10.04.2013 г. ~ ЛИС ~ упр.

! Първо контролно
задачи - р-ия

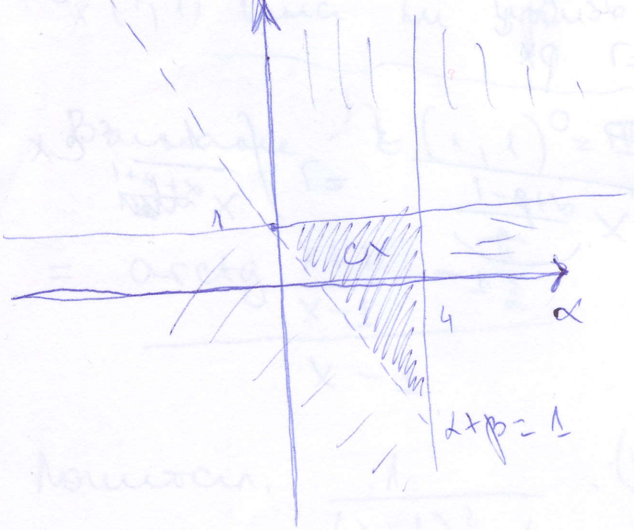
(4.) а) $\displaystyle\int_0^{\infty} \frac{\operatorname{arctg}^2 x}{x^{\alpha}|1-x|^{\beta}} \ln|1-x|\, dx$

Особености: 1) т. 0

$$\frac{\operatorname{arctg}^2 x \cdot \ln|1-x| \cdot x}{x^2 \cdot x^{\alpha-2} |1-x|^{\beta} \cdot x} \sim \frac{1}{x^{\alpha-3}}$$

$\alpha - 3 < 1$ – сх.

$\alpha < 4$



2) т. 1

$$\frac{\operatorname{arctg}^2 x \cdot \ln|1-x|}{x^2 \cdot x^{\alpha-3} |1-x|^{\beta}}$$

$$\frac{\ln|1-x|}{|1-x|^{\beta}}$$

ако $\beta = 1$ – разх.

$\beta > 1$, $\dfrac{\ln|1-x|}{|1-x|^{\beta}} > \dfrac{\ln|1-x|}{|1-x|}$

разх. $\Leftarrow$ р^x

$x < 1$

$p \quad \frac{p+1}{2} \quad 1$

$p < 1 \quad \frac{\ln|1-x|\,|1-x|^{\frac{1-p}{2}}}{|1-x|^{\frac{p+1}{2}}} \xrightarrow[x\to 1]{} 0, \quad \frac{p+1}{2} < \frac{1}{2}$

сх.

б) т. ∞:

$$\frac{\operatorname{arctg}^2 x \cdot \ln|1-x|}{x^2 \cdot x^{\alpha-3}|1-x|^{\beta}} \sim \frac{\ln|1-x|}{x^{\alpha+\beta}\left(1-\frac{1}{x}\right)^{\beta}} \sim \frac{\ln x}{x^{\alpha+\beta}}$$

$\boxed{* \ \frac{\ln x}{\ln(x-1)} \to 1}$

$\to \alpha+\beta = 1, \quad \frac{\ln x}{x} \quad px \Rightarrow px$

$\to \alpha+\beta < 1, \quad x^{\alpha+\beta} < x$

$$\frac{\ln x \cdot 1}{1\,x} < \frac{1 \cdot \ln x}{x^{\alpha+\beta}}$$

$px \Rightarrow px$

$\to \alpha+\beta > 1, \quad \underbrace{\frac{\ln x}{x^{\frac{\alpha+\beta+1}{2}}}}_{>0} \to 0 \cdot x^{\overbrace{\frac{\alpha+\beta-1}{2}}^{>1}} \Rightarrow \int \frac{1}{x^{\frac{\alpha+\beta+1}{2}}}$ сх

$1 \quad \alpha+\beta$

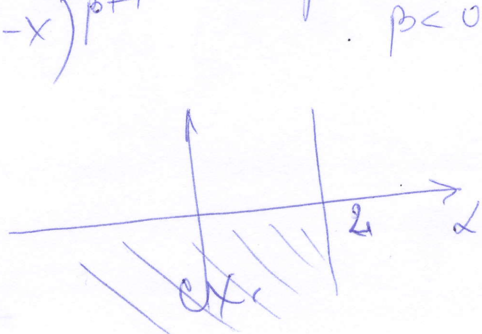
б). $$\int_0^{\frac{\pi}{2}} \frac{\operatorname{tg} x}{x^{\alpha}\left(\frac{\pi}{2}-x\right)^{\beta}}$$

Особености: т. 0 и $\frac{\pi}{2}$

1) т. 0: $\frac{\operatorname{tg} x}{x \cdot x^{\alpha-1}\underbrace{\left(\frac{\pi}{2}-x\right)^{\beta}}_{\to\frac{\pi}{2}}} \sim \frac{1}{x^{\alpha-1}}$ $\quad \alpha < 2$ сх.

2) т. $\frac{\pi}{2}$: $\frac{1 \cdot \sin x \cdot \left(\frac{\pi}{2}-x\right)}{x^{\alpha}\cos x\left(\frac{\pi}{2}-x\right)^{\beta+1}} \sim \frac{1}{\left(\frac{\pi}{2}-x\right)^{\beta+1}}$ $\quad \beta+1 < 1$ сх. $\beta < 0$

$\boxed{* \ \frac{\cos x}{\frac{\pi}{2}-x} \xrightarrow[x\to\frac{\pi}{2}]{} \sin\frac{\pi}{2} = 1}$



(5) $z(x,y) = \operatorname{arctg}\dfrac{x+y}{x-y}$; $z''_{xy}(2,1) = ?$

$$z'_x = \frac{1}{1+\left(\frac{x+y}{x-y}\right)^2} \cdot \frac{x-y-x-y}{(x-y)^2} = -\frac{\cancel{2}y}{\cancel{2}(x^2+y^2)}$$

$$z'_{xy} = -\frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\boxed{z'_{xy}(2,1) = -\frac{3}{25}}$$

$z'_x(1,1)$ има ли произв.? $= \dfrac{z(x,1)-z(1,1)}{x-1} \xrightarrow[x\to 1]{} z'_x(1,1)$

Взимаме $z(1,1) = \dfrac{\pi}{2}$:

$$= \frac{\operatorname{arctg}\frac{x+1}{x-1} - \frac{\pi}{2}}{x-1} \xrightarrow[x\to 1]{} ?$$

Лопитал. $\dfrac{1}{\left(\frac{x+1}{x-1}\right)^2+1} \cdot \dfrac{(x-1)-(x+1)}{(x-1)^2} = \dfrac{-2}{2(x^2+1)} \xrightarrow[x\to 1]{} -\dfrac{1}{2}$

$\Rightarrow$ Има ч. произв. в т. $(1,1)$

Дали е непрек. в т. $(1,1)$

$x_n \to$
$y_n \to 1$, $f(x_n, y_n) \to \dfrac{\pi}{2}$

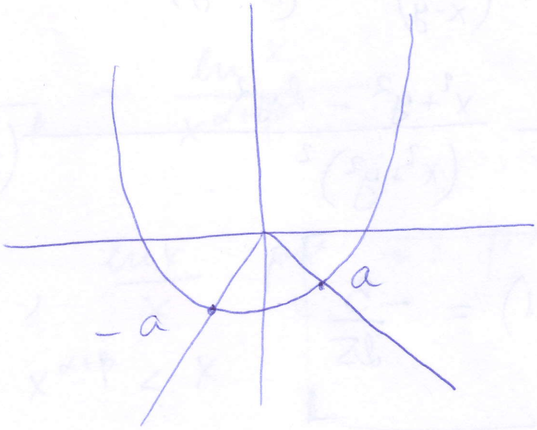
$x_n \to 1$
$y_n \to 1$, но $x_n > y_n \to \dfrac{\pi}{2}$ ✓

$x_n < y_n \to -\dfrac{\pi}{2}$ X

$\Rightarrow$ ф-ията е прек. в т. O

1. $x^2 - 8 \le y \le -|x|$

$|x| = \frac{-1 \pm \sqrt{33}}{2}$, $x = \pm \frac{-1 + \sqrt{33}}{2} = \pm a$



a) $S = \int_{-a}^{a} (-|x| - x^2 + 8)\,dx =$

$= 2\int_0^a (-x - x^2 + 8)\,dx =$

$= 2\left(-\frac{a^2}{2} - \frac{a^3}{3} + 8a\right)\Big|_0^a$

$a^2 = 8 - a$

$\Rightarrow -\frac{a^2}{2} - \frac{a^3}{3} + 8a = -\frac{8-a}{2} - \frac{a(8-a)}{3} + 8a =$

$= -4 + \frac{a}{2} + 8a - \frac{8a}{3} + \frac{8-a}{3} = \dots = \frac{33a-8}{6}$

б) $P = 2a\sqrt{2} + 2\int_0^a \sqrt{1+4x^2}\,dx = 2a\sqrt{2} + \int_0^{\frac{a}{2}} \sqrt{1+x^2}\,dx$

1) 2. $\int_0^{2\pi} \frac{dx}{3+\cos^2 x} = 2\int_0^{\pi} \dots = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \dots = 4\int_0^{\frac{\pi}{2}} \dots \Rightarrow$

2) (*) $\int_a^{a+\omega} f(x)\,dx = \int_0^{\omega} f(x)\,dx$

$t = \operatorname{tg} x$

$x = \operatorname{arctg} t$

$$= \int_0^\infty \frac{dt}{\left(3+\frac{1}{1+t^2}\right)(1+t^2)} = \int_0^\infty \frac{dt}{4+3t^2} = \frac{2}{4\sqrt{3}} \int_0^\infty \frac{d\,\frac{t}{2}\sqrt{3}}{1+\left(\frac{\sqrt{3}t}{2}\right)^2} =$$

$$= \frac{1}{2\sqrt{3}} \operatorname{arctg} t\frac{\sqrt{3}}{2} \Big|_0^\infty = \frac{\pi}{4\sqrt{3}}$$

3. $\int_0^\infty \frac{dx}{\sqrt{x}(x+1)}$

т. 0 : $\frac{1}{\sqrt{x}(x+1)} \sim \frac{1}{x^{\frac{1}{2}}}$ сх.

т. ∞ : $\frac{1}{x^{\frac{3}{2}}\left(1+\frac{1}{x}\right)} \sim \frac{1}{x^{\frac{3}{2}}}$ сх.

I н.) $\int_0^\infty \frac{dx}{\sqrt{x}(x+1)} = \int_0^\infty \frac{2t\,dt}{t(t^2+1)} = 2\operatorname{arctg} t \Big|_0^\infty = \pi$

$\sqrt{x} = t$

$x = t^2$

II н.) $\int_\varepsilon^p \frac{dx}{\sqrt{x}(x+1)} = \int_{\sqrt{\varepsilon}}^{\sqrt{p}} \frac{2\cancel{t}\,dt}{\cancel{t}(t^2+1)} = 2(\operatorname{arctg}\sqrt{p} - \operatorname{arctg}\sqrt{\varepsilon})$

$$\lim_{p\to\infty}\left(\lim_{\varepsilon\to 0}\left(\_\_ + \_\_\right)\right) = \pi$$