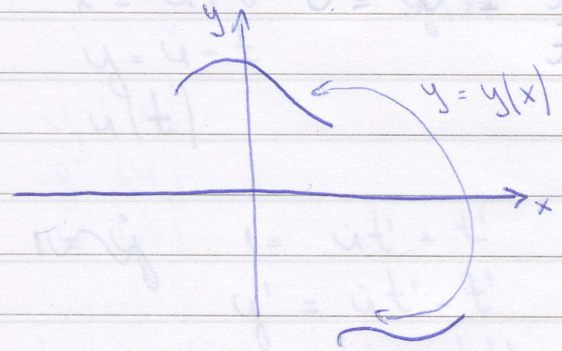
17.04.2013г. ДИС - упражнение

Смяна на променливите

$F(x, y, u, t) = 0$
$G(x, y, u, t) = 0$ $\Rightarrow x = f(u, t)$
$y = g(u, t)$

$\underline{y(f(u,t)) = g(u,t)}$



$y = y(x) \quad u = u(t)$

$x = f(u(t), t)$
$t(x)$

$u(x, y)$
$v(x, y)$
$y(x)$
$t(x)$

Задача

$(1+x^2)^2 y'' + 2x(1+x^2)y' + y = 0$

$x = tg\, t$

$\left| \begin{array}{l} x - tg\, t = 0 \\ y = u \end{array} \right| \quad \begin{array}{l} x = tg\, t \\ y = u \end{array}$

$u(t) \quad t(x)$ заместв. и получаване тъждествата

$x - tg(t(x)) \equiv 0$

$y(x) \equiv u(t(x))$

$1 - \frac{1}{\cos^2 t} t' = 0 \Rightarrow t' = \cos^2 t$

$\boxed{\dot{u}(t)} = u'(t)$ знак за улеснение

$y' = \dot{u} t' = \dot{u} \cos^2 t$

$y'' = (\ddot{u} \cos^2 t - \dot{u} 2\cos t \sin t) t' = \ddot{u} \cos^4 t - 2\dot{u} \cos^3 t \sin t$

$\dot{u}(t)$

заместване с u и t в $(1+x^2)y'' + 2x(1+x^2)y' + y = 0$

~~$1+tg^2 t$~~

$$\frac{1}{\cos^4 t}\left(\ddot{u}\cos^4 t - 2\dot{u}\cos^3 t \sin t\right) + \frac{2\sin t}{\cos t \cdot \cos^2 t} \cdot \dot{u}\cos^2 t + u = 0$$

$$\ddot{u} \cancel{\cos t} - 2\dot{u}\frac{\cancel{\cos t}\sin t}{\cos t} + \frac{2\dot{u}\sin t}{\cos t} + u = 0$$

~~$\ddot{u}\cos^2 t - \dot{u}\sin 2t + \dot{u}\frac{2\sin t}{\cos t} = 0$~~

$$\ddot{u} - 2\dot{u}\frac{\sin t}{\cos t} + \frac{2\dot{u}\sin t}{\cos t} + u = 0$$

$$\ddot{u} + u = 0$$

задача

$$y'' + (e^x - x)y'^3 = 0$$

смяна на променливите, като и вземе независимата променлива за зависимата и обратно

$x = u$

$y = t$

$y(x) \quad u(t) \qquad x = u(t) \quad t(x)$

$x = u(t(x)) \xrightarrow{\text{по } x} 1 = \dot{u}t'$

$y = t(x) \qquad y' = t' = \frac{1}{\dot{u}(t)}$

$$y'' = -\frac{1}{\dot{u}^2} \cdot t' = -\frac{1}{\dot{u}^2} \cdot \frac{1}{\dot{u}} = -\frac{1}{\dot{u}^3} = -\frac{\ddot{u}}{\dot{u}^2}t' = -\frac{\ddot{u}}{\dot{u}^3}$$

$$-\frac{\ddot u}{\dot u^3} + (e^u - u)\cdot\frac{1}{\dot u^3} = 0$$

$$-\ddot u + e^u - u = 0$$

~~интеграл~~

<u>задача</u>

$$y'\cdot y''' - 3y''^2 = 0$$
$$x = u$$
$$y = t$$

... от миналата задача

$$y''' = -\frac{(\dddot u \dot u^3 - \ddot u . 3\dot u^2 \ddot u)}{\dot u^6}.t' = -\frac{\dddot u \dot u^3 - \ddot u 3\dot u^2 \ddot u}{\dot u^6}.\frac{1}{\dot u} =$$

$$=$$

$$\frac{1}{\dot u}.\frac{3\ddot u^2 - \dot u.\dddot u}{\dot u^5} - 3\frac{\ddot u^2}{\dot u^6} = 0$$

$$\dot u.\dddot u = 0 \Rightarrow \text{едно от двете} = 0 \quad (1)\ \dot u = 0$$

$$\Rightarrow u - \text{const}$$

$$(2)\ \dddot u = 0$$

$$\Rightarrow \ddot u - \text{const}$$

$$u = At^2 + Bt + C$$

$$\Rightarrow u = At^2 + Bt + C$$

$$x = Ay^2 + By + C$$

$y(x)$

Задача

$$y'' + (x+y)(1+y')^3 = 0$$

$x = u + t \quad y(x) \quad t(x)$

$y = u - t$

$u(t)$

$1 = \dot{u}t' + t' \qquad 1 = t'(\dot{u}+1) \qquad t' = \frac{1}{\dot{u}+1}$

$y' = \dot{u}t' - t'$

$y' = (\dot{u}-1)t' =$

$= \frac{(\dot{u}-1)}{\dot{u}+1}$

$$y'' = \frac{\ddot{u}(\dot{u}+1) - (\dot{u}-1)\ddot{u}}{(\dot{u}+1)^2} \cdot t' = \frac{1}{\dot{u}+1}$$

$$y'' = \frac{\ddot{u}(\dot{u}+1-\dot{u}+1)}{(\dot{u}+1)^2(\dot{u}+1)} = \frac{2\ddot{u}}{(\dot{u}+1)^3}$$

$$\frac{2\ddot{u}}{(\dot{u}+1)^3} + 2u\left(1 + \frac{\dot{u}-1}{\dot{u}+1}\right)^3 = 0$$

$$\frac{2\ddot{u}}{(\dot{u}+1)^3} + 2u\left(\frac{\dot{u}+1+\dot{u}-1}{\dot{u}+1}\right)^3 = 0$$

$$\frac{2\ddot{u}}{(\dot{u}+1)^3} + \frac{2u \cdot (2\dot{u})^3}{(\dot{u}+1)^3} = 0$$

$$2\ddot{u} + 2u \cdot 8\dot{u}^3 = 0$$

$$2\ddot{u} + 16u\dot{u}^3 = 0$$

$$\ddot{u} + 8u\dot{u}^3 = 0$$

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Задача

$$y' = \frac{x+y}{x-y}$$

$$x = \varrho\cos\varphi \qquad \varrho(\varphi)$$
$$y = \varrho\sin\varphi$$

$y(x)$
$\varrho(\varphi)$
$\varphi(\lambda)$

$$1 = (\dot\varrho\cos\varphi - \varphi\sin\varphi)\varphi'$$
$$y' = (\dot\varrho\sin\varphi + \varphi\cos\varphi)\varphi'$$

$$y' = \frac{\dot\varrho\sin\varphi + \varrho\cos\varphi}{\dot\varrho\cos\varphi - \varrho\sin\varphi} = -\frac{\cos\varphi + \sin\varphi}{\cos\varphi - \sin\varphi} =$$

$$\cancel{\dot\varrho\sin\varphi\cos\varphi - \varrho\sin^2\varphi + \varrho\cos^2\varphi\sin\varphi - \varrho\cos\varphi\sin\varphi - \varrho\cos\varphi}$$

$$-\frac{\cos\varphi + \sin\varphi}{\cos\varphi - \sin\varphi} = \frac{\varrho\cos\varphi + \varrho\sin\varphi}{\varrho\cos\varphi - \varrho\sin\varphi}$$

$$-(\cos\varphi + \sin\varphi)(\varrho\cos\varphi - \varrho\sin\varphi) = (\varrho\cos\varphi + \varrho\sin\varphi)(\cos\varphi - \sin\varphi)$$

в пространството

$F(x, y, z, u, v, w) = 0$
$G(x, y, z, u, v, w) = 0$ $\Longleftrightarrow$ $x = f(u, v, w)$
$H(x, y, z, u, v, w) = 0$

$y = g(u, v, w)$
$z = h(u, v, w)$

$x = f(u, v, w(u, v))$
$y = g(u, v, w(u, v))$

$u(x, y)$
$v(x, y)$

$F(x, y, z(x, y), u(x, y), v(x, y), w(u(x, y), v(x, y))) \equiv 0$

$x = f(u, v)$
$y = g(u, v)$
$z(x, y)$ $z = w$

задача

$y z'_x - x z'_y = 0$ (1)
$u = x$
$v = x^2 + y^2$

$z(x, y)$
$w = z$
$z(x, y) = w(u(x, y), v(x, y))$

$z'_x = w'_u \cdot u'_x + w'_v \cdot v'_x = w'_u \cdot 1 + 2w'_v x$ (2)

$\left.\begin{array}{l} u'_x = 1 \\ v'_x = 2x \end{array}\right\} \nearrow$

заместване (2) + (3) $\to$ (1)

$z'_y = w'_v \cdot 2y$ (3)
$u'_y = 0$
$v'_y = 2y$

$y(w_u + 2w'_v x) - x(w'_v \cdot 2y) = 0$
$y \cdot w'_u = 0$
$w'_u = 0$