11.04.2013 г.

Неявни ф-ии

$F_1(x, y, z, u, v) = 0$

$F_2(x, y, z, u, v) = 0$

$F_3(x, y, z, u, v) = 0$

x, y – парам., с-мата се решава ~~x,y~~ z, u, v. $(z(x,y), u(x,y), v(x,y))$

$$\begin{vmatrix} F'_z, & F'_u, & F'_v \\ G'_z, & G'_u, & G'_v \\ H'_z & H'_u & H'_v \end{vmatrix} \neq 0$$

$\boxed{Th}$ Ако в някоя т. са изп. рав-та и усл. за det, то с-мата може да се реши спрямо избраните параметри

- $x = t + \frac{1}{t}$ → $y(x), z(x), t(x)$ – решаване у-ията отн. x

$y = t^2 + \frac{1}{t^2}$

$z = t^3 + \frac{1}{t^3}$

$t + \frac{1}{t} - x = 0$

$y - t^2 - \frac{1}{t^2} = 0$

$z - t^3 - \frac{1}{t^3} = 0$

$$\rightarrow \begin{vmatrix} 1 - \frac{1}{t^2} & 2t - \frac{2}{t^3} & 3t^2 - \frac{3}{t^4} \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 - \frac{1}{t^2} \neq 0$$

$\Rightarrow t \neq \pm 1$ (за тези ст-ти на t с-мата може да се реши)

$x = t + \frac{1}{t}$

$y = \left(t + \frac{1}{t}\right)^2 - 2 = x^2 - 2$

$z = \left(t + \frac{1}{t}\right)^3 - 3\left(t + \frac{1}{t}\right) = x^3 - 3x$

$t^2 - tx + 1 = 0$

$\boxed{*}\ y''(x) = 2$

$t = \dfrac{x \pm \sqrt{x^2-4}}{2}$

Зад. $y''(x) = ?$, ако $x = t(x) + \dfrac{1}{t(x)}$

$y(x) = t^2(x) + \dfrac{1}{t^2(x)}$

$z(x) = t^3(x) + \dfrac{1}{t^3(x)}$

$y'(x) = \left(2t - \dfrac{2}{t^3}\right) \cdot t'(x)$

$x': 1 = t' - \dfrac{1}{t^2} \cdot t' \Rightarrow t' = \dfrac{t^2}{t^2-1}$

$\Rightarrow y'(x) = \dfrac{2(t^4-1)}{t^3} \cdot \dfrac{t^2}{t^2-1} = 2\,\dfrac{t^2+1}{t}$

$y'' = \left(\text{———}\right)'_t \cdot t'$

Зад.
$x = u + v$
$y = u^2 + v^2$
$z = u^3 + v^3$

(x, y) . Решаване отн. u, v, z

$$\begin{vmatrix} 1 & 1 & 0 \\ 2u & 2v & 0 \\ 3u & 3v & -1 \end{vmatrix} \left(\begin{array}{l} \text{I ред} - x \text{ спрямо } u, v \\ \text{II р.} - y \text{ спр. } u, v \\ \text{III р.} - z \text{ спр. } u, v \end{array}\right) = -2v + 2u = 2(u - v)$$

$\Rightarrow$ за $u \neq v$ с-мата ще има ф-ия спрямо парам.

$u(x, y)$
$v(x, y)$
$z(x, y) = u^3(x, y) + v^3(x, y)$

$z'_x = 3u^2(x, y) \cdot u'_x + 3v^2(x, y) \, v'_x$

$x = u(x,y) + v(x,y)$

$y = u^2(x,y) + v^2(x,y)$

диф. по x и двете:

$u'_x = 1 - v'_x$

$1 = u'_x + v'_x$

$0 = 2u(x,y) . u'_x + 2v(x,y) . v'_x$

$$u'_x = \frac{\begin{vmatrix} 1 & 1 \\ 0 & 2v \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2u & 2v \end{vmatrix}} = \frac{v}{v-u}$$

$$v'_x = \frac{1-u}{v-u}$$

$$\Rightarrow z'_x = \frac{3u^2v - 3v^2u}{v-u} = -3uv = 3u^2 . u'_x + 3v^2 . v'_x =$$

$z''_{xy} = -3u'_y . v - 3v'_y . u$

$0 = u'_y + v'_y$

$1 = 2u . u'_y + 2v . v'_y$

$= u'_y(2u - 2v)$

от $x = u(x,y) + v(x,y)$

$y = u^2(x,y) + v^2(x,y)$

диф. по y

$$u'_y = -v'_y = \frac{1}{2(u-v)}$$

зад. 1 $\begin{cases} uv - x\cdot y = 5 \\ x\cdot u + y\cdot v = 0 \end{cases}$

$u(x,y)$ и $v(x,y)$ – да се опр.?

$u''_{xx}(1,-1) = ?$

$v''_{xy}(1,-1) = ?$

зам. (x,y) с $(1,-1)$ в с-мата: $\begin{cases} uv + 1 = 5 \\ u - v = 0 \end{cases}$

$u = v$

$v^2 = 4$, $v = \pm 2 = u$

1) т. $(1,-1,2,2)$ и 2) $(1,-1,-2,-2)$

$\begin{vmatrix} v & u \\ x & y \end{vmatrix} = v\cdot y - x\cdot u \neq 0 \quad = +4$ в 1 т.

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad -4$ в 2-рата т.

$\begin{cases} u(x,y)\cdot v(x,y) - xy = 5 \\ x\cdot u(x,y) + y\cdot v(x,y) = 0 \end{cases}$

дифер. по x:

$\begin{cases} u'_x \cdot v(x,y) + u(x,y)\cdot v'_x - y = 0 \\ u(x,y) + x\cdot u'_x + y\cdot v'_x = 0 \end{cases}$

$\begin{cases} u'_x = -y\cdot v'_x - u(x,y) \\ u'_x = \dfrac{y - u(x,y)\cdot v'_x}{v(x,y)} \end{cases}$

$y - u(x,y)\cdot v'_x = v(x,y)(-y)v'_x - v(x,y)\cdot u(x,y)$

$$u'_x \cdot V + u \cdot v'_x = y$$

$$u'_x \cdot x + y \cdot v'_x = -u$$

$$u'_x = \frac{y - u \cdot v'_x}{v} = \frac{-u - y \cdot v'_x}{x}$$

$$\boxed{v'_x = -\frac{yx + uv}{yv - ux}} \leftarrow, \quad u'_x = \frac{y}{v} + \frac{uyx + u^2 v}{v(yv - ux)} = \frac{y^2 v + u^2 v}{y \cdot v^2 - u \cdot vx} =$$

$$= \frac{y^2 + u^2}{y \cdot v - u \cdot x}$$

$$u'_x(1,-1) = \frac{1 + 4}{-4} = -\frac{5}{4}$$

$$v'_x(1,-1) = \frac{3}{4}$$

$$u''_{xx} = \frac{2u \cdot u'_x (yv - ux) - (y^2 + u^2)(yv'_x - u'_x \cdot x - u)}{(y \cdot v - u \cdot x)^2}$$

-1.2 -2.1

$$u''_{xx}(1,-1) = \frac{4 \cdot -\frac{5}{4}(-4) - (1+4)\left(-\frac{3}{4} + \frac{5}{4} - 2\right)}{16} = \frac{+20 - 5 \cdot -\frac{3}{2}}{16} = \frac{\frac{+40+15}{2}}{16} =$$

$$= \frac{+55}{32}$$

~~Зад.~~ $\boxed{\text{Зад.}}$ $h(s,t)$

$F = h(xu, yz) - x \cdot z$

$$\begin{cases} F(x,y,z,u) = 0 \\ F'_u(x,y,z,u) = 0 \end{cases}$$

при какви у-ия от с-мата може да се опр. неявна ф-ия на $z(x,y)$ и $u(x,y)$

$\boxed{\text{Зад.}}$ ДА, че $(x+z) z''_{xx} + {z'_y}^2 = z''_{xy}$ $\rightarrow$ $\boxed{10.14 \quad 2)}$