04.VII.2013г. Консултация - Божилов

(1) $\sigma_1 = x_1 + x_2 + \dots + x_n$

$\sigma_2 = x_1x_2 + x_1x_3 + \dots + x_1x_n + x_2x_3 + \dots + x_2x_n + \dots + x_{n-1}x_n =$

$= \sum_{i<j} x_i x_j$

$\sigma_3 = x_1x_2x_3 + \dots = \sum_{i<j<k} x_i x_j x_k$

— — — — — — — — — —

$\sigma_n = x_1 x_2 \dots x_n$

$x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n} > x_1^{\beta_1} x_2^{\beta_2} \dots x_n^{\beta_n}$

$(\underbrace{x_1 x_1 \dots x_1}_{\alpha_1}, \underbrace{x_2 \dots x_2}_{\alpha_2} \mid \dots \underbrace{x_n \dots x_n}_{\alpha_n} \mid \underbrace{x_1 \dots x_1}_{\beta_1} \underbrace{x_2 \dots x_2}_{\beta_2} \mid - \underbrace{x_n \dots x_n}_{\beta_n}$

$x_n \dots x_k \, x_{k+1}$

$\parallel \quad \vee$

$x_n \dots x_k \, x_{k+1}$

$f_k : \begin{cases} \alpha_1 = \beta_1 \\ \alpha_{k-1} = \beta_{k-1} \\ \alpha_k > \beta_k \end{cases}$

Lm (старшия едночл. f) x (старшия едночл. g) =

= (старшия едночл. fg)

f - симетричен полином $x_1^{\alpha_1} \dots x_n^{\alpha_n}$ - старшия едночлен на f

$\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$

$f \to A x_1^{\alpha_1} \dots x_n^{\alpha_n}$ - старши едночлен

$f_1 = f - A \underbrace{\sigma_1^{\alpha_1 - \alpha_2} \sigma_2^{\alpha_2 - \alpha_3} \dots \sigma_{n-1}^{\alpha_{n-1} - \alpha_n} \sigma_n^{\alpha_n}}_{\text{~~старшия~~ } ! \text{ степен}}$

$x_1^{\alpha_1 - \alpha_2} (x_1 x_2)^{\alpha_2 - \alpha_3} \dots (x_1 \dots x_{n-1})^{\alpha_{n-1} - \alpha_n} (x_1 \dots x_n)^{\alpha_n}$

старшия едночлен f_1 < старшия едночлен f

$f = \sum f_i$ $\forall$ едночлени в f_i са от една и съща степен

f_i-хомогенни полинома)

$\left(f(tx_1 \dots tx_n) = t^n f(x_1 \dots x_n) \right)$

$\boxed{\text{б.о.о}}$ f-хомогенни

$Ax_1^{\alpha_1} \dots x_n^{\alpha_n}$ -старши едночлени

$\alpha_1 \dots \alpha_n$

П.Н. $\forall$ $\beta_1 \dots \beta_n \longrightarrow \sigma_1^{\beta_1 - \beta_2} \dots \sigma_{n-1}^{\beta_{n-1} - \beta_n} \sigma_n^{\beta_n}$

$\sum \beta_i = \sum \alpha_i$

$$f = \sum_{\beta_1 \dots \beta_n} \boxed{C_{\beta_1 \dots \beta_n}} \sigma_1^{\beta_1 - \beta_2} \dots \sigma_{n-1}^{\beta_{n-1} - \beta_n} \sigma_n^{\beta_n}$$

$f(x_1, x_2, x_3) = \boxed{x_1^3} + x_2^3 + x_3^3 = A\sigma_1^3 + B\sigma_1\sigma_2 + C\sigma_3$ $A = 1$

$x_1^3 \to 3\,0\,0 \to \sigma_1^{3-0}\sigma_2^{0-0}\sigma_3^0 = \sigma_1^3$

$2\,1\,0 \to \sigma_1^{2-1}\sigma_2^{1-0}\sigma_3^0 = \sigma_1\sigma_2$

$\underset{x}{1}\,\underset{y}{1}\,\underset{z}{1} \to \sigma_1^{1-1}\sigma_2^{1-1}\sigma_3^1 = \sigma_3$

$f = \sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3$

$f_1 = f - \sigma_1^3 \leftarrow x_1 x_2$

$f_2 = f_1 - \square\,\sigma_1\sigma_2$

$f_3 = f_2 - \square\,\sigma_3 = 0$

$x_1 = x_2 = 1$ $\quad \sigma_1 = 2, \sigma_2 = 1, \sigma_3 = 0, f = 2$

$x_3 = 0$ $\quad 2 = 2^3 + B.2.1 + C.0, B = -3$

$x_1 = x_2 = x_3 = 1$ $\quad \sigma_1 = \sigma_2 = 3, \sigma_3 = 1, f = 3$

$3 = 3^3 - 3.3.3 + C.1 \Rightarrow C = 3$

1) $x \le y \le z$

2) $x + y + z = 2 + 1 + 0 = 3$

3) $\forall_{n+1}$

(2)

② $x_{1,2,3}: x^3+px+q=f$

$$F=\frac{x_1^2}{6+x_2x_3}+\frac{x_2^2}{6+x_1x_3}+\frac{x_3^2}{6+x_1x_2}$$

$$f=x^n+\dots=(x-x_1)\dots(x-x_n)$$

$$f(a)=(a-x_1)\dots(a-x_n)$$

$$\sim \ln f=\sum \ln(x-x_i) \qquad (\cancel{2}\dots)'$$

$$\frac{f'}{f}=\sum\frac{1}{x-x_i} \xrightarrow{(\ )'} \boxed{\sum\frac{1}{a-x_i}=\frac{f'(a)}{f(a)}}$$

Доказва се лесно, че $f'=\sum \frac{f}{(x-x_i)}$

$$\frac{f''f-f'^2}{f^2}=-\sum\frac{1}{(x-x_i)^2}$$

$$\sum\frac{1}{(x-x_i)^2}=\frac{f'^2-f\cdot f''}{f^2}$$

$$\boxed{\sum\frac{1}{(a-x_i)^2}=\frac{(f'(a))^2-f(a)\cdot f''(a)}{(f(a))^2}}$$

$x_1+x_2+x_3=0$ | $f(x_2)=0$

$x_1x_2+x_1x_3+x_2x_3=p$ | $x_1^3+\underbrace{px_1+q}=0$

$x_1x_2x_3=-q$ | $f'=3x^2+p$

$$\frac{x_1^2}{6+x_2x_3}=\frac{x_1^2}{6+\frac{-q}{x_1}}=\frac{x_1^3}{6x_1-q}=\frac{-px_1-q}{6x_1-q}=\cancel{-\frac{p}{6}\cdot\frac{\frac{q}{p}+x_1}{\frac{q}{6}-x_1}}$$

$$=-\frac{p}{6}\left(\frac{x_1+\frac{q}{p}\cancel{-\frac{q}{6}+\frac{q}{6}}}{x_1-\frac{q}{6}}\right)=-\frac{p}{6}-\frac{p}{6}\cdot\frac{\frac{q}{p}+\frac{q}{6}}{x_1-\frac{q}{6}}=$$

$$=-\frac{p}{6}+\frac{\cancel{p}q}{6}\cdot\frac{p+6}{\cancel{p}\cdot 6}\cdot\frac{1}{\frac{q}{6}-x_1}$$

$$F = -\frac{3p}{6} + \frac{q(p+6)}{6^2} \sum \frac{1}{\frac{q}{6} - x_i} = -\frac{3p}{6} + \frac{q(p+6)}{6^2} \cdot \frac{-3\left(\frac{q}{6}\right)^2 + p}{\left(\frac{q}{6}\right)^3 + p\frac{q}{6} + q} =$$

$$= -\frac{3p}{6} + \frac{q(p+6)}{6} \cdot \frac{3q^2+36p}{q^3 + 36pq + 216q} = \frac{-3p(q^2+36p+216) + (3pq^2 + 36p^2 + 18q^2 + \dots}{6(q^2+36p+216)}$$

$$= \frac{+216p}{---} = \frac{-72p^2 - 432p + 18q^2}{6(q^2+36p+216)}$$

③ $\begin{cases} x \equiv 7 \pmod{10} \\ x \equiv 2 \pmod{5} \\ x \equiv 8 \pmod{9} \end{cases} \longrightarrow x = 7 + 10y$

$\begin{cases} 10y + 7 = 2\ (5) \ ✓ \\ 10y + 7 \equiv 8\ (9) \end{cases}$

$10y \equiv 1\ (9)$

$y \equiv 1\ (9)$

$y = 1 + 9z$

$x = 7 + 10(1 + 9t) =$

$= 17 + 90t$

$x \equiv 17\ (90)$

взаимно прости → НОК

(единствено решение, може и чрез налучкване)

④ $\boxed{1.1\ g)}$ $U = \{ z \in \mathbb{C} \mid |z| = 1 \}$

група относно умножение

I начин: директно от дефиницията:

0) $z_1, z_2 \in U \Rightarrow z_1 z_2 \in U$

$\Downarrow$ $\Uparrow$

$|z_1| = |z_2| = 1$ $|z_1 z_2| = 1$

$3 + 4i$

-1

$1 + i$

i

$1 + \sqrt{2}i$

1) $U \subset \mathbb{C} \Rightarrow$ асоциативен

2) $\exists e \in U : \exists \boxed{1} \in U : (|1| = 1)$

$\boxed{1} z = z \cdot \boxed{1} = z \quad \forall z \in U \qquad \checkmark \Leftarrow$ от $\mathbb{C}$

3) $\forall z \in U \quad \exists z^{-1} \in U:$

$z \cdot z^{-1} = z^{-1} z = 1 \qquad \checkmark \Longleftarrow$ от $\mathbb{C}$

④ $z^{-1} = \frac{1}{z} \qquad z \in U \Rightarrow |z| = 1 \Rightarrow |z^{-1}| = \left|\frac{1}{z}\right| = \frac{1}{1} \Rightarrow z^{-1} \in U$

$H < G \iff \left| \begin{array}{l} H \subset G \\ H \text{ е група относно операциите в } G \end{array} \right.$

$\Updownarrow$

$\left| \begin{array}{l} H \subset G \\ h_1 h_2,\ h^{-1} \in H \quad \forall h_{1,2}, h \in H \end{array} \right.$

$\Updownarrow$

$\left| \begin{array}{l} H \subset G \\ h_1^{-1} h_2 \in H \ (h_1 h_2^{-1} \in H) \end{array} \right.$

$\Updownarrow (+)$

$h_1 - h_2 \in H$

R - пръстен

$K < R$ \
подпръстен

$\iff \left| \begin{array}{l} K \subset R \\ K \text{ - подпръстен} \ldots \end{array} \right. \iff \left| \begin{array}{l} K \subset R \\ k_1 k_2 \quad k_1 - k_2 \in K \\ \forall k_1, k_2 \in K \end{array} \right.$

II начин: $U \subset \mathbb{C}^*$ (съобразяваме го)

$\mathbb{C}^* = \mathbb{C} \setminus \{0\}$

$(\mathbb{C}^*, \cdot)$ - група

$\forall z_1, z_2 \in U \qquad |z_1^{-1} z_2| = \left| \frac{z_2}{z_1} \right| = \frac{|z_2|}{|z_1|} = \frac{1}{1} = 1 \Rightarrow z_1^{-1} z_2 \in U$

$\Rightarrow U < \mathbb{C}^* \Rightarrow U$ - група

$H \triangle G \underset{def.}{\iff} Hg = gH \quad \forall g \iff \boxed{ghg^{-1} \in H \quad \forall h \in H, \forall g \in G}$

$\subset$
$\supset$

$I \triangle R \overset{def.}{\iff} \left| \begin{array}{ll} i_1 - i_2 \in I & \forall i_1, i_2 \in I \\ ir, ri \in I & \forall i \in I, \forall r \in R \end{array} \right.$

$f \in \mathbb{Z}[x]$

f - разложим над $\mathbb{Q} \iff f$ разложим над $\mathbb{Z}$

$\deg f = 3 \qquad f \in \mathbb{Z}[x]$

f - неразложим $\iff$ няма рационални корени

(5) $\deg f = 4$, $f = f_1 f_2$

$\deg f_1 = \cancel{0}, 1, 2, \cancel{3}, \cancel{4}$

$\downarrow$ f_1-лин. $\searrow$ f_2-лин.

f-неразложим $\Leftrightarrow$ | няма рационални корени
| $\nexists f_1, f_2$: | $\deg f_1 = \deg f_2 = 2$
| $f = f_1 f_2$

$\boxed{9.9}$ - критерий на Айзенщайн

$\boxed{9.10}$, $\boxed{9.11}$ > модификации на критерия на Айзенщайн

(6) $\boxed{5.1\ б)}$

$f = gq_1 + r_1$

$g = r_1 q_2 + r_2$

$r_1 = r_2 q_3 + r_3$

$r_2 = r_3 q_4 + r_4$

$\vdots$

$d = (f, g)$

$- d / f,\ d / g$

$- d' / f,\ d' / g \Rightarrow d' / d$

$f / g,\ g / f \Rightarrow \exists c : g = c.f$

$r_1 = f - gq_1$

$r_2 = g - r_1 q_2 = g - (f - gq_1)q_2 = (-q_2)f + (1 + q_1 q_2)g$

$r_3 = r_1 - r_2 q_3 = (f - q_1 g) - ((-q_2)f + (1 + q_1 q_2)g)q_3 =$

$= (1 + q_2 q_3)f - (q_1 + q_3 + q_1 q_2 q_3)g$

(7) G_1, G_2 - групи

$\varphi : G_1 \to G_2$.

(хомоморфизъм) ХММ: $\varphi(gh) = \varphi(g(\varphi(h))$ $\forall g, h \in G_1$

$\mathrm{Ker}\ \varphi = \{ g_1 \in G_1 \mid \varphi(g_1) = 1_{G_2} \} \triangle G_1$

$\mathrm{Im}\ \varphi = \{ g_2 \in G_2 \mid \exists g_1 \in G_1 : \varphi(g_1) = g_2 \} < G_2$

Теорема за ХММ: $G_1 / \mathrm{Ker}\varphi \cong \mathrm{Im}\varphi$

$H \triangle G$

$G/H = \{ gH \mid g \in G \}$, $\cdot$

$g_1 H . g_2 H = (g_1 g_2) H$

$gH = \{ gh \mid h \in H \}$

$H < G$

$g_1 \sim g_2 \overset{def}{\Leftrightarrow} g_1^{-1} g_2 \in H \Leftrightarrow$

$\Leftrightarrow g_1 H = g_2 H \mid \forall g\ (gH) = (H)$

$\bar{g} = \{ x \mid g \sim x \} = gH$

(6)

R_1, R_2 – пръстени

$\varphi : R_1 \to R_2$

ХММ: $\left| \begin{array}{l} \varphi(a+b) = \varphi(a) + \varphi(b) \\ \varphi(ab) = \varphi(a)\varphi(b) \end{array} \right.$

$\operatorname{Ker}\varphi = \{ r_1 \in R_1 \mid \varphi(r_1) = 0_{R_2} \} \triangle R_1$

$\operatorname{Im}\varphi = \{ r_2 \in R_2 \mid \exists r_1 \in R_1 : \varphi(r_1) = r_2 \} < R_2$

Теорема: $R_1 / \operatorname{Ker}\varphi \cong \operatorname{Im}\varphi$

$I \triangle R$

$((I,+) \triangle (R,+))$

$R/I = \{ r + I \mid r \in R \}$

$(r_1 + I) + (r_2 + I) := (r_1 + r_2) + I$

$(r_1 + I)(r_2 + I) := r_1 \cdot r_2 + I$

8) $\mathbb{Z}^{\downarrow G, R}$

$H = I = (n) = \{ nz \mid z \in \mathbb{Z} \}$

$(H,+) \triangle (\mathbb{Z},+)$ – група 1) $nz_1, nz_2 \Rightarrow nz_1 - nz_2 = n(z_1 - z_2) \in H$

$(I,+,\cdot) \triangle (\mathbb{Z},+,\cdot)$ – подгрупа 2) $nz_1 \in H$, $z_2 \in \mathbb{Z}$ $(nz_1)z_2 = n(z_1 z_2) \in H$

$\mathbb{Z}/H$ което множество $\{ z + (n) \mid z \in \mathbb{Z} \}$

$\mathbb{Z}/I$

$z_1 + (n) = z_2 + (n) \Leftrightarrow z_1 - z_2 \in (n) \Leftrightarrow n \mid z_1 - z_2 \Leftrightarrow z_1 \equiv z_2 \pmod{n}$

z_1 пети в 1 клас

$\boxed{Hg_1 = Hg_2 \Leftrightarrow g_1 g_2^{-1} \in H}$

$z = nq + r, \quad 0 \le r < n, \quad z \equiv r(n) \Rightarrow z + (n) = r + (n)$

$\left. \begin{array}{l} r_1 + (n) = r_2 + (n) \Rightarrow n / r_1 - r_2 \\ 0 \le r_1, r_2 < n \longrightarrow (r_1 - r_2) < n \end{array} \right\} r_1 - r_2 = 0 \Rightarrow r_1 = r_2$

~~Коректност:~~ $\boxed{(z_1 + (n)) \dot{+} (z_2 + (n)) = (z_1 \dot{+} z_2) + (n)}$

Коректност: $r_1+(n)=r_1'+(n) \Rightarrow r_1 \equiv r_1'$
$r_2+(n)=r_2'+(n) \Rightarrow r_2 \equiv r_2'$

$\overline{r_1 \dot{+} r_2 \equiv r_1' \dot{+} r_2' (n)}$

$\overline{r_1} \dot{+} \overline{r_2} = \overline{r_1 \dot{+} r_2}$

$\Updownarrow$

$\bar{a} + \bar{b} = \bar{c}$

$\Updownarrow$

$a+b \equiv c\ (n)$

- ХММ
- върху $\rightarrow \mathrm{Im}\,\varphi = \mathbb{Z}_n$
- $\mathrm{Ker}\,\varphi = \{ a \in \mathbb{Z} \mid \underbrace{\varphi(a) = 0+(n)}_{\Updownarrow\ n/a} \} = (n)$

$a+(n)$

$\varphi : \mathbb{Z} \rightarrow \mathbb{Z}_n$
$a \longmapsto a+(n)$

$\bar{1} = 1+(n) = \{ 1+nz \mid z \in \mathbb{Z} \}$

$\mathbb{Z}_n \cong \mathbb{Z}/(n)$

ч. + ч. = ч. $\quad \bar{0}+\bar{0}=\bar{0}$

н. + ч. = н. $\quad \bar{0}+\bar{1}=\overline{0+1}=\bar{1}$

н + н. = ч. $\quad \bar{1}+\bar{1}=\bar{2}=\bar{0}$

$GL_n(F) = \{ A \in M_n(F) \mid \det A \neq 0 \}$

$SL_n(F) = \{ A \in M_n(F) \mid \det A = 1 \} < GL_n(F)$

$\det(A.B) = \det A . \det B$

$1 = \det(A.A^{-1}) = \det A . \det A^{-1} \Rightarrow \frac{1}{\det A}$

$H \triangle G$

$g \in G,\ h \in H\ (\det h = 1)$

$\det(ghg^{-1}) = \underbrace{\det g \cdot \overset{=1}{\det h} \cdot \det g^{-1}}_{1} \Rightarrow 1 \Rightarrow H \triangle G$

$g_1 H = g_2 H \Leftrightarrow g_1^{-1} g_2 \in H \Leftrightarrow \det(g_1^{-1} g_2) = 1 \Leftrightarrow \det g_1 = \det g_2$

$G/H \cong F^*$

⑧