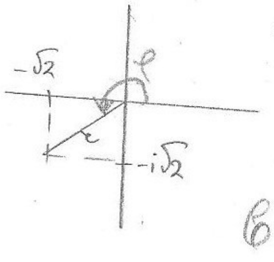


$$r = \sqrt{1+3} = 2$$

$$\frac{\sqrt{3}}{2} = \frac{\pi}{6} \Rightarrow \varphi = \frac{\pi}{6}+\frac{\pi}{2} = \frac{2\pi}{3}$$

$$\underline{a = 2e^{i\frac{2\pi}{3}}}$$

$b = -\sqrt{2}-i\sqrt{2}$



$$r = \sqrt{2+2} = 2$$

$$\frac{\sqrt{2}}{2} = \frac{\pi}{4} \Rightarrow \varphi = \frac{\pi}{4}+\pi = \frac{5\pi}{4} = -\frac{3\pi}{4}$$

$$\underline{b = 2e^{-i\frac{3\pi}{4}}}$$

$$ab = 2e^{i\frac{2\pi}{3}}\,2e^{-i\frac{3\pi}{4}} = 4e^{i\left(\frac{2\pi}{3}-\frac{3\pi}{4}\right)} = \underline{4e^{-i\frac{\pi}{12}}}$$

$$\frac{a}{b} = \frac{2e^{i\frac{2\pi}{3}}}{2e^{-i\frac{3\pi}{4}}} = e^{i\left(\frac{2\pi}{3}+\frac{3\pi}{4}\right)} = e^{i\frac{17\pi}{12}} = e^{-i\frac{7\pi}{12}}$$

2б) $z^3 = -\sqrt{2} - i\sqrt{2}$ $\qquad r = \sqrt{2+2} = 2$

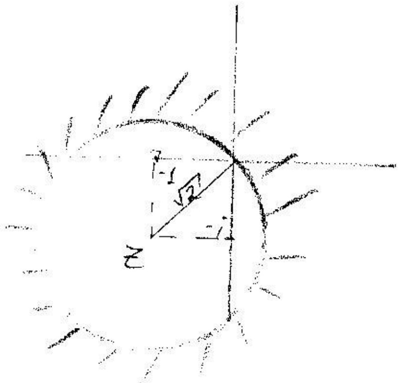
$$z_k = \sqrt[3]{2}\, e^{i\left(\frac{-\frac{3\pi}{4} + 2k\pi}{3}\right)}, \quad k = 0, 1, 2$$

$$a^7 + (\bar{a})^7 = (2\,\mathrm{Re}\,a)^7 = -2(\sqrt{3})^7$$

3 а) $|z+1+i| > \sqrt{2}$

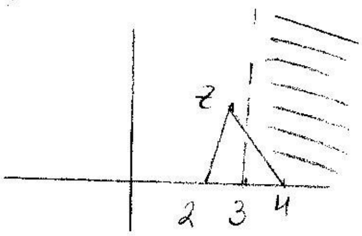
$z = -1 - i$

област



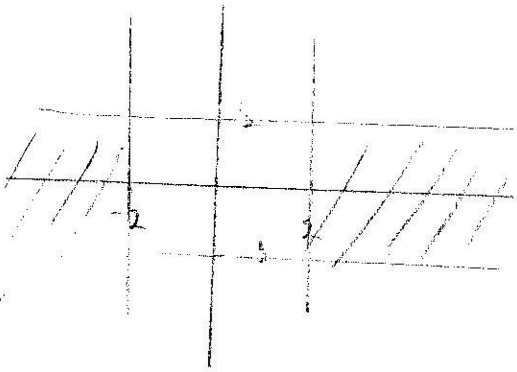
б) $|z-4| \le |z-2|$, $\Leftrightarrow |z-3|$

не е област



в) $|\mathrm{Re}\,z| > 2$ $|\mathrm{Im}\,z| < 1$

Не е област



г) $0 < \arg z < \pi$, $1 < |z| < 2$

област