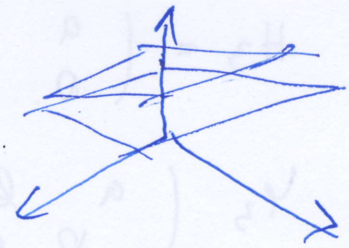
$$g^m = \cos\left(\frac{5\pi m}{6}\right) + i\sin\left(\frac{5\pi m}{6}\right) = 1$$



$$\frac{5\cancel{\pi} m}{6} = 2k\cancel{\pi}$$

$$5m = 12k \Rightarrow 12/m \Rightarrow m = 12$$

б) в) $g = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

$$\cos\left(\frac{5\pi m}{6}\right) + i\sin\left(\frac{5\pi m}{6}\right) = 0$$

$$\cos\left(\frac{5\pi m}{6}\right) = 0 \quad = \sin\left(\frac{5\pi m}{6}\right) \text{ не}$$

в) $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$

$(1\ 2\ 3) \quad (4\ 5) \qquad 3.2 = 6$

г) $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 1 & 6 \end{pmatrix}$

$(1\ 2\ 3\ 4\ 5)(6) \qquad 5.1 = 5$

д) $g = \begin{pmatrix} -1 & a \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} -1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{ред } 2$$

е) $g = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

$$g^2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(iii) $H_3 = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$

$\varphi_3 \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = c$

$\varphi_3 \left(\begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ 0 & c_2 \end{pmatrix} \right) = \varphi_3 \left(\begin{pmatrix} a_1 a_2 & a_1 b_2 + b_1 c_2 \\ 0 & c_1 c_2 \end{pmatrix} \right) = c_1 c_2$

$\varphi_3 \left(\begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix} \right) \cdot \varphi_3 \begin{pmatrix} a_2 & b_2 \\ 0 & c_2 \end{pmatrix} = c_1 . c_2$

(iv) $\varphi_4 \left(\begin{pmatrix} a_1 & b_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ 0 & 1 \end{pmatrix} \right) = \varphi_4 \begin{pmatrix} a_1 a_2 & a_1 b_2 + b_1 \\ 0 & 1 \end{pmatrix}$

$\parallel$

$a_1 b_2 + b_1$

$\varphi_4 \begin{pmatrix} a_1 & b_1 \\ 0 & 1 \end{pmatrix} \cdot \varphi_4 \begin{pmatrix} a_2 & b_2 \\ 0 & 1 \end{pmatrix} = b_1 . b_2 \quad \# \text{ не}$

7. a) $g = -\frac{\sqrt{3}}{2} + \frac{1}{2} i = \cos \frac{5\pi}{6} + \sin \frac{5\pi}{6} i$

$g^2 = \left(\cos \frac{5\pi}{6} + \sin \frac{5\pi}{6} i \right)^2 = \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}$

$g^m = \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)^m = \cos \frac{5\pi m}{6} + i \sin \frac{5\pi m}{6}$

~~1: $\frac{5\pi}{6}$, 2: $\frac{5\pi}{3}$, 3: $\frac{5\pi}{2}$, 4: $\frac{10}{3}\pi$, 5: $\frac{25}{6}\pi$, 6: 5π, 7: $\frac{35\pi}{6}$, 8: $\frac{40}{6}\pi$,~~

$\frac{\pi}{6}$ $\quad \frac{\pi + 5\pi}{6}$

~~9: $\frac{45\pi}{6}$ 10: $\frac{50\pi}{6}$ 11: $\frac{55\pi}{6}$ 12~~

$g + H = \cancel{H} + \cancel{g}$

Ако $\begin{array}{l} a_1 \equiv b_1 \pmod{m_1} \\ a_1 \equiv b_1 \pmod{m_2} \\ \vdots \\ a_1 \equiv b_1 \pmod{m_n} \end{array}$ $\Rightarrow a_1 \equiv b_1 \pmod{m_1 \cdot m_2 \cdot \ldots \cdot m_n}$

$a_1 \equiv b_1 \pmod{[m_1, \ldots, m_n]}$

$(m_1, m_2, \ldots, m_n) \neq 1$

5. a) $f(z) = |z|$ $z = a + bi$

$|z| = \sqrt{a^2+b^2}$

$f(z_1 z_2) = |z_1 . z_2| = |z_1| . |z_2| = f(z_1) . f(z_2)$ да

(ii) $f(z) = 2|z|$

$f(z_1 . z_2) = 2|z_1 . z_2| \neq 2|z_1| . 2|z_2| = f(z_1) . f(z_2)$ не

(iii) $= f(z) = \frac{1}{|z|} = \frac{1}{|z_1| . |z_2|} = f(z_1) . f(z_2)$ да

(iv) $f(z_1 . z_2) = 1 + \cancel{f}|z_1 . z_2| \neq 2 + \cancel{f}|z_1| + \cancel{f}|z_2| = f(z_1) f(z_2)$ не

(v) $f(z_1 . z_2) = |z_1 . z_2|^2 = |z_1|^2 |z_2|^2 = f(z_1) . f(z_2)$ да

(vi) $f(z_1 . z_2) = 1 \neq 1 \cdot 1 = f(z_1) \cdot f(z_2)$ да

(vii) $f(z_1 . z_2) = 2 \neq 2 . 2 = f(z_1) . f(z_2)$ не

6. $H_1 = \left\{ \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \middle| \; a \in \mathbb{R} \right\} \longrightarrow (\mathbb{R}, +)$

(i) $\varphi_1 \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} + \varphi_1 \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = a + b$

$\varphi_1 \left(\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \right) = \varphi_1 \left(\begin{pmatrix} 2 & a+b \\ 0 & 2 \end{pmatrix} \right) = a + b$

(ii) $\varphi_2 \left(\begin{pmatrix} a_1 & b_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ 0 & 1 \end{pmatrix} \right) = \varphi_2 \begin{pmatrix} a_1 a_2 & a_1 b_2 + b_1 \\ 0 & b_2 + 1 \end{pmatrix}$

$= a_1 \cdot a_2$; $= \varphi_2 \begin{pmatrix} a_1 & b_1 \\ 0 & 1 \end{pmatrix} \varphi_2 \begin{pmatrix} a_2 & b_2 \\ 0 & 1 \end{pmatrix}$

$a_1 . a_2$

$12.\ 3x + 12 \equiv 0 \pmod{35} \longrightarrow x \equiv -12 \pmod{35}$

$7x \equiv 11 \pmod{20}$

$(7,20) = 1$

$\exists u, v : 7 \cdot u + 20v = 1$

$20 = 7 \cdot 2 + 6$
$7 = 6 \cdot 1 + 1$

$1 = 7 - 6 \cdot 1 = 7 - (20 - 7 \cdot 2) = 7 \cdot 3 - 20$

$7 \cdot 3x \equiv 33 \pmod{20}$
$x \equiv 13 \pmod{20}$

$\left| \begin{array}{l} x \equiv 13 \pmod{20} \\ x \equiv -12 \pmod{35} \end{array} \right.$ $x = 35y - 12$

$x \equiv 1 \pmod{140}$

$35y - 12 \equiv 13 \pmod{20}$
$35y \equiv 25 \pmod{20}$
$7y \equiv 5 \pmod{4}$
$(7,4) = 1$
$1 = 7u + 4v$
$7 = 4 \cdot 1 + 3$
$4 = 3 \cdot 1 + 1$

$1 = 4 - 3 \cdot 1 = 4 - (7 - 4 \cdot 1) \cdot 1 = 4 \cdot 2 - 7$
$-1 \cdot 7y \equiv 5 \cdot -1 \pmod{4}$
$y_0 \equiv -5 \pmod{4} \equiv -1 \pmod{4} \equiv 3 \pmod{4}$

$y_i = y_0 + i\,4 \pmod{20}$ 0,
$y_i \equiv 3 \pmod{4}$ 0, 1, 2, 3
$y_i \equiv 7 \pmod{20}$
y_i

Задачи от контролното от вариант

1) $\begin{cases} 5x+1 \equiv 0 \pmod{24} \\ 4x \equiv 19 \pmod{21} \end{cases}$

$(5,24)=1$

$\exists u,v: 5u+24v=1$

$24 = 5.4+4$
$5 = 1.4+1$

$1 = 5-1.4 = 5-1.(24-5.4) = 5.5-24$

$5.5x \equiv -5 \pmod{24}$
$x \equiv -5 \pmod{24} \equiv 19 \pmod{24}$

$(4,21)=1$ $\qquad 4x \equiv 19 \pmod{21}$

$\exists u,v: 4u+21v=1$

$21 = 5.4+1$

$1 = 21-5.4$

$-5.4x \equiv -5.19 \pmod{21}$

$x \equiv -95 \pmod{21} \equiv -14 \pmod{21} = 7 \pmod{21}$

$\begin{cases} x \equiv 19 \pmod{24} \longrightarrow x = 24.y+19 \\ x \equiv 7 \pmod{21} \longrightarrow x = 21.z+7 \end{cases}$

~~$x \equiv 26 \pmod{24.7}$~~

$2(2y+1) = 21.z$
$4(2y+1) = 7z$
$y=3,\ z=4$
$x=91$

$91 \mid 84$

2) $\begin{cases} 3x+1 \equiv 0 \pmod{35} \\ 7x \equiv 11 \pmod{20} \end{cases}$

$\exists u,v: 3u+35v=1$

~~$35 = 3.11+2$~~
~~$11 = 5.2+1$~~

~~$1 = 11-5.2 = 11-5.(35-3.11) = \ldots -5.35+66.3$~~

~~$66.3+66$~~

$35 = 3.11+2$
$3 = 2.1+1$

$1 = 3-2.1 = 3-(35-3.11) = -35+3.12$

2. а) $2x \equiv 3 \pmod 5$

I начин: $(2,5)=1 \longrightarrow$ имаме ! решение

$$2x-3 \equiv 0 \pmod 5$$

$$x \equiv 4 \pmod 5$$

II начин: $(2,5)=1$

$\exists u,v \in \mathbb{Z} : 2u+5v=1$

$\boxed{5} = 2.\boxed{2}+1 \longrightarrow$ остатък 1 спираме

$$1 = 1.5 - 2.2$$

$$-2.2x \equiv -2.3 \pmod 5$$

$$x \equiv -1 \pmod 5$$

б) $2x \equiv 1 \pmod 4$

$(2,4) = 2$

$(1,4) = 1$ $\Rightarrow$ н.р.

в) $6x \equiv 3 \pmod 9$

$2x \equiv 1 \pmod 3$

$(2,3)=1$

$\exists u,v \in \mathbb{Z} : 2u+3v=1$

$3 = 2.1+1$

$1 = 3 - 2.1$

$-1.2x = -1.1 \pmod 3$

$x = -1 \pmod 3 = 2 \pmod 3$

$x_i \equiv x_0 + i\,3 \pmod 9$, $0 \le i \le 2$

$x_1 \equiv -1 \pmod 9$

$x_2 \equiv 2 \pmod 9$

$x_3 \equiv 5 \pmod 9$

15.

$$\begin{cases} 2x \equiv -1 \pmod{3} \quad \cdot 2 \\ 3x \equiv -1 \pmod{5} \end{cases}$$

$$\begin{cases} x \equiv -2 \pmod{3} \\ x \equiv -2 \pmod{5} \end{cases}$$

$$x = 3k - 2$$

$$\cancel{3k - 2 \equiv -2 \pmod{3}}$$

$$\cancel{3k \equiv 0 \pmod{3}}$$

$$3k - \cancel{2} \equiv -\cancel{2} \pmod{5}$$

$$3k \equiv 0 \pmod{5}$$

$$k \equiv 0 \pmod{5}$$

$$x = 15t - 2$$

$$x = -2 \pmod{15}$$

(ii) $$\begin{cases} 5x \equiv 1 \pmod{6} \\ 5x \equiv 6 \pmod{7} \end{cases} \longrightarrow \begin{cases} x \equiv 5 \pmod{6} \\ x \equiv 18 \pmod{7} \end{cases}$$

$$x \equiv 4 \pmod{7}$$

$$x = 7k + 4$$

$$\begin{cases} x \equiv 5 \pmod{6} \\ x \equiv 4 \pmod{7} \end{cases}$$

$$7k + 4 \equiv 5 \pmod{6}$$

$$7k \equiv +1 \pmod{6}$$

$$k \equiv +1 \pmod{6}$$

$$k = 6t + 1$$

$$x = 7(6t+1) + 4 = 42t + 11$$

$$x \equiv 11 \pmod{42}$$

$$\begin{cases} 2x \equiv 1 \pmod{3} & \cdot\ 2 \\ 3x \equiv 4 \pmod{5} & \cdot\ 2 \\ 5x \equiv -2 \pmod{7} & \cdot\ 3 \end{cases}$$

$$\begin{cases} x \equiv 2 \pmod{3} \\ x \equiv 3 \pmod{5} \\ x \equiv 1 \pmod{7} \end{cases} \qquad x \equiv 8 \pmod{3.5}$$

Проект по Софтуерни Технологии

На тема:

„Форматна конверсия на е-книги"

Изготвили:

Борис Ненов

Теодор Мердийски

Иван Ванев

$R = \{ a + b\sqrt{3} \mid a, b \in \mathbb{Z} \}$

$I = (3+2\sqrt{3}) = \{ (3+2\sqrt{3})r \mid r \in R \}$

$K = \{ a + b\sqrt{3} \in R \mid a \equiv 0 \pmod 3 \}$

Тъй като I е идеал

$\Rightarrow r \in R,\ a \in I \quad \Rightarrow ra \in I$

$$(x + y\sqrt{3}).(3 + 2\sqrt{3}) = 3x + 6y + (2x + 3y)\sqrt{3} =$$

$$= a + b\sqrt{3} \in I$$

$$a = 3x + 6y \equiv 0 \pmod 3$$

$\Rightarrow K \subseteq I \qquad I \subseteq K$

Искаме да докажем, че

$$(x_1 + y_1\sqrt{3})(3 + 2\sqrt{3}) = \underbrace{3x_1 + 6y_1}_{a} + \underbrace{(2x_1 + 3y_1)}_{b}\sqrt{3} \in K$$

$$a \equiv 0 \pmod 3$$

$$\begin{cases} 3x_1 + 6y_1 = a \\ 2x_1 + 3y_1 = b \end{cases}$$

$$\begin{cases} x_1 + 2y_1 = \frac{a}{3} \\ 2x_1 + 3y_1 = b \end{cases} \quad \begin{matrix} \rightarrow x_1 = \frac{a}{3} - 2y_1 \\ \rightarrow y_1 = \frac{b - 2x_1}{3} \end{matrix}$$

$\cancel{y_1 = b - \frac{2a}{3} - 2y_1}$

$$3y_1 = b - \frac{2a}{3} \rightarrow y_1 = \underbrace{\frac{b}{3} - \frac{2a}{9}}_{9} = \frac{3b - 2a}{9}$$

$$x_1 = \frac{a}{3} - \frac{2}{3}(b - 2x_1) = \frac{a}{3} - \frac{2}{3}b + \frac{4}{3}x_1$$

$$-\frac{1}{3}x_1 = \frac{a}{3} - \frac{2}{3}b$$

$$x_1 = -a + 2b \in \mathbb{Z}$$

$$y_1 = \frac{b}{3} - \frac{2}{3}(-a + 2b) = \frac{2a}{3} + \frac{b}{3} - \frac{4b}{3} = \frac{2a}{3} - b \in \mathbb{Z}$$

$K \supseteq I$

$(3+2\sqrt{3})(x+y\sqrt{3}) = (3x+6y) + (2x+3y)\sqrt{3}$ c $3x+6y \equiv 0 \pmod{3}$

$I \subseteq \{a+b\sqrt{3} \in R \mid a \equiv 0 \pmod{3}\}$

Нека $a, b \in \mathbb{Z}$, $a \equiv 0 \pmod{3}$

$(\overset{3x}{3}+2\sqrt{3})(x+y\sqrt{3}) = (3x+6y) + (2x+3y)\sqrt{3}$

$3x+6y = a$

$2x+3y = b$

$x = -a$

$(3+2\sqrt{3}) = \{(3+2\sqrt{3})r \mid r \in R\}$

1) $I \subseteq (3+2\sqrt{3})$

$(3+2\sqrt{3})(x+y\sqrt{3}) = (3+2\sqrt{3})r$

// $x \equiv 0 \pmod{3}$

$x+2y \equiv b \pmod{3}$

$\begin{cases} 3x+6y = 3r \\ 2x+3y = 2r \end{cases}$

$(3x+6y) + (2x+3y)\sqrt{3} = (3+2\sqrt{3})r$

$\boxed{I}$ е идеал в R

$a = 0$

$(3+2\sqrt{3})(x+y\sqrt{3}) = a + b\sqrt{3}$

$\begin{cases} 3x+6y = a \\ 2x+3y = b \end{cases} \to x+y = \frac{a}{3} \to x = \frac{a}{3} - y$

$2\left(\frac{a}{3} - y\right) + 3y = b$

$\frac{2a}{3} - 2y + 3y = b$

$y = b - \frac{2a}{3} \in \mathbb{Z}$

$x = \cancel{b-2}\, \frac{a}{3} - b + \frac{2a}{3} = a - b \in \mathbb{Z}$

$R = \{a + b\sqrt{3} \mid a, b \in \mathbb{Z}\}$

$K = \{a + b\sqrt{3} \in R \mid a \equiv 0 \pmod 3\}$.

$(3+2\sqrt{3}) = \{(3+2\sqrt{3})r,\ r \in R\}$

$$(3+2\sqrt{3})(\cancel{x}+y\sqrt{3}) = 3x + 3y\sqrt{3} + 2x\sqrt{3} + 2.3.y =$$

$$= 6x + 6y + (3y+2x)\sqrt{3} \quad \in R$$

$$6x+6y \equiv 0 \pmod 3$$

$\Rightarrow K \subset I$

$a \equiv 0 \pmod 3$

$(a+b\sqrt{3})(x+y\sqrt{3})$

$3+2\sqrt{3}$ $\qquad I = \{a+b\sqrt{3} \in R \mid a \equiv 0 \pmod 3\}$

$(3+2\sqrt{3})(x+y\sqrt{3}) = 3x + 3y\sqrt{3} + 2x\sqrt{3} + 6y =$

$= \underbrace{3x+6y} + (2x+3y)\sqrt{3}$

$3x+6y \equiv 0 \pmod 3$

$\Rightarrow I = \{a+b\sqrt{3} \in R \mid a \equiv 0 \pmod 3\}$

$3+2i$ $\qquad i^2 = -1$

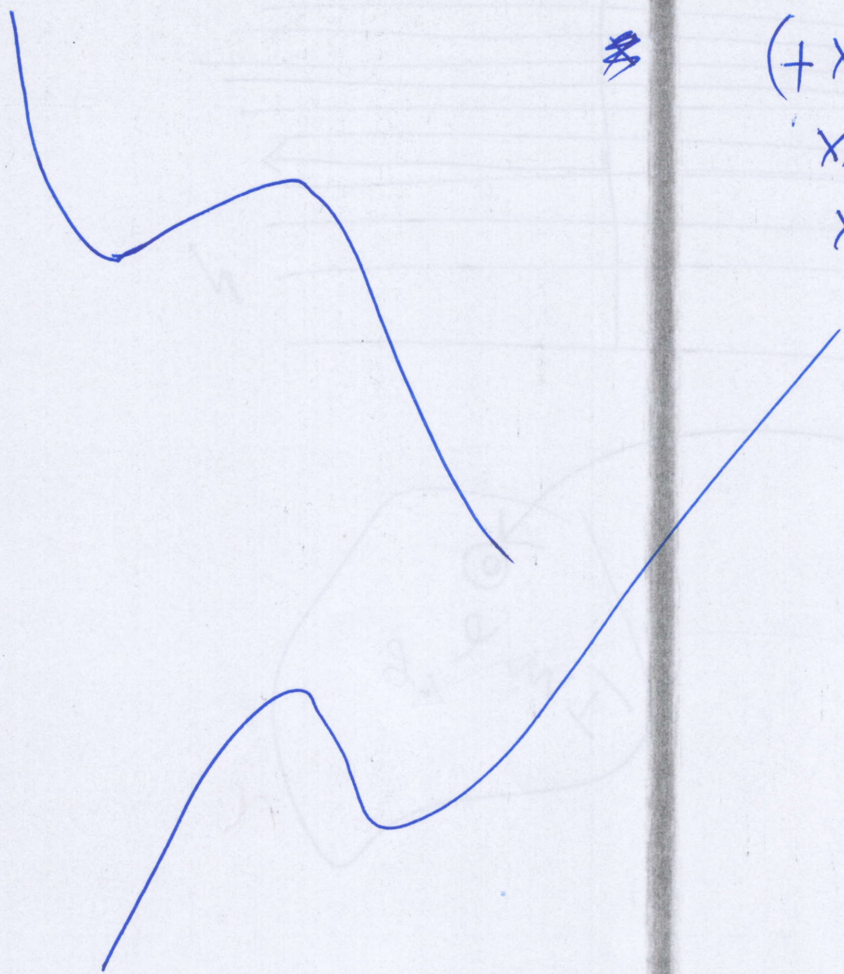
$(3+2i)(x+yi) = 3x + 3yi + 2ix + \underset{\underset{-2y}{\shortparallel}}{2i^2y} =$

$= \underbrace{(3x-2y)}(3y+2x)i$

$3x-2y \equiv 0 \pmod{13}$

$\Rightarrow I = \{a+bi \in \mathbb{Z}[i] \mid 2a-3b \equiv 0 \pmod{13}\};$

$x^3 - p.x - 5 = 0$



$(+x_1)(+x_2)(+x_3) = 5$

$x_1 . x_2 + x_2 x_3 + x_1 x_3 = +p$

$x_1 + x_2 + x_3 = 0$

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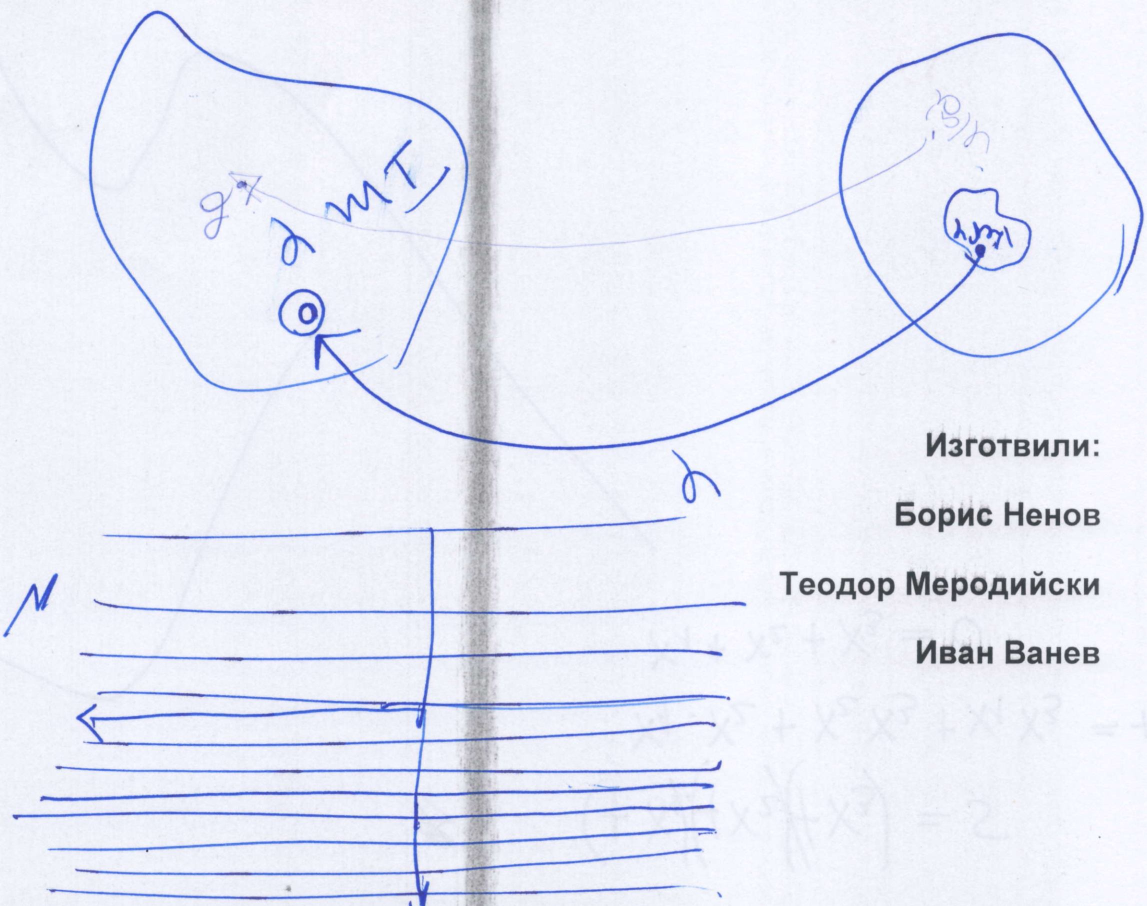

Изготвили:

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Иван Ванев

$$g^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad 3$$

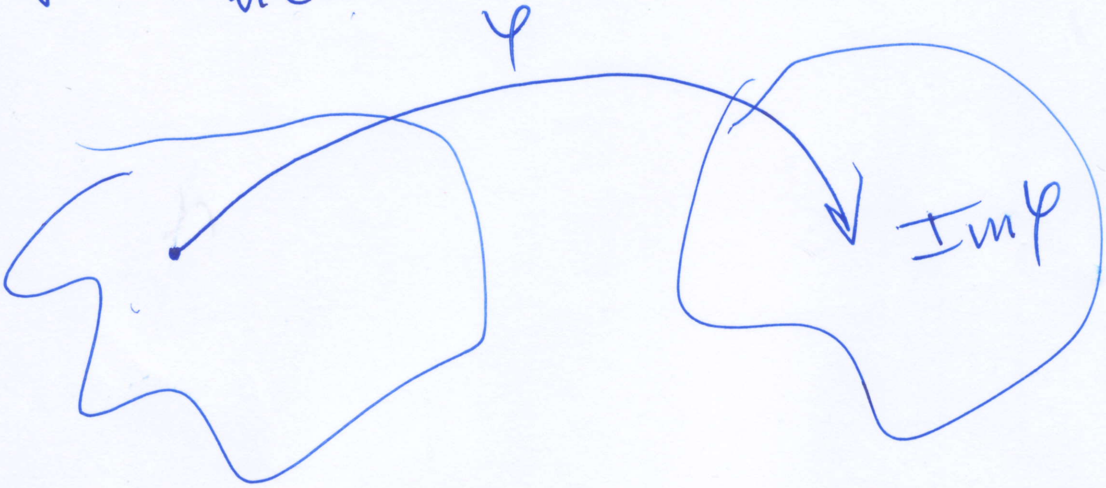
4 ред

$u, v \in H_1$

$u \circ v \in H_1$

$u^{-1} \in H_1$ $\qquad u \circ u^{-1} = 1 \in H_1$

$g^{-1} h g \in H \quad g \in G, \; h \in H$ $\qquad H \trianglelefteq G$



4.

a e нулев елемент

+	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

$b.e = (e+d)e = e.e + d.e = e + d.e$

$c.c = (e+e)c = ec + ec = c + c = e$

$d.c = (b+c).c = b.c + c.c = c + e = a$

$f.c = (c+d)c = c.c + d.c = e + a = e$

$\cancel{d.e = (c+b)e = c.e + b.e = e +}$

$d.e = (f+e).e = f.e + e.e = c + e = a$

$b.e = e + a = e$

Очевидно $a = \theta$
$b = 1$

Обратният на $b \quad e \quad f$
$c \quad e \quad e$