1. Ако $f\left(x\right)=\sqrt[5]{\left(\frac{1-x}{1+x^2}\right)^3}$, пресметнете $f'(0)$.

Решение: Нека $g\left(x\right)=1-x$, $h\left(x\right)=1+x^2$.

$f\left(x\right)=\left(\frac{g\left(x\right)}{h\left(x\right)}\right)^{\frac{3}{5}}$. Тогава $f'(0)=\frac{3}{5}\left[f\left(0\right)\right]^{-\frac{2}{5}}.\frac{g'(0)h(0)-g\left(0\right)h'(0)}{h^2\left(0\right)}$.

От $f\left(0\right)=1$, $g'\left(0\right)=-1$, $h\left(0\right)=1$, $h'\left(0\right)=0$, получаваме $f'\left(0\right)=\frac{3}{5}(1)\frac{-1.1-g\left(0\right).0}{1^2}=-\frac{3}{5}$.

Забележка: Не е необходимо да се пресметне $f'\left(x\right)$ и чак тогава $f'(0)$.

2. Пресметнете производната на $f\left(x\right)=\ln\left(x+\sqrt{1+x^2}\right)$.

$$f'\left(x\right)=\frac{1}{x+\sqrt{1+x^2}}\left(x+\sqrt{1+x^2}\right)'=\frac{1}{x+\sqrt{1+x^2}}\left(1+\frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}\right)=\frac{1}{x+\sqrt{1+x^2}}\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}=$$

$$=\frac{1}{\sqrt{x^2+1}}.$$

Добре е да запомните, че производната на $\ln\left(x+\sqrt{1+x^2}\right)$ е $\frac{1}{\sqrt{x^2+1}}$!

3. Пресметнете производната на $f\left(x\right)=\operatorname{arctg}\frac{1+x}{1-x}$.

$$f'\left(x\right)=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\left(\frac{1+x}{1-x}\right)'=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\frac{\left(1-x\right)+\left(1+x\right)}{\left(1-x\right)^2}=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\frac{2}{\left(1-x\right)^2}=$$

$$=\frac{2}{\left(1-x\right)^2+\left(1+x\right)^2}=\frac{2}{2+2x^2}=\frac{1}{1+x^2}.$$