3.06.2013г. ВА-упражнение

<u>задача</u>

формулите на Виет (сложните)

$f = x^4 - x^3 + \lambda x^2 - x - 6 \in \mathbb{C}[x]$ $\lambda = ?$

$x_1 + x_2 + x_3 + x_4 = +1$

$x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 +$

$x_1x_2x_3 + x_1x_2x_4 + x_2x_3x_4 + x_1x_3x_4 =$

$x_1x_2x_3x_4 = -6$

$x_1 + x_2 = 1$

$\underbrace{x_1 + x_2}_{1} + x_3 + x_4 = 1$

$x_3 + x_4 = 0$

$x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 = \lambda$

$x_1x_2 + x_1(x_3 + x_4) + x_2(x_3 + x_4) + x_3x_4 = \lambda$

$(\underbrace{x_3 + x_4}_{0})(\underbrace{x_1 + x_2}_{1}) = \lambda - x_1x_2 - x_3x_4$

$x_1x_2 + x_3x_4 = \lambda$

$x_1x_2x_3 + x_1x_2x_4 + x_2x_3x_4 + x_1x_3x_4 = 1$

$x_1x_2(\underbrace{x_3 + x_4}_{0}) + x_3x_4(\underbrace{x_1 + x_2}_{1}) = 1$

$x_3x_4 = 1$

$\underline{x_1x_2}\,\underbrace{x_3x_4}_{1} = -6$

$x_1x_2 = -6$

$\cancel{x_1 = -\frac{6}{x_2}}$

$-6 + 1 = \lambda$

$\lambda = -5$

обратно

$\underline{x_1x_2}\,x_3x_4 = -6$

$-6\,x_3x_4 = -6$

$x_3x_4 = 1$

$-x_4^2 = 1$

$x_4^2 + 1 = 0$

$x_4^2 = -1 = i^2$

$x_4 = -i$

<u>задача</u>

$f = x^3 - 5x^2 + 8x + \lambda \in \mathbb{C}[x]$

$\lambda = ?$

$x_1 + x_2 + x_3 = 5$

$x_1x_2 + x_1x_3 + x_2x_3 = 8$

$x_1x_2x_3 = -\lambda$

$x_1 + x_2 = x_1x_2$

$x_1x_2 + (x_1+x_2)x_3 = 8$ (Виет)

$x_1x_2 + x_1x_2x_3 = 8$

$x_1x_2 = 8+\lambda = x_1 + x_2$

$x_1 + x_2 + x_3 = 8 + \lambda + x_3 = 5$

$x_3 = 3 - \lambda$

$x_1x_2x_3 = (8+\lambda)(3+\lambda) = \lambda$

$\lambda^2 + 10\lambda + 24 = 0$

$\lambda_1 = -6$

$\lambda_2 = -4$

ако един полином е от нечетна степен, един от корените му (поне) ще е чисто реален.

нека $\lambda = -4$

$f = x^3 - 5x^2 + 8x - 4$

$x_1 + x_2 + x_3 = 5$

$x_1x_2 + x_1x_3 + x_2x_3 = 8$

$x_1x_2x_3 = 4$

	1	-5	8	-4
+1	1	-4	4	0
2	1	-2	0	

$f = (x-1)(x-2)^2$

$x_1 = 1 \quad x_2 = 2$

[illegible]

задача

$f = x^4 + 4x^3 + 9x^2 + 12x + \lambda \in \mathbb{C}[x]$ $\quad \cancel{x_1} + x_2 + x_3 + x_4 = -4$

за кой λ $\quad x_1x_2 = x_3x_4$ $\quad x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4 = -9$

$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 + \cancel{x_1} - 12$

$x_1x_2(x_3 + x_4) + x_3x_4(x_1 + x_2) = -12$ $\quad x_1x_2x_3x_4 = \lambda$

$x_1x_2(\underbrace{x_1 + x_2 + x_3 + x_4}_{-4}) = -12$

$x_1x_2 = 3 = x_3x_4$

$x_1x_2x_3x_4 = 9 = \lambda$

обратно

$x^4 + 4x^3 + 9x^2 + 12x + 9$

$\rightsquigarrow$

$f \in F[x]$, $K \geq F$, $\lambda \in K$ $\quad N \in \mathbb{N}$ $\quad f(x) = (x-\lambda)^k g(x)$

λ - к-кратен на $f(x) \Leftrightarrow$

$f(\lambda) = f'(\lambda) = f''(\lambda) = \ldots = f^{k-1}(\lambda) = 0$

$f^k(\lambda) \neq 0$

$\operatorname{char} F = 0$

λ - кратност?

a) $\lambda = 2$, $f = x^5 - 5x^4 + 7x^3 - 2x^2 + 4x - 8$

$f(2) = 32 - 80 + 56 - 8 + 8 - 8 = 0$

$f'(2) = 80 - 160 + 84 - 8 + 4 = 0$

$f''(x) = 20x^3 - 60x^2 + 42x - 4$ $\quad \Rightarrow \lambda = 2$ трикратен корен

$f''(2) = 160 - 240 + 84 - 4 = 0$

$f'''(2) = 240 - 240 + 42 \neq 0$

δ) $\lambda = 1$, $f = x^{2n+1} - (2n+1)x^{n+1} + (2n+1)x^{n} - 1 \quad (n \in N)$

$f(1) = 0$

$f'(x) = (2n+1)x^{2n} - (2n+1)(n+1)x^{n} + (2n+1)n\,x^{n-1}$

$f'(1) = (2n+1) - (2n+1)(n+1) + (2n+1)n =$

$= (2n+1)[1 - 1 - n + n] = 0$

$f''(x) = (2n+1)2n\,x^{2n-1} - (2n+1)(n+1)n\,x^{n-1} + (2n+1)n(n-1)x^{n-2} =$

$f''(1) = (2n+1)n(\cancel{2} - \cancel{n} - \cancel{1} + \cancel{n} - \cancel{1}) = 0$

$f'''(x) = (2n+1)2n(2n-1)x^{2n-2} - (2n+1)(n+1)n(n-1)x^{n-2} + (2n+1)n(n-1)(n-2)x^{n-3}$

$f'''(3) = n(2n+1)[2n(2n-\cancel{1}) - \cancel{n}^2 + 1 + n^2 + 2n + \cancel{2}] =$

$n(2n+1)(4n+1) \neq 0$

$\Rightarrow$ $\lambda = 1$ е трикратен корен

задача

$a, b \in F$ 1- двукратен корен

$f = ax^{n+1} + bx^{n} + 1 \qquad n \in \mathbb{N}$

$f' = a(n+1)x^{n} + bn\,x^{n-1}$

$f'(1) = a(n+1) + bn = an + bn + a = 0$

$f'' = a(n+1)n\,x^{n-1} + bn(n-1)x^{n-2}$

$f''(1) = a(n+1)n + bn(n-1) = an^2 + an + bn^2 - bn \neq 0$

$$\begin{cases} f(1) = 0 \\ f'(1) = 0 \\ f''(1) \neq 0 \end{cases} \qquad \left(\begin{array}{cc|c} 1 & 1 & -1 \\ n+1 & n & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 1 & 0 & n \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & n \\ 0 & 1 & -n-1 \end{array}\right)$$

$f''(1) = (n+1)(n^2) - n(n^2-1) = n^2 + n = n(n+1) \neq 0 \qquad a = n, \quad b = -n-1$

Задача

$\lambda = ?$

$f = x^4 + \lambda x^3 + 27 \in \mathbb{C}[x]$ - кратен корен

$f(\alpha) = \alpha^4 + \lambda\alpha^3 + 27 = 0$

$f' = 4x^3 + \lambda 3x^2 + \cancel{0}$

$f'(\alpha) = 4\alpha^3 + 3\lambda\alpha^2 = 0$

$f'' = 12x^2 + 6\lambda x \cancel{+0}$

$f''' = 24x + 6\lambda$

$\begin{cases} f(\alpha) = 0 \\ f'(\alpha) = 0 \end{cases}$ $\quad \alpha^2(4\alpha + 3\lambda) = 0$

$\alpha = 0$

$\alpha = -\frac{3\lambda}{4}$

$\cancel{\begin{cases} \alpha^4 + \lambda\alpha^3 + 27 = 0 \\ \alpha^2(4\alpha + 3\lambda) = 0 \end{cases}}$

$\cancel{\alpha_1 = 0 \quad \alpha_2 = -\frac{3}{4}}$

~~не може~~

I, сл. $\alpha = 0$ $\quad f(\alpha) \neq 0$

$\Rightarrow$ не

II, сл. $\alpha = -\frac{3}{4}$ $\quad \cancel{f(\alpha) = 0 \quad \lambda =}$

$\cancel{\alpha^3(\alpha + \lambda) + 27 = 0}$

$\cancel{-\frac{27}{64}\left(-\frac{3}{4} + \lambda\right) = -27}$

$\cancel{-\frac{3}{4} + \lambda = 27 \cdot \frac{64}{27}}$

$\cancel{\lambda = 64 + \frac{3}{4} = \frac{259}{4}}$

II, сл.

$\alpha^3(\alpha + \lambda) + 27 = 0$

$\left(-\frac{3\lambda}{4}\right)^3\left(-\frac{3\lambda}{4} + \lambda\right) + 27 = 0$

$-\frac{27\lambda^3}{64}\left(\frac{\lambda}{4}\right) + 27 = 0$

$\frac{-27\lambda 4}{256} = -27$

$\lambda 4 = 256$

$\lambda = (-4), 4, 4i, -4i$

задача 6.11 от сборника за упражнение

задача

$f = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}$ – няма кратни корени

доп. че има кратни

$f(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} = 0$

$f(\alpha) = 1 + \frac{\alpha}{1} + \frac{\alpha^2}{2!} + \dots + \frac{\alpha^n}{n!}$

$f'(\alpha) =$

$f'(x) = 1 + \frac{x}{1!} + \dots + \frac{x^{n-1}}{(n-1)!}$

$f'(\alpha) = 1 + \alpha + \dots + \frac{\alpha^{n-1}}{(n-1)!} = 0$

$\Rightarrow \frac{\alpha^n}{n!} = 0 \Leftrightarrow \alpha = 0$, но $f(0) = 1 \Rightarrow$ >:<

<u>задача</u>

остатък при делени на f с g

$f = gq + r \qquad \deg r < \deg g$

(остатка)

$r = ax^2 + bx + c$

а) $f = x^n - 1$

$g = x^3 - 2x^2 + x$

$g = x(x^2 - 2x + 1) =$

$= x(x-1)^2$

$f(0) = \underbrace{g(0).q(0)}_{0} + r(0) = c \Rightarrow \boxed{c = -1}$

$0 = f(1) = \underbrace{g(1)q(1)}_{0} + r(1) = a + b + c \Rightarrow a + b = 1$

$n = f'(1) = \underbrace{g'(1)q(1)}_{0} + \underbrace{g(1)q'(1)}_{0} + r'(1) = 2a + b = n$

$\begin{cases} a + b = 1 \\ 2a + b = n \end{cases} \qquad \begin{array}{l} a = n-1 \\ b = 2-n \end{array}$

$\Rightarrow r = (n-1)x^2 + (2-n)x - 1$

б) $f = x^n - 2x^{n-1} + 2x$

$g = x^3 - 5x^2 + 8x - 4$

$f = gq + r \qquad r = ax^2 + bx + c$

търсим корените на g от Виет

$$g = x^3 - 5x^2 + 8x - 4$$

$$x_1 + x_2 + x_3 =$$

$$x_1x_2 + x_1x_3 + x_2x_3 =$$

$$g = (x-1)(x-2)^2$$

$$1 = f(1) = \underbrace{g(1)q(1)}_{0} + r(1) = a + b + c$$

$$a + b + c = 1 \qquad r(1) = 1$$

$$= f(2) = 2^n - 2^n + 4 = 4$$

$$4 = f(2) = \underbrace{g(2)q(2)}_{0} + r(2) = 4a + 2b + c$$

$$f'(x) = n(x^{n-1}) - 2(n-1)x^{n-2} + 2$$

$$f'(2) = n(2^{n-1}) - 2(n-1)2^{n-2} + 2 =$$

$$= \cancel{n2^{n-1}} - \cancel{n2^{n-1}} + 2^{n-1} + 2 =$$

$$= \cancel{2n}$$

$$2^{n-1} + 2 = f'(2) = \underbrace{g'(2)q(2)}_{0} + r'(2) = 2ax + b$$

$$\cancel{4a + b} =$$

$$2^{n-1} + 2 = 4a + b$$

$$\begin{cases} a + b + c = 1 \\ 4a + 2b + c = 4 \\ 4a + b = 2^{n-1} + 2 \end{cases}$$

$$a + b = 1 - c \qquad c = 1 - a - b$$

$$4a + 2b + 1 - a - b = 4$$

$$3a + b = 3$$

$$b = 3 - 3a$$

$$4a + 3 - 3a = 2^{n-1} + 2$$

$$a = 2^{n-1} - 1$$

$$b = 3 - 3 \cdot 2^{n-1} - 3 = -3 \cdot 2^{n-1}$$

$$c = 1 - \underline{2^{n-1}} + 1 + \underline{3 \cdot 2^{n-1}} = 2 + 2 \cdot 2^{n-1} = 2 + 2^n$$

$$= 2^{n-1}(3-1)$$

~~от~~

$$r = (2^{n-1} - 1)x^2 + -3 \cdot 2^{n-1} x + 2 + 2^n$$