

$$\begin{aligned}
 \textcircled{1} \int \frac{x^4+1}{x^2+1} dx &= \int \frac{(x^2-1)(x^2+1)+1}{(x^2+1)} dx = \\
 &= \int \frac{\cancel{(x^2+1)}(x^2-1)}{x^2+1} dx + \int \frac{dx}{x^2+1} = \int x^2 dx - \int dx + \arctan x \\
 &= \frac{x^3}{3} - x + \arctan x + c
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cdot \cos^2 x} dx = \\
 &= \int \frac{1}{\sin^2 x} - \int \frac{dx}{\cos^2 x} = -\cot x - \tan x + c
 \end{aligned}$$

$$\textcircled{3} \int x e^{-x^2} dx = \frac{-1}{2} \int e^{-x^2} dx^2 = \frac{1}{2} e^{-x^2} + C$$

$$\begin{aligned}
 \textcircled{4} \int x^3 \ln x dx &= \cancel{x^3 \ln x} - \int \cancel{\ln x} x dx^2 \\
 &= \frac{1}{4} \int \ln x dx^4 = \frac{1}{4} x^4 \ln x - \frac{1}{4} \int x^4 d \ln x = \\
 &= \frac{1}{4} x^4 \ln x - \frac{1}{4} \int x^3 dx = \frac{1}{4} x^4 \ln x - \frac{x^4}{4} + C
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \int \arctan x dx &= x \arctan x - \int x d \arctan x = \\
 &= \arctan x \cdot x - \int \frac{x}{1+x^2} dx = \arctan x \cdot x - \frac{1}{2} \int \frac{1}{1+x^2} dx^{2+1} \\
 &= \arctan x \cdot x - \frac{1}{2} \ln(1+x^2)
 \end{aligned}$$

$$\textcircled{6} \int x \cdot \operatorname{arctg}^2 x \, dx = \frac{1}{2} \int \operatorname{arctg}^2 x \, dx^2 =$$

$$= \frac{1}{2} x^2 \operatorname{arctg}^2 x - \frac{1}{2} \int x^2 d \operatorname{arctg}^2 x = \frac{1}{2} x^2 \operatorname{arctg}^2 x -$$

$$- \frac{1}{2} \int \frac{x^2 d \operatorname{arctg} x}{1+x^2} dx = \frac{1}{2} \operatorname{arctg}^2 x \cdot x^2 -$$

$$- \frac{1}{2} \int \frac{\operatorname{arctg} x}{1+x^2} dx^3 = \frac{1}{2} \operatorname{arctg}^2 x \cdot x^2 -$$

$$= \frac{1}{2} \operatorname{arctg}^2 x \cdot x^2 - \frac{1}{2} \int \frac{x \operatorname{arctg} x}{1+x^2} dx^2 =$$

$$= \int - \frac{1}{2} \int x \operatorname{arctg} x d \ln(1+x^2) = \int - \frac{1}{2}$$

$$= \int - \frac{1}{2} x \operatorname{arctg} x \cdot \ln(1+x^2) + \frac{1}{2} \int \ln(1+x^2) d x \operatorname{arctg} x$$

$$= \int_1 + \frac{1}{2} \int \ln(1+x^2) \operatorname{arctg} x \, dx + \frac{1}{2} \int \frac{\ln(1+x^2) \cdot x}{1+x^2} dx =$$

$$= \int_1 + \frac{1}{2} \int \ln(1+x^2)$$

$$\textcircled{6} \int x \operatorname{arctg}^2 x \, dx = \frac{1}{2} \operatorname{arctg}^2 x \, d(x^2+1) =$$

~~$$= \frac{\operatorname{arctg}^2 x \cdot x^2}{2} - \frac{1}{2} \int \frac{x^2 \cdot \operatorname{arctg} x \, dx}{1+x^2} =$$~~

~~$$= \frac{\operatorname{arctg}^2 x \cdot x^2}{2} - \frac{1}{2} \int \frac{x \operatorname{arctg} x \, d(x^2+1)}{1+x^2} =$$~~

~~$$= \frac{\operatorname{arctg}^2 x \cdot x^2}{2}$$~~

$$= \underbrace{\frac{1}{2} (x^2+1) \operatorname{arctg}^2 x}_J - \frac{1}{2} \int \frac{(x^2+1) \cdot x \operatorname{arctg} x \, dx}{x^2+1} =$$

$$= J - \int \operatorname{arctg} x \, dx$$

Ръководство по математически анализ
 първа част — Киколова, Келевски
 (Упражнения)

$$dg(x) = g'(x) \, dx$$

$$\textcircled{7} \int e^{g(x)} dg(x) = e^{g(x)} + C$$

$$\textcircled{8} \text{ a) } \int e^{ax} \cos bx \, dx = \frac{1}{a} \int \cos bx \, de^{ax} =$$

$$\text{b) } \int e^{ax} \sin bx \, dx = \frac{1}{a} \int \sin bx \, de^{ax} =$$

$$\text{a) } = \frac{e^{ax} \cos bx}{a} - \frac{1}{a} \int e^{ax} d \cos bx =$$

$$\text{b) } = \frac{e^{ax} \sin bx}{a} - \frac{1}{a} \int e^{ax} d \sin bx =$$

$$\text{a) } = \frac{e^{ax} \cos bx}{a} + \frac{b}{a} \int e^{ax} \sin bx \, dx$$

$$\text{b) } = \frac{e^{ax} \sin bx}{a} - \frac{b}{a} \int e^{ax} \cos bx \, dx$$

$p = ?$
 $q = ?$

$$\int f(x) \, dx \Rightarrow \int f(\varphi(t)) \, d\varphi(t) = \int f(\varphi(t)) \cdot \varphi'(t) \, dt$$

$x = \varphi(t)$

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$$\textcircled{9} \int e^{\sqrt{x}} dx \Rightarrow \int e^t dt^2 = \int e^t \cdot 2t dt =$$

$$x = t^2 \Rightarrow t = \sqrt{x}$$

$$= 2 \int t de^t = 2te^t - 2 \int e^t dt = 2te^t - 2e^t + c =$$

$$= 2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$$

$$(e^t)' = e^t \cdot 1$$

$$\textcircled{10} \int \frac{dx}{\sqrt{x}(1+\sqrt[3]{x})} = \int \frac{dt^6}{t^3(1+t^2)} = \int \frac{6t^{2+1-1} dt}{t^3(1+t^2)} =$$

$$x = t^6 \quad t = \sqrt[6]{x}$$

$$= 6 \int \frac{t^2+1}{1+t^2} dt - 6 \int \frac{1}{1+t^2} dt = 6t - 6 \arctan t + c$$

$$= 6\sqrt[6]{x} - 6 \arctan \sqrt[6]{x} + c$$

$$\textcircled{11} \int \frac{x dx}{\sqrt{2-x}} = \int \frac{(2-t^2) d}{\sqrt{2-x}} \cdot (-2) \cdot t dt = 2 \int t^2 - 2 dt =$$

$$\sqrt{2-x} = t$$

$$2-x = t^2$$

$$x = 2-t^2$$

$$dx = d(2-t^2) = -2t dt$$

$$= \frac{2}{3} t^3 - 4t + c =$$

$$= \frac{2}{3} \sqrt[3]{(2-x)^3} - 4\sqrt{2-x} + c$$

cyf.

1) $\int \frac{dx}{1+e^x}$

2) $\int \arcsin x dx$
no result

1) $\int \frac{dx}{1+e^x} = \int \frac{d \ln t}{1+t} = \int \frac{\frac{1}{t} dt}{1+t} = \int \frac{dt}{t(t+1)}$
non: ~~$t = e^x$~~
 ~~$x = \ln t$~~
 ~~$dx = d(\ln t) = \frac{1}{t} dt$~~

non: $e^{x+1} = t$
 $e^x = t-1$
 $x = \ln(t-1)$
 $dx = d(\ln(t-1)) = \frac{1}{t-1} dt$

$= \int \frac{\frac{1}{t-1} dt}{t} = \int \frac{dt}{t(t-1)} = \int \frac{dt}{t} - \int \frac{dt}{t-1} = \ln|t| - \ln|t-1| + C$

$= \ln(e^{x+1}) - \ln(e^x) = x+1 - x = 1 + C$

2)

2) $\int \arcsin x dx = \int x \arcsin x dx = \int x d \arcsin x =$

$= x \arcsin x - \int x d \arcsin x =$

$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx = x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} dx =$

$= x \arcsin x - \ln|x^2-1| + C$

⑥ $= x \arcsin x + \int \frac{d(x^2+1)}{\sqrt{1-x^2}} = x \arcsin x + \int d\sqrt{1-x^2} =$
 $= x \arcsin x + \sqrt{1-x^2} + C$

$$Q: \left\{ \frac{p}{q} : q \neq 0, p, q \in \mathbb{Z} \right\}$$

$$\begin{aligned} \text{Cyc. } P &= a_n x^n + \dots + a_1 x + a_0 \\ q &= a_N x^m + \dots + a_1 x + a_0 \end{aligned}$$

$$\frac{P(x)}{Q(x)}$$

$$\deg P \geq \deg Q \quad \frac{5}{3} = 1 \frac{2}{3}$$

$$\int S(x) \quad \text{---} \quad \int \frac{T(x)}{Q(x)}$$

$$\begin{array}{r} x^6 - x^5 \\ - (x^6 - 2x^5 + 2x^4 - 4x^3 + x^2 - 2x) \\ \hline x^5 - 2x^4 + 2x^3 - 2x^2 + 3x + 1 \\ - (x^5 - 2x^4 + 2x^3 - 4x^2 + 2x + 1) \\ \hline 2x^2 + 2x + 1 \end{array}$$

$\frac{2x^2 + 2x + 1}{x^5 - 2x^4 + 2x^3 - 4x^2 + x - 2}$

$$\int x dx + \int dx + \int \frac{2x^2 + 2x + 1}{x^5 - 2x^4 + 2x^3 - 4x^2 + x - 2} dx =$$

$$= \frac{x^2}{2} + x + \dots$$

2	1	-2	2	-4	1	-2
2	4	0	2	0	1	0

$$(x-2)(x^4 + 2x^2 + 1) = (x-2)(x^2 + 1)^2$$

$$2x^2 + 2x + 13$$

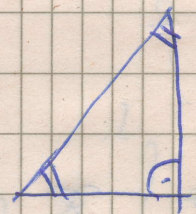
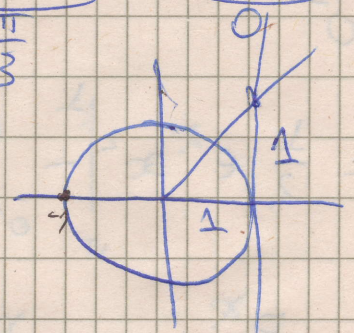
$$\Delta = 1 - 26 = -25 < 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{-25}}{2}$$

10.11.2013.

① $\int_0^{\sqrt{3}} \frac{dx}{1+x^2} = \arctg x \Big|_0^{\sqrt{3}} = \arctg \sqrt{3} - \arctg 0 = \frac{\pi}{3}$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
	0	30	45	60	90
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0



② $\int_0^2 |1-x| dx = -\int_0^1 (1-x) dx + \int_1^2 (x-1) dx$

$= -\left[\frac{(1-x)^2}{2} \right]_0^1 + \left[\frac{(x-1)^2}{2} \right]_1^2 = -\left[\frac{(x-1)^2}{2} \right]_0^1 + \left[\frac{(x-1)^2}{2} \right]_1^2$

~~$= -\left[\frac{(1-x)^2}{2} \right]_0^1 + \left[\frac{(x-1)^2}{2} \right]_1^2 = -\frac{1}{2}$~~

$-0 + \frac{1}{2} + \frac{1}{2} - 0 = 1$

$$\textcircled{3} \int_0^1 x e^x dx = \frac{1}{2} \int_0^1 e^x dx^2 = \frac{1}{2} e^x x^2 \Big|_0^1 - \frac{1}{2} \int_0^1 x^2 de^x$$

$$= \frac{1}{2} e^x x^2 \Big|_0^1 - \int_0^1 e^x dx = \frac{1}{2} e^x x^2 \Big|_0^1 - e^x \Big|_0^1 =$$

$$= \frac{e^x}{2} - e^x + 1 = 1 - \frac{e^x}{2}$$

$$= \frac{1}{2} \int_0^1 x de^x = e^x x \Big|_0^1 - \int_0^1 e^x dx = e^x x \Big|_0^1 - e^x \Big|_0^1$$

$$= e - 0 - e + 1 = 1$$

$$\textcircled{4} \int_{-1-\varepsilon}^{1-\varepsilon} \sqrt{1-x^2} dx = x \sqrt{1-x^2} \Big|_{-1-\varepsilon}^{1-\varepsilon} - \int_{-1-\varepsilon}^{1-\varepsilon} x d \sqrt{1-x^2} =$$

$$= x \sqrt{1-x^2} \Big|_{-1-\varepsilon}^{1-\varepsilon} - \int_{-1-\varepsilon}^{1-\varepsilon} x \cdot \frac{1}{2} (1-x^2)^{-\frac{1}{2}} (-2x) dx =$$

$$= (1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} - (-1-\varepsilon) \sqrt{1-(-1-\varepsilon)^2} +$$

$$+ \int_{-1-\varepsilon}^{1-\varepsilon} dx = (1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} + (1-\varepsilon) \sqrt{1-(1+\varepsilon)^2} +$$

$$+ x \Big|_{-1-\varepsilon}^{1-\varepsilon} = 2(1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} + (-1-\varepsilon) - (-1-\varepsilon) =$$

$$\textcircled{5} = 2(1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} - 2(1-\varepsilon) =$$

$$= 2(1-\varepsilon) \left(\right)$$

$$= 2(1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} + \int_{-1+\varepsilon}^{1-\varepsilon} x^2 (1-x^2)^{-\frac{1}{2}} dx =$$

$$= 2(1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} + \frac{1}{2} \int_{-1+\varepsilon}^{1-\varepsilon} x (1-x^2)^{-\frac{1}{2}} d(-x^2+1) =$$

$$= 2(1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} - \frac{1}{2} \int_{-1+\varepsilon}^{1-\varepsilon} x (1-x^2)^{-\frac{1}{2}} d \sqrt{1-x^2} =$$

$$= 2(1-\varepsilon) \sqrt{1-(1-\varepsilon)^2} - \frac{1}{2} x \sqrt{1-x^2} \Big|_{-1+\varepsilon}^{1-\varepsilon} + \frac{1}{2} \int_{-1+\varepsilon}^{1-\varepsilon} \frac{1}{\sqrt{1-x^2}} dx =$$

$$\int \frac{x^2}{\sqrt{1-x^2}} dx = \int \frac{-1+x^2+1}{\sqrt{1-x^2}} dx$$

$$= - \int \frac{1-x^2}{\sqrt{1-x^2}} dx + \int \frac{dx}{\sqrt{1-x^2}} =$$

$$I \stackrel{!}{=} \arcsin x + I + \arcsin x$$

$$I = \frac{1}{2} \left(\arcsin x + \arcsin x \right)$$

$$I = \frac{1}{2} \left(2(1-8) \sqrt{1-(1-8)^2} + \arcsin \left(\frac{1-8}{3+1} \right) \right) =$$

$$= \frac{1}{2} \left(2(1-8) \sqrt{1-(1-8)^2} + \arcsin(-1+8) + \arcsin(1-8) \right)$$

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$$= \frac{1}{2} \left(0 + \frac{\pi}{2} + \frac{\pi}{2} \right) = \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{2}$$

2.6

$$\int_0^{\pi} \frac{1+\cos 2x}{2} dx = \int_0^{\pi} \frac{1+\cos^2 x + \sin^2 x}{2} dx =$$

$$= \int_0^{\pi} \frac{\sqrt{2} \cos^2 x}{\sqrt{2}} dx = \frac{1}{\sqrt{2}} \int_0^{\pi} |\cos x| dx =$$

~~$$= \frac{1}{\sqrt{2}} \sin x \Big|_0^{\pi} = \frac{1}{\sqrt{2}} \cdot 2 \cdot \frac{\sqrt{2}}{2}$$~~

~~$$= \frac{1}{\sqrt{2}} \sin x \Big|_0^{\pi} - \frac{1}{\sqrt{2}} \sin x \Big|_0^{\pi} =$$~~

~~$$= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \cos x dx + \frac{1}{\sqrt{2}} \int_{\frac{\pi}{2}}^{\pi} \cos x dx =$$~~

~~$$= \frac{1}{\sqrt{2}} \sqrt{2} \int_0^{\frac{\pi}{2}} \cos x dx = \sqrt{2}$$~~

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$$= \frac{1}{\sqrt{2}} \sin x \Big|_0^{\frac{\pi}{2}} - \frac{1}{\sqrt{2}} \sin x \Big|_{\frac{\pi}{2}}^{\pi} = \frac{2}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$\textcircled{6} \int_0^{\pi} \frac{dx}{1+2\sin^2 x} = \int_0^{\pi} \frac{dx}{1+2(1-\cos 2x)} =$$

$$\int_0^{\pi} \frac{dx}{2-\cos 2x} =$$

$$= \frac{1}{2} \int_0^{\pi} \frac{dx}{1-\frac{1}{2}\cos 2x} = \frac{\pi}{2} - \frac{1}{2} \int_0^{\pi} \frac{dx}{\cos 2x} =$$

$$= \int_0^{\pi} \frac{dx}{2-\frac{1-\tan^2 x}{1+\tan^2 x}} = \int_0^{\pi} \frac{dx}{\frac{2+\tan^2 x-1+\tan^2 x}{1+\tan^2 x}} = \int_0^{\pi} \frac{dx}{\frac{1+3\tan^2 x}{1+\tan^2 x}} =$$

$$= \int_0^{\pi} \frac{(1+\tan^2 x) dx}{1+3\tan^2 x} = \int_0^{\pi} \frac{dx}{1+3\tan^2 x} + \int_0^{\pi} \frac{\tan^2 x dx}{1+3\tan^2 x} =$$

$$= \int_0^{\pi} \frac{dx}{1+3\tan^2 x} + \frac{1}{3} \int_0^{\pi} \frac{d(3\tan^2 x)}{1+3\tan^2 x} + 1 = \frac{1}{3} \int_0^{\pi} \frac{dx}{1+3\tan^2 x} + \frac{1}{3} \int_0^{\pi} dx =$$

$$= \int_0^{\pi} \frac{dx}{1+3\tan^2 x} + \frac{\pi}{3}$$

$\ln^3 x = \frac{e}{x}$
 $\ln^2 x = 2 \frac{1}{x}$

$$\textcircled{7} \int_1^e \ln^3 x dx = x \ln^3 x \Big|_1^e - \frac{1}{3} \int_1^e x d \ln^3 x =$$

$$= x \ln^3 x \Big|_1^e - \frac{1}{3} \int_1^e \frac{3x \ln^2 x}{x^2} dx = x \ln^3 x \Big|_1^e - \int_1^e \frac{\ln^2 x}{x} dx =$$

$$= x \ln^3 x \Big|_1^e - \int_1^e \ln^2 x dx =$$

$$= x \ln^3 x \Big|_1^e - \int_1^e \ln^2 x dx =$$

$$2 \int_1^e x \ln^3 x \, dx = \int_1^e x \ln^2 x \, dx + \int_1^e x \ln^2 x \, dx$$

$$2 \int_1^e x \ln^3 x \, dx = \int_1^e x \ln^2 x \, dx + 2 \int_1^e x \ln x \, dx =$$

$$2 \int_1^e x \ln^3 x \, dx = \int_1^e x \ln^2 x \, dx + \frac{2}{x} \Big|_1^e =$$

$$2 \int_1^e x \ln^2 x \, dx = \left(\ln x \Big|_1^e - 1 \right) + \frac{2}{e} - 2 =$$

$$2(e \ln^2 e - 1)(\ln e - 2) + \frac{2}{e} - 2 =$$

$$2 \int_1^e x \ln^3 x \, dx = \int_1^e x \ln^2 x \, dx + 2 \int_1^e x \ln x \, dx - 2 \int_1^e dx =$$

$$= x \ln x \Big|_1^e \left(\ln^2 x \Big|_1^e - \ln^3 x \Big|_1^e + 2 \right) - 2e + 2 =$$

$$2(e \cdot \frac{1}{2} - 1)(e - 1 - e + 1 + 2) - 2(e + 1) =$$

$$= 2e - 2 - 2e - 2 = -4$$

22.11.2013.

$$(1) \int_0^2 |1-x| dx = \int_0^1 (1-x) dx + \int_1^2 (x-1) dx = 1$$

$$- \int_0^1 (1-x) d(-x+1) = - \left. \frac{(1-x)^2}{2} \right|_0^1 = +\frac{1}{2}$$

$$+ \int_1^2 (x-1) d(-x+1) = + \left. \frac{(1-x)^2}{2} \right|_1^2 = \frac{1}{2}$$

$$(2) \int_0^\pi \sqrt{\frac{1+\cos 2x}{2}} dx = \int_0^\pi |\cos x| dx = 2$$

$$\int_0^{\frac{\pi}{2}} \cos x dx = \sin x \Big|_0^{\frac{\pi}{2}} = 1$$

$$- \int_0^{\frac{\pi}{2}} x d \cos x =$$

$$2 \cos x \cdot x \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} x \sin x dx$$

$$2 \sin x \Big|_0^{\frac{\pi}{2}} = 1$$

$$- \int_{\frac{\pi}{2}}^\pi \cos x dx = - \sin x \Big|_{\frac{\pi}{2}}^\pi = 1$$

$$\boxed{\begin{aligned} \frac{1+\cos 2x}{2} &= \cos^2 x \\ \frac{1-\cos 2x}{2} &= \sin^2 x \end{aligned}}$$

$$(8) \int_0^a \sqrt{a^2 - x^2} dx = \int_0^{\frac{\pi}{2}} \sqrt{a^2(1 - \sin^2 t)} da \cdot \sin t =$$

$$\text{non } x = a \cdot \sin t$$

$$\begin{aligned} 0 &= a \sin t & \Rightarrow t &= 0 \\ a \cos t & & \Rightarrow t &= \frac{\pi}{2} \end{aligned}$$

$$\frac{1 + \cos 2t}{2} = \cos^2 t$$

$$= a^2 \int_0^{\frac{\pi}{2}} \cos t \cdot \cos t dt = a^2 \int_0^{\frac{\pi}{2}} \cos^2 t dt =$$

$$= a^2 \cos^2 t \Big|_0^{\frac{\pi}{2}} + a^2 \int_0^{\frac{\pi}{2}} t \cdot \cos^2 t \cdot 2 \cdot \cos t \cdot \sin t dt =$$

$$= a^2 \cos^2 t \Big|_0^{\frac{\pi}{2}} + a^2 \int_0^{\frac{\pi}{2}} t \cdot \sin 2t dt = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} \sin 2t dt$$

$$= \frac{a^2}{2} \int_0^{\frac{\pi}{2}} \sin 2t dt = \frac{a^2}{2} \cdot \sin 2t \cdot t \Big|_0^{\frac{\pi}{2}} - \frac{a^2}{2} \int_0^{\frac{\pi}{2}} t^2 d \sin 2t$$

$$= \frac{a^2}{2} \sin + \frac{a^2}{2} \int_0^{\frac{\pi}{2}} t^2 \cos 2t dt =$$

$$= \frac{a^2}{2} t^2 \Big|_0^{\frac{\pi}{2}} \cdot \cos 2t dt =$$

$$= a^2 \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} dt = a^2 \int_0^{\frac{\pi}{2}} \frac{1}{2} dt + \frac{a^2}{4} \int_0^{\frac{\pi}{2}} \cos 2t d2t =$$

$$= \frac{a^2}{2} t \Big|_0^{\frac{\pi}{2}} + \frac{a^2}{4} \sin 2t \Big|_0^{\frac{\pi}{2}} = \frac{a^2 \pi}{4}$$

$$\textcircled{4} \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{x\sqrt{1-x^2}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{d\sin t}{\sin t \sqrt{1-\sin^2 t}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos t dt}{\sin t \cos t} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin t} dt = \dots$$

$$x = \sin t \quad \left[\frac{\pi}{6}, \frac{\pi}{3} \right] \\ = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin t dt}{\sin^2 t} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dt}{\sin t} = \frac{1}{2} \ln \left| \frac{\cos t + 1}{\cos t - 1} \right|$$

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \left\{ \ln \left| \frac{\frac{\sqrt{3}}{2} + 1}{\frac{\sqrt{3}}{2} - 1} \right| - \frac{1}{2} \left\{ \ln \left| \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1} \right| \right\} \right\}$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \left(\frac{1}{x-a} - \frac{1}{x+a} \right) = \frac{1}{2a} \int \frac{d(x-a) - d(x+a)}{(x-a)(x+a)} = \frac{1}{2a} \int \frac{d(x-a)}{x-a} - \frac{1}{2a} \int \frac{d(x+a)}{x+a} =$$

$$= \frac{1}{2a} (\ln|x-a| - \ln|x+a|) = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$\textcircled{5}$ $f(x)$ — непрерывная в $[-a, a]$

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx, \quad f(x) \text{ четкая}$$

$$\int_{-a}^a f(x) dx = 0, \quad f(x) \text{ нечеткая}$$

$$\int_{-a}^a f(x) dx = - \int_{+a}^0 f(-t) dt + \int_0^a f(-t) dt$$

$$\text{non: } x = -t$$

$$f(x) = f(-x) \text{ четкая} \\ -f(x) = f(-x) \text{ нечеткая}$$

$$\int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx =$$

nono $x = -t$

$$= + \int_a^0 f(-t) d(-t) + \int_0^a f(t) dt =$$

$\Rightarrow t = a$
 $0 = -t \quad t = 0$

$$\int_a^0 f(x) dx = - \int_a^0 f(x) dx$$

$$= - \int_a^0 f(-t) dt - \int_0^a f(-t) dt = \int_0^a f(-t) dt - \int_0^a f(t) dt$$

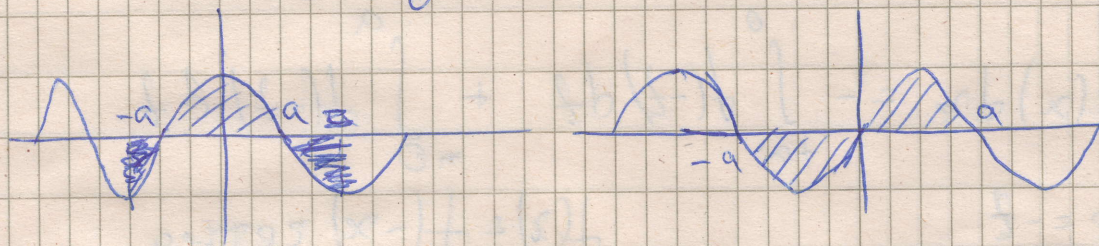
$$= \int_0^a f(-t) dt + \int_0^a f(t) dt = \int_0^a (f(-t) + f(t)) dt$$

f -parita $\Rightarrow \int_0^a f(t) + f(t) dt = 2 \int_0^a f(t) dt$

f -imparita $\Rightarrow \int_0^a -f(t) + f(t) dt = 0$

$$\int_0^a 0 dt = 0$$

(5) $\int_{-a}^a \cos x dx = 2 \int_0^a \cos x dx = 2 \sin x \Big|_0^a$

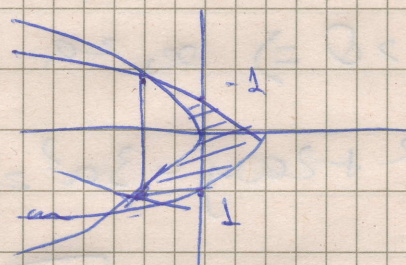


$$\textcircled{6} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} x^8 \sin^9 x dx = 0$$

$x^8 - \sin^9 x$ - нечетная

$$-f(x) = f(-x) \text{ - четная}$$

$$\textcircled{7} \begin{cases} x = -2y^2 \\ x = 1 - 3y^2 \end{cases}$$

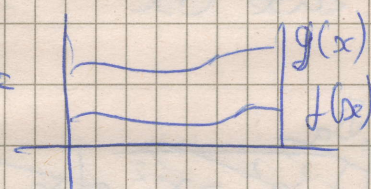


$$\begin{aligned} 1 - 3y^2 &= -2y^2 \\ 1 &= y^2 \\ y &= \pm 1 \end{aligned}$$

$$\begin{aligned} y &= f(x) \\ x &= f(y) \end{aligned}$$

$$\int_{-1}^1 (1 - 3y^2 - (-2y^2)) dy =$$

$$= \int_{-1}^1 (1 - y^2) dy = y \left|_{-1}^1 - \frac{y^3}{3} \right|_{-1}^1 =$$

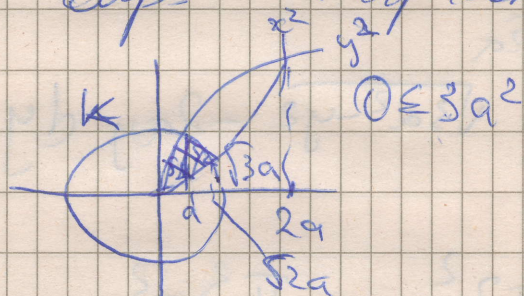


$$\int g(x) - f(x) dx$$

$$= 2 - \frac{1}{3} - \frac{1}{3} = \frac{4}{3}$$

$$\textcircled{8} \text{ Квадрат, ограниченный от дуги и 2 парабол}$$

$$\begin{cases} x^2 + y^2 = 3a^2 \\ x^2 = 2ay \\ y^2 = 2ax \end{cases}$$



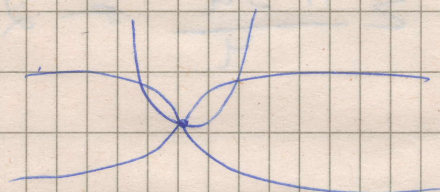
$$\begin{aligned} x^2 &= 2ay \\ y^2 &= 2ax \end{aligned}$$

$$\Rightarrow y^2 = \frac{x^4}{4a^2}$$

$$2ax = \frac{x^4}{4a^2}$$

$$x^3 = 8a^3$$

$$x = 2a$$



$$x^2 + 2ax = 3a^2$$

$$x^2 + 2ax - 3a^2 = 0$$

$$x_{1,2} = \frac{-a \pm \sqrt{a^2 + 3a^2}}{2} = -a \pm 2a$$

$$x > 0 \Rightarrow x_1 = a$$

$$y^2 + 2ay - 3a^2 = 0$$

$$y_{1,2} = \frac{-a \pm \sqrt{a^2}}{2} = -a \pm a = a$$

$$x^2 = 2a^2$$

$$x = \sqrt{2}a$$

$$y = \frac{x^2}{2a}$$

$$S_2 = \int_0^a 2ax \, dx - \int_0^a \frac{x^2}{2a} \, dx$$

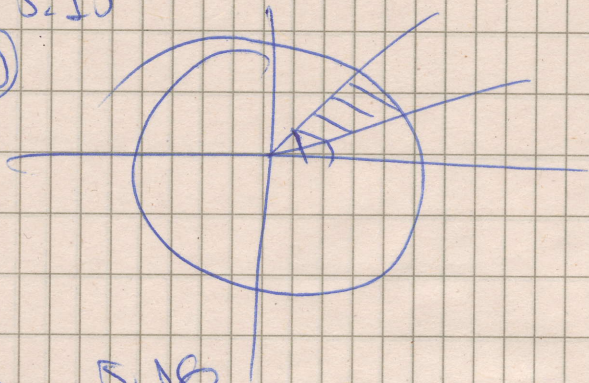
~~$$S_2 = \int_a^{\sqrt{3}a} \sqrt{3a^2 - x^2} \, dx - \int_{\sqrt{2}a}^a \sqrt{3a^2 - x^2} \, dx$$~~

$$\int_0^{\sqrt{2}a} \sqrt{3a^2 - x^2} - 2ax \, dx = J$$

$$\int_0^{\sqrt{2}a} \sqrt{3a^2 - y^2} - 2ay \, dy = J$$

$$\frac{\pi 2^2}{4} = \frac{\pi 3a^2}{4} = 2J$$

5.15
(9)



$$\pi r^2$$

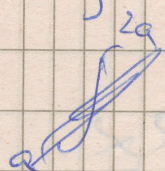
$$\frac{(p_2 - p_1)}{2} \cdot r^2$$

(10)

5.16

$$p \geq a$$

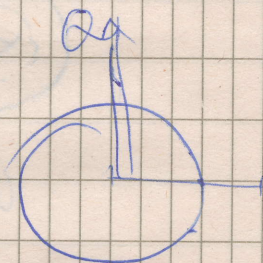
$$p = 2a \cdot \cos^3 \theta$$



$$a > 0$$

$$p = 2a$$

~~$$p = 2a$$~~



$$\begin{aligned} a &\leq \theta \leq b \\ 0 &\leq \phi \leq f(\theta) \end{aligned}$$

$$I(\phi) = \frac{1}{2} \int_a^b f^2(\theta) d\theta$$

$$\frac{1}{2} \int_{\pi/9}^{\pi/3} 4a^2 \cos^3 \theta d\theta =$$

$$a = 2a \cdot \cos^3 \theta$$

$$\frac{1}{2} = \cos^3 \theta$$

$$3\theta = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{9}$$

$$\cos^3 \theta = 1$$

$$3\theta = \pi$$

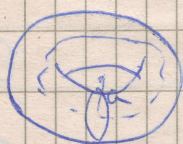
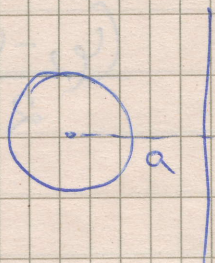
$$\theta = \frac{\pi}{3}$$

$$= \frac{2a^2}{3} \int_{\pi/9}^{\pi/3} \cos^3 \theta d\theta = \frac{2a^2}{3} \cdot \sin 3\theta \Big|_{\pi/9}^{\pi/3} =$$

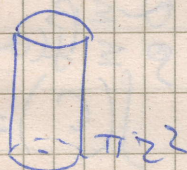
$$= \frac{2a^2}{3} \cdot \frac{\sqrt{3}}{2} = + \frac{\sqrt{3}a^2}{3}$$

(21)

5.17
 11



~~8\pi a~~ $C = 2\pi a$

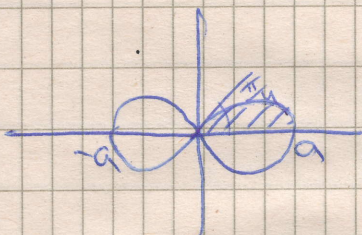


$$2\pi a \cdot \pi a^2 = 2a\pi^2 a^2$$

29. III. 2013.

① $\rho^2 = a^2 \cos 2\theta$ $a > 0$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$

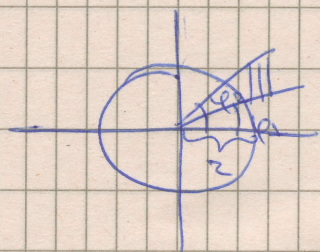


$$4 \cdot \frac{1}{2} \int_0^{\frac{\pi}{4}} a^2 \cos 2\theta d\theta =$$

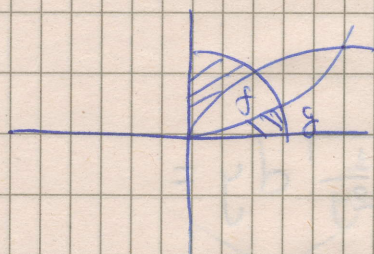
$$= 2a^2 \int_0^{\frac{\pi}{4}} \cos 2\theta d2\theta = a^2 \sin 2\theta \Big|_0^{\frac{\pi}{4}} = a^2$$

22

②



$$\frac{1}{2} \int_{\varphi_1}^{\varphi_2} z^2 d\varphi = \frac{1}{2} \int_{\varphi_1}^{\varphi_2} z^2 (\varphi_2 - \varphi_1)$$



$$\int_0^{\sqrt{3}a} f - g dx = J$$

③ $\int_0^1 \sqrt{x^2 + 1} dx$

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{\sin^2 t + 1}}{\cos^2 t} \cdot \frac{1}{\cos^2 t} dt =$$

$$x = \tan t$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sqrt{\sin^2 t + 1}}{\cos^2 t} \cdot \frac{1}{\cos^2 t} dt = \int_0^{\frac{\pi}{4}} \frac{1}{\cos^3 t} dt$$

и на упрощение

④ l-кривая

$$y^2 = x^3$$

$$x \in [0, 4]$$

$$y = f(x)$$

$$\int_a^b \sqrt{1 + f'(x)^2} dx$$

$$y = f(t)$$

$$t \in [\alpha, \beta]$$

$$x = g(t)$$

$$\int_a^b \sqrt{f'(t)^2 + g'(t)^2} dt$$

$$y = x^{\frac{3}{2}}$$

$$y' = \frac{3}{2} x^{\frac{1}{2}}$$

$$\frac{1}{9} \int_0^4 \sqrt{\frac{9}{4} x^{\frac{1}{2}} + 1} dx = \frac{3}{2} \cdot \frac{1}{9} \left(\frac{9x+4}{\frac{1}{2} \cdot 4} \right)^{\frac{3}{2}} \Big|_0^4 =$$

⑤

$$2\frac{8}{27} - (10 \cdot \sqrt{10} = 1)$$

⑤ l-urubara $x = \frac{1}{4}y^2 - \frac{1}{2}\ln y$

$$y \in [1, 2]$$

~~$$\int_1^2 \sqrt{1 + f'(y)^2} dy = \int_1^2 \sqrt{1 + \frac{3}{4}y^3 - \frac{1}{2y}} dy =$$

$$= \int_1^2 \sqrt{1 + \frac{1}{2}y(\frac{3}{2}y - 1)} dy = 4 \int_1^2 \sqrt{1 + \frac{1}{4}y} \frac{dy}{2} =$$

$$= \frac{2}{3} \left[\frac{1}{4}y + 1 \right]^{\frac{3}{2}} \Big|_1^2 = 1$$~~

~~$$\int_1^2 \sqrt{1 + \left(\frac{1}{2}y - \frac{1}{2y}\right)^2} dy = \int_1^2 \sqrt{1 + \left(\frac{1}{2}\right)^2 \left(\frac{y^2 - 1}{y}\right)^2} dy =$$

$$= \int_1^2 \sqrt{1 + \frac{1}{4y^2}(y^2 - 2y + 1)} dy = \int_1^2 \sqrt{1 + \frac{1}{4} - \frac{1}{2y} + \frac{1}{4y^2}} dy =$$

$$= \int_1^2 \left(\frac{3}{4} - \frac{1}{2y} \left(1 - \frac{1}{2y}\right) \right) dy$$~~

$$\int_1^2 \sqrt{1 + \left(\frac{1}{2}y - \frac{1}{2y}\right)^2} dy = \int_1^2 \sqrt{1 + \frac{1}{4}y^2 - \frac{1}{4y} + \frac{1}{4y^2}} dy =$$

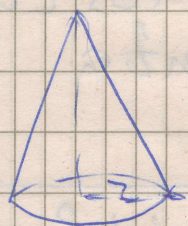
$$= \int_1^2 \sqrt{\frac{2y^2 + y^4 + 1}{4y^2}} dy = \int_1^2 \frac{\sqrt{y^4 + 2y^2 + 1}}{2y} dy =$$

$$= \int_1^2 \frac{y^2 + 1}{2y} dy = \frac{1}{2} \int_1^2 y dy + \frac{1}{2} \int_1^2 \frac{1}{y} dy = \left[\frac{ab}{c} = \frac{a}{c} + \frac{b}{c} \right]$$

$$= \frac{1}{2} \frac{y^2}{2} \Big|_1^2 + \frac{1}{2} \ln y \Big|_1^2 = 1 - \frac{1}{4} + \frac{1}{2} \ln 2 - 0 =$$

$$= \frac{3}{4} + \frac{1}{2} \ln 2$$

⑥ V- h-вие. нрав кривоу кривоу
2-рагуну

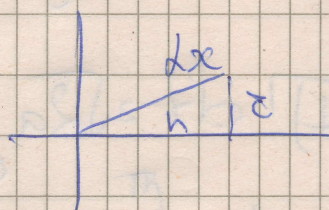


$$V = \pi \int_a^B f(x)^2 dx$$

$$r = \frac{dh}{dx} \Rightarrow \frac{dr}{dx} = \frac{r}{h}$$

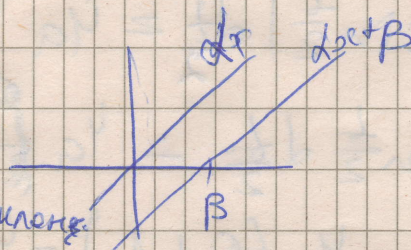


$$dx + \beta$$



$$B \geq 0$$

$(dx)'$ - наклон

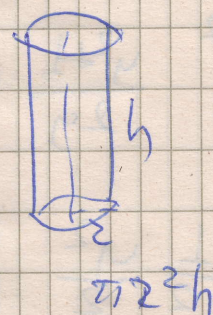


$$V = \pi \int_0^h \left(\frac{z}{h} \cdot x \right)^2 dx = \pi \frac{z^2}{h^2} \int_0^h x^2 dx = \pi \frac{z^2}{h^2} \left[\frac{x^3}{3} \right]_0^h = \pi \frac{z^2 h}{3} =$$

$$= \frac{1}{3} \pi z^2 h$$

⑦ Цилиндр

$$\pi \int_0^h z^2 dx = \pi z^2 h$$



⑧ l - кривая $t \in [0, 2\pi]$ $a \geq 0$

$$2\cos^2 x = 1 + \cos 2x$$

$$2\sin^2 x = 1 - \cos 2x$$

урав. $x = a(t - \sin t)$
 $y = a(1 - \cos t)$

$$x'(t) = a - \cos t$$

$$y'(t) = \sin t$$

$$l = \int_0^{2\pi} \sqrt{(a - a\cos t)^2 + a^2 \sin^2 t} dt = \int_0^{2\pi} \sqrt{a^2 - 2a^2 \cos t + a^2 \cos^2 t + a^2 \sin^2 t} dt =$$

$$= \int_0^{2\pi} \sqrt{a^2(1 - 2\cos t + 1)} dt = \sqrt{2}a \int_0^{2\pi} \sqrt{1 - \cos t} dt =$$

$$= \frac{1}{2}a \int_0^{2\pi} \left| \sin \frac{t}{2} \right| dt = 4a \int_0^{\pi} \sin \frac{t}{2} \frac{dt}{2} =$$

$$= 4a \int_{\pi}^{2\pi} \sin \frac{t}{2} \frac{dt}{2} = 4a \left(\cos \frac{t}{2} \right) \Big|_0^{\pi} - 4a \left(\cos \frac{t}{2} \right) \Big|_{\pi}^{2\pi} =$$

$$= 4a(0 + 1) - 4a(1 + 0) = 8a$$

②6

③ ③ упражнение

$$\int_0^1 \sqrt{1+x^2} dx = \int_0^{\frac{\pi}{4}} \sqrt{1 + \frac{\sin^2 t}{\cos^2 t}} d \operatorname{tg} t =$$

нон $x = \operatorname{tg} t$

$$= \int_0^{\frac{\pi}{4}} \frac{\sqrt{\cos^2 t + \sin^2 t}}{\cos t} dt = \int_0^{\frac{\pi}{4}} \frac{1}{\cos^3 t} dt =$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin^2 t + \cos^2 t}{\cos^3 t} dt = \int_0^{\frac{\pi}{4}} \frac{\sin^2 t}{\cos^3 t} dt + \int_0^{\frac{\pi}{4}} \frac{1}{\cos t} dt =$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin^2 t}{\cos t} d \operatorname{tg} t = \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos t} - \cos t \right) d \operatorname{tg} t =$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos t} - \cos t \right) d \operatorname{tg} t = \frac{\operatorname{tg} t}{\cos t} \Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \operatorname{tg} t d \cos t =$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin^2 t}{\cos t} dt = \int_0^{\frac{\pi}{4}} \frac{1}{\cos t} dt + \int_0^{\frac{\pi}{4}} \cos t dt =$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin^2 t}{\cos t} d \cos t + \int_0^{\frac{\pi}{4}} \frac{1}{\cos t} dt = - \int_0^{\frac{\pi}{4}} \frac{1}{\cos t} dt +$$

$$+ \int_0^{\frac{\pi}{4}} \cos t dt - \int_0^{\frac{\pi}{4}} \frac{1}{\cos t} dt = \int_0^{\frac{\pi}{4}} \cos t dt = \sin t \Big|_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2}$$



$$\text{Bsp. } \int_0^1 \sqrt{1+x^2} dx = ?$$

$$\sqrt{1+x^2} = x+t$$

$$1+x^2 = x^2 + 2xt + t^2$$

$$2xt = -1 + t^2$$

$$x = \frac{1-t^2}{2t}$$

$$t=1$$

$$t^2 + 2t - 1 = 0$$

$$t_{1,2} = \pm 1$$

$$0 = \frac{1-t^2}{2t} \quad t=1$$

$$dx = d\left(\frac{1-t^2}{2t}\right) = \frac{-2t - 2t - (1-t^2)2}{4t^2} dt = \frac{-4t^2 - 2 + 2t^2}{4t^2} dt =$$

$$= \frac{-2t^2 - 2}{4t^2} dt = \frac{-t^2 - 1}{2t^2} dt =$$

$$-\int_{+1}^{-1} \left(\frac{1-t^2}{2t} + t\right) \frac{t^2+1}{2t^2} dt = \int_{-1}^1 \frac{(1-t^2+2t^2)(t^2+1)}{4t^3} dt =$$

$$= \int_{-1}^1 \frac{(1+t^2)^2}{4t^3} dt = \frac{1}{4} \int_{-1}^1 \frac{(t^2+1)^2}{t^3} dt = \frac{1}{4} \int_{-1}^1 \frac{t^4 + 2t^2 + 1}{t^3} dt =$$

$$= \frac{1}{4} \int_{-1}^1 t dt + \frac{1}{2} \int_{-1}^1 \frac{dt}{t} + \frac{1}{4} \int_{-1}^1 \frac{dt}{t^3} = \frac{1}{4} \frac{t^2}{2} \Big|_{-1}^1 +$$

$$+ \frac{1}{2} \ln|t| \Big|_{-1}^1 - \frac{1}{4} - \frac{1}{2t^2} \Big|_{-1}^1 =$$

$$= \frac{1}{8} - \frac{1}{8} + \frac{1}{2} + 0 - 0 - \frac{1}{8} + \frac{1}{8} = 0$$

~~2~~

$$2 \int_{-1}^1 \frac{1}{t^2} dt = \frac{1}{4} \int_{-1}^1 t dt + \frac{1}{2} \int_{-1}^1 \frac{1}{t} dt + \frac{1}{4} \int_{-1}^1 \frac{dt}{t^3} + \frac{1}{4} \int_{-1}^1 \frac{dt}{t^3}$$

$$0 = \frac{1-t^2}{2t}$$

$$t^2 = 1$$

$$t \neq 0 \quad t = \pm 1$$

$$1 = \frac{1-t^2}{2t}$$

$$t^2 + 2t - 1 = 0$$

$$(t+1)^2 - 2 = 0$$

$$(t+1)^2 = 2$$

$$t+1 = \sqrt{2}$$

$$t = \sqrt{2} - 1$$

$$t = \sqrt{2} - 1$$

$$t = \sqrt{2} - 1$$

$$1 = \frac{1-2t^2}{2t}$$

$$2t = 1 - 2t^2$$

$$2t^2 + 2t - 1 = 0$$

$$0 = 1 - 2(-1) = 3$$

$$t_{1,2} = \frac{-1 \pm \sqrt{3}}{2}$$

$$\sqrt{1+x^2} = \pm x \quad t \pm 1$$

$$x \pm 1$$

$$1+x^2 = x^2 + 2xt + 1$$

$$x^2(1-t^2) = 2xt$$

$$x = \frac{2t}{1-t^2}$$

$$0 \Rightarrow t = 0$$

$$1 \Rightarrow 1 - t^2 - 2 = 0$$

$$\textcircled{9} \int_0^{\varphi} \frac{d\varphi}{(\cos \varphi - \cos \vartheta)^\alpha} = \int_0^{\varphi} \frac{d\varphi}{\left(2 \sin \frac{\varphi + \vartheta}{2} \cdot \frac{\sin \frac{\varphi - \vartheta}{2} \cdot \frac{\varphi - \vartheta}{2}\right)^\alpha}$$

$\vartheta \in [0, \varphi]$

~~$$\int_0^{\varphi} \frac{d\varphi}{(\varphi - \vartheta)^\alpha}$$~~

$$\int_0^{\varphi} \frac{1}{(\varphi - \vartheta)^\alpha} = \int_0^{\varphi} \frac{1}{(\sin \frac{\varphi}{2})^\alpha} d\varphi$$

$\downarrow$
1

$$\int_0^{\varphi} \frac{d\varphi}{(\varphi - \vartheta)^\alpha}$$

$$\int \frac{dx}{x^\alpha}$$

$\varphi - \vartheta = 1$
 $\varphi = 1 + \vartheta$
 $\alpha > 1$ npu
 $\alpha < 1$ csogyu

$$\textcircled{10} e^x - e^{-x} = x \left(1 + \frac{o(x)}{x} \right)$$

$$\frac{1}{x^\alpha (e^x - e^{-x})} \sim \frac{1}{x^{\alpha+1} \left(1 + \frac{o(x)}{x} \right)} \sim \frac{1}{x^{\alpha+1}}$$

$\downarrow$
1

02 IV 2013 2.

$$\begin{aligned}
 \textcircled{1} \int \frac{dx}{3+\cos^2 x} &= \int \frac{dx}{3\sin^2 x + 4\cos^2 x} = \int \frac{dx}{\cos^2 x (3\tan^2 x + 4)} = \\
 &= \int \frac{d\tan x}{3\tan^2 x + 4} = \frac{1}{\sqrt{3}} \int \frac{d\sqrt{3}\tan x}{\sqrt{3}\tan^2 x + 4} = \frac{1}{\sqrt{3}} \left(\frac{1}{2} \arctg \frac{\sqrt{3}\tan x}{2} + C \right) = \\
 &= \frac{1}{2\sqrt{3}} \arctg \frac{x}{2} + C
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \int_0^1 x \sqrt{1+x^2} dx &= \frac{1}{2} \int_0^1 \sqrt{1+x^2} d(x^2+1) = \frac{1}{2} \int_0^1 (1+x^2)^{\frac{1}{2}} d(x^2+1) = \\
 &= \frac{1}{2} \cdot \frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^1 = \frac{(x^2+1)^{\frac{3}{2}}}{3} \Big|_0^1 = \frac{2\sqrt{2}}{3} - \frac{1}{3} = \frac{2\sqrt{2}-1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \int_{\pi}^{\frac{5\pi}{4}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx &= \int_{\pi}^{\frac{5\pi}{4}} \frac{\sin 2x dx}{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x} = \\
 &= \int_{\pi}^{\frac{5\pi}{4}} \frac{\sin 2x dx}{1 - \frac{\sin^2 2x}{2}} = \int_{\pi}^{\frac{5\pi}{4}} \frac{2\sin 2x dx}{2 - \sin^2 2x} = \int_{\pi}^{\frac{5\pi}{4}} \frac{\sin 2x d2x}{1 + \cos^2 2x} = \\
 &= - \int_{\pi}^{\frac{5\pi}{4}} \frac{d \cos 2x}{1 + \cos^2 2x} = - \arctg(\cos 2x) \Big|_{\pi}^{\frac{5\pi}{4}} = \\
 &= - \arctg\left(\cos \frac{5\pi}{2}\right) + \arctg(\cos 2\pi) = -0 + \frac{\pi}{4} = \frac{\pi}{4}
 \end{aligned}$$

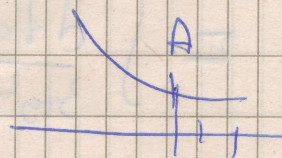
$$\int_{-\infty}^{\infty} f(x) dx = \lim_{A \rightarrow \infty} \int_{-A}^A f(x) dx$$

$$\int_0^{\infty} x dx$$

$$\int_0^{\infty} \frac{1}{x} dx$$

$$\forall \varepsilon > 0 \quad \exists A = m, n > A$$

$$\Rightarrow \left| \int_m^n f(x) dx \right| < \varepsilon$$



$$(3) \int_0^{\infty} \frac{dx}{x^{\lambda}}$$

$$\frac{1}{x} \xrightarrow{x \rightarrow \infty} 0 \quad \int_0^{\infty} \frac{dx}{x} = \ln x \Big|_0^{\infty} - \text{разходящ}$$

$$\lambda < 0 \quad \int_0^{\infty} dx = x \Big|_0^{\infty} - \text{разходящ}$$

$$\lambda < 0 \\ \text{non } \lambda = -\mu \\ \mu > 0$$

$$\int_0^{\infty} x^{\mu} dx = \text{разходящ}$$

$$\lambda \in (0, 1)$$

$$\int_0^{\infty} \frac{dx}{x^{\lambda}}$$

$$\int \frac{dx}{x^{\lambda}}$$

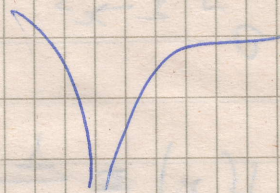
$$\lambda > 1 - \text{сх.} \\ \lambda \leq 1 - \text{разх.}$$

$$\int_{-\infty}^{+\infty} f(x) dx$$

$$\int_a^b f(x) dx$$

$$\int_{-1}^1 \frac{f(x)}{x^n} dx$$

$$\int_1^2 \frac{f(x)}{x^n} dx$$



$$f(x) \leq g(x)$$

$$\int_{-\infty}^{+\infty} g(x) dx - \text{e crog.}$$

$$\Rightarrow \int_{-\infty}^{+\infty} f(x) dx - \text{crog.}$$

$$f(x) \text{ e para.} \Rightarrow \int_{+\infty}^{-\infty} g(x) dx \text{ para.}$$

$$|f(x)| \leq \frac{c}{x^\lambda}$$

$$c > 0$$

$$\int_{-\infty}^{+\infty} f(x) dx$$

$$\text{npn } \lambda \geq 1 \text{ e crog.}$$

$$\text{npn } \lambda \leq 1 \text{ e para.}$$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{\frac{1}{x^\lambda}} = c > 0 < \infty$$

$$\frac{c}{x^\lambda} \leq |f(x)|$$

$$\text{npn } \lambda \leq 1 \text{ e para.}$$

$$\lambda > 1 \text{ e crog.}$$

$$\lambda \leq 1 \text{ e para.}$$

$$\int_a^b f(x) dx$$

$$\lim_{x \rightarrow b} \frac{f(x)}{\frac{1}{(b-x)^\lambda}} = c > 0$$

$$\text{npn } \lambda < 1 \text{ crog.}$$

$$\text{npn } \lambda \geq 1 \text{ para.}$$

Прог несомненно

$$(4) \int_0^1 \frac{dx}{1-x^3} = ?$$

$$f(x) = \frac{1}{1-x^3} \quad \lim_{x \rightarrow 1} \frac{\frac{1}{1-x^3}}{\frac{1}{1-x}} = \lim_{x \rightarrow 1} \frac{1-x}{1-x^3} =$$

$$= \lim_{x \rightarrow 1} \frac{(1-x)}{(1-x)(x^2+x+1)} = \frac{1}{3} \Rightarrow \text{разнообразие}$$

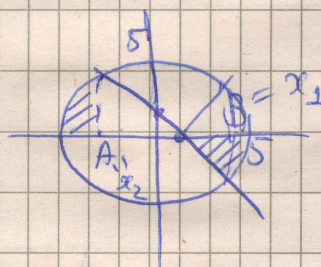
$$(5) \int_0^{\pi} \frac{\sin x}{x^2} dx$$

$$\lim_{x \rightarrow a} \frac{f(x)}{(x-a)^x}$$

$x=0$ e осод. можа

$$\lim_{x \rightarrow 0} \frac{\frac{\sin x}{x^2}}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \rightarrow 1 > 0 \Rightarrow \text{разн.}$$

$$(6) \begin{cases} y = -x + 1 \\ x^2 + y^2 = 25 \end{cases}$$



$$\begin{aligned} x^2 + (1-x)^2 &= 25 \\ x^2 + 1 - 2x + x^2 - 25 &= 0 \\ 2x^2 - 2x - 24 &= 0 \\ x^2 - x - 12 &= 0 \end{aligned}$$

$$\begin{aligned} x_1 &= 4 \\ x_2 &= -3 \end{aligned}$$

$$- \int_1^4 (-x+1) dx = - \int_1^4 x-1 dx = -1 =$$

$$= - \frac{1}{2} (x-1)^2 \Big|_1^4 =$$

$$\begin{aligned} \int f(x) dx &> 0 \\ - \int f(x) dx &< 0 \end{aligned}$$

$$= - \frac{1}{2} (x^2 - 2x + 1) \Big|_1^4 = - \frac{1}{2} \cdot 9 \quad S_2 = \frac{9}{2}$$

(7)

$$-\int_4^5 \sqrt{25-x^2} dx =$$

$$y^2 = 25 - x^2$$

$$y = \pm \sqrt{25-x^2}$$

$$= + \int_4^5 x d \sqrt{25-x^2} - \sqrt{25-x^2} x \Big|_4^5 =$$

$$= \frac{1}{2} 12 - \int_4^5 x \cdot \frac{1}{2} (25-x^2)^{-\frac{1}{2}} 2x dx =$$

$$= 12 - \int_4^5 \frac{x^2 dx}{\sqrt{25-x^2}} = 12 - \int_4^5 \frac{25-x^2}{\sqrt{25-x^2}} dx + 25 \int \frac{dx}{\sqrt{25-x^2}}$$

$$= 12 - \frac{5}{\sqrt{25}} \int \frac{d \frac{x}{5}}{\sqrt{1 - (\frac{x}{5})^2}} = 12 -$$