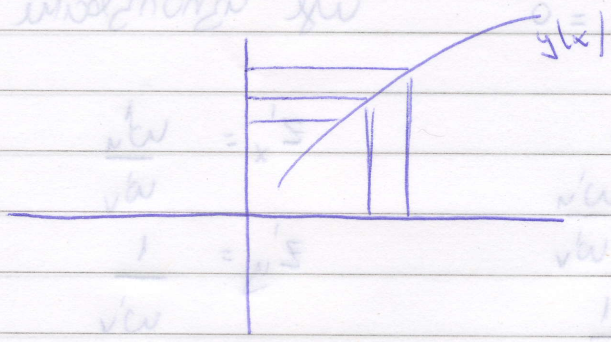
18.04.2013г. ДИС - упражнение



Задача

$$z'_x + z^2 z'_y = 0$$

$x = u$

$y = w$

$z = v$

u, v, w

$w(u,v)$

$$w(u,v) \to \begin{cases} x = u \\ y = w \end{cases} \to u(x,y) \quad v(x,y)$$

$$\begin{cases} x = u(x,y) \\ y = w(u(x,y), v(x,y)) \\ z(x,y) = v(x,y) \end{cases}$$

по x
$$\begin{cases} 1 = u'_x \\ 0 = w'_u u'_x + w'_v v'_x \\ z'_x = v'_x \end{cases} \qquad \begin{aligned} 0 &= w'_u + w'_v v'_x \\ z'_x &= v'_x = -\frac{w'_u}{w'_v} \end{aligned}$$

по y
$$\begin{cases} 0 = u'_y \\ 1 = w'_y u'_y + w'_v v'_y \\ z'_y = v'_y \end{cases} \qquad z'_y = v'_y = \frac{1}{w'_v}$$

$$-\frac{w'_u}{w'_v} + v^2 \cdot \frac{1}{w'_v} = 0$$

$w'_u = v^2$ $\qquad w = u \cdot v^2 + f(v)$ $\qquad y = x \cdot z^2 + f(z)$

$z(x,y)$

задача

$z''_{xx} z'_{yy} - z''^2_{xy} = 0$ ще използваме неща от предната задача

$u'_x = 1 \quad u'_y = 0$

$v'_x = z'_x = -\frac{w'_u}{w'_v}$

$v'_y = z'_y = \frac{1}{w'_v}$

$z'_x = \frac{w'_u}{w'_v}$

$z'_y = \frac{1}{w'_v}$

$z''_{xx} = \frac{(w''_{uu} u'_x + w''_{vv} v'_x) w'_v - w'_u (w''_{uv} u'_x + w''_{vv} v'_x)}{w'^2_v} =$

$= (w''_{uu} 1 + w''_{vv} \cdot (-\frac{w'_u}{w'_v})) w'_v - w'_u w'_{uv} = w'_u w''_{vv} v'_x - \frac{w'_u}{w'_v}$

$= w''_{uu} w'_v - w''_{vv} w'_u - w'_u w'_{uv} + \frac{w'^2_u w''_{vv}}{w'_v} =$

$= w'_{uu} w'_v$ още си нямам.

$z''_{xy} = w''_{uu} \cdot u'_y + w''_{uv} \cdot v'_y w' \qquad = \frac{-w''_{uv} w'_v + w''_{..}}{(w'_v)^3}$

$z'_{yy} = -\frac{w''_{uv} u'_y + w'_{vv} v'_y}{w'^2_v} \qquad = -\frac{w''_{vv}}{w'^3_v}$

Задача

$$z'_x z''_{yy} - z'_y z''_{xx} = 0$$

$x = w$

$y = u$

$z = v$

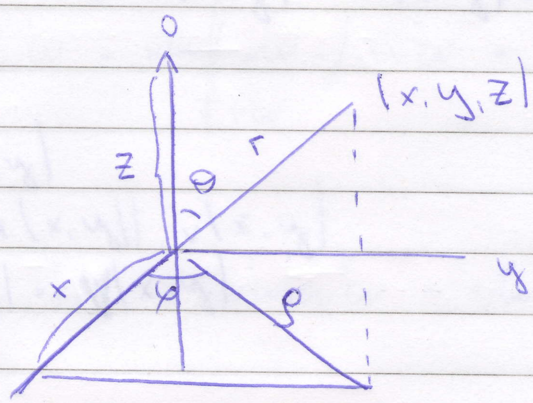
$w(u,v)$

$u(x,y)$

$v(x,y)$

$$yz'_x - xz'_y = 0$$

$$\begin{cases} x = r\cos\varphi\sin\theta \\ y = r\sin\varphi\sin\theta \\ z = r\cos\theta \\ 0 \le \varphi \le 2\pi \\ 0 \le \theta \le \pi \end{cases}$$



$z(\varphi, \theta)$