29.05.2013 ВА - упраж.

Полиноми на една променлива

$(f, g) = d = ?$ и u и $v = ?$

$\deg g$

$f = x^4 - 2x^3 + 2x - 4$

$x^4 - 2x^3 + 4x^2 - 8x$

$$\begin{array}{l} f = x^4 - 2x^3 + 2x - 4 \\ - \; x^4 - 2x^3 + 4x^2 - 8 \\ \hline r_1 = -4x^2 + 10x - 4 \end{array} \quad \Bigg| \begin{array}{l} g = x^3 - 2x^2 + 4x - 8 \\ \hline q_1 = x \end{array}$$

$\deg r_1 < \deg g$

$$\begin{array}{l} g = x^3 - 2x^2 + 4x - 8 \\ -x^3 - \frac{5x^2}{2} + x \\ \hline \frac{1}{2}x^2 + 3x - 8 \\ - \; \frac{1}{2}x^2 - \frac{5}{4}x + \frac{1}{2} \\ \hline r_2 = \frac{17x}{4} - \frac{17}{2} \end{array} \quad \Bigg| \begin{array}{l} r_1 = -4x^2 + 10x - 4 \\ \hline q_2 = -\frac{1}{4}x - \frac{1}{8} \end{array}$$

$\deg r_2 < \deg r_1$

$F = \mathbb{Q}$

$$\begin{array}{l} r_1 = -4x^2 + 10x - 4 \\ - \; -4x^2 + 8x \\ \hline 2x - 4 \\ - \; 2x - 4 \\ \hline 0 \end{array} \quad \Bigg| \begin{array}{l} r_2 = \frac{17}{4}x - \frac{17}{2} \\ \hline q_3 = \frac{16x}{17} + \frac{8}{17} \end{array}$$

$(f, g) = \frac{4}{17} \cdot r_2 = x - 2$

$uf + vg = (f, g)$ $\quad u, v = ?$

$r_2 = g - r_1 q_2$

$r_2 = g + \left(\frac{1}{4}x + \frac{1}{8}\right) r_1$

$r_1 = f - g \cdot q_1$

$r_2 = g + \frac{1}{4}\left(x + \frac{1}{8}\right)\left(f - gx\right) = \left(\frac{1}{4}x + \frac{1}{8}\right) f - \left(\frac{1}{4}x^2 + \frac{1}{8}x - 1\right) g$

$\cancel{u = \frac{4}{17}}$.

$u = \frac{1}{17}x + \frac{1}{34}$

$v = \frac{1}{17}x^2 - \frac{1}{34}x + \frac{4}{17}$

зад)

$$\begin{array}{l} f = x^5 - 3x^3 + x^2 - 3x + 1 \\ - \; x^5 - 3x^3 + x^2 \\ \hline \qquad x^3 \qquad - 3x + 1 \\ \quad - \; x^3 \qquad - 3x + 1 \\ \hline \end{array} \quad \begin{array}{|l} g = x^3 - 3x + 1 \\ \hline q = x^2 + 1 \end{array}$$

$F = Q$

$(f, g) = g$

$uf + vg = (f, g)$

$u = 0$

$v = 1$

зад. 5.1 в)

$f = \bar{3}x^5 + x^4 + \bar{3}x^3 + \bar{4}$ $\quad g = \bar{2}x^4 + \bar{2}x^3 + \bar{2}x + \bar{3}$ $\quad F = \mathbb{Z}_5$

$-\bar{3}x^5 + \bar{3}x^4 + \bar{3}x^2 + \bar{2}x$ $\quad q = \bar{4}x + \bar{4}$

$\bar{3}x^4 + \bar{3}x^3 - \bar{3}x^2 - \bar{2}x + 4$

$= \bar{3}x^4 + \bar{3}x^3 + \bar{3}x + \bar{2}$

$r_1 = -\bar{3}x^2 + \bar{2}$

$\bar{2}x^4 + \bar{2}x^3 + \bar{2}x + \bar{3} \quad : (-\bar{3}x^2 + 2) \quad q_2 = -\bar{4}x^2 - \bar{4}x - \bar{1}$

$-\bar{2}x^4 + \bar{3}x^2$

$\bar{2}x^3 - \bar{3}x + \bar{2}x + 3$

? $-\bar{2}x^3 - \bar{3}x$

$-\bar{3}x^2 + \bar{3}$

$-\; -3x^2 - \bar{2}$

$\bar{0}$

$(f, g) = r_1(-\bar{2}) = r_1\bar{3}$

$uf + vg = \bar{3}r_1$

$r_1 = (f - g(\bar{4}x + \bar{4}))\bar{3} =$

$= \bar{3}f - g\bar{2}x + g\bar{2} = \bar{3}f - g(\bar{2}x + \bar{2})$

$u = \bar{3}$

$g = \bar{2}x + \bar{2}$

зад1

$f = x^4 + x^3 - \bar{4}x^2 + x + \bar{1}$ $\qquad g = x^4 - x^3 + x - \bar{3}$, $\qquad F = \mathbb{Z}_7$

$-\ x^4 \cancel{+} x^3 \qquad + x - \bar{3}$ $\qquad q_1 = +\bar{1}$

$r_1 = \bar{2}x^3 - \bar{4}x^2 + \bar{4}$

$g : r_1$ $\quad \deg r_1 < \deg g$, но $r_1 \neq 0 \Rightarrow g : r_1$

$-\ x^4 - x^3 + x - \bar{3}$ $\qquad r_1 = \bar{2}x^3 - \bar{4}x^2 + \bar{4}$

$\bar{1}x^4 - \bar{2}x^3 + \bar{2}x$ $\qquad q_2 = \bar{4}x + \bar{4}$

$x^3 - \bar{1}x - \bar{3}$

$-\ x^3 - \bar{2}x^2 + \bar{2}\cancel{x}$

$\cancel{x + 2x}$

$r_2 = +\bar{2}x^2 - x - \bar{5} \neq 0$

$r_1 : r_2$

$\bar{2}x^3 - \bar{4}x^2 + \bar{4}$ $\qquad r_2 = \bar{2}x^2 - x - 5$

$-\ 2x^3 - x^2 - \bar{5}x$ $\qquad q_1 = x + \bar{2}$

$\cancel{-\bar{3}x^2 + \bar{5}x + \bar{4}}$

$+\bar{4}x^2 + \bar{5}x + \bar{4}$

$\bar{4}x^2 - \bar{2}x + 4$

$r_3 = 0$

$(f, g) = (\bar{2}x^2 - x - \bar{5})\bar{4} =$

$= x^2 - \bar{4}x + \bar{1}$

$uf + vg = \bar{2}x^2 - \bar{4}x - \bar{6}$

$f = gq_1 + r_1$ $\qquad \bar{2}x^2 - x - 5 = g - r_1 q_2 =$

$r_1 = f - gq_1 = f - g$ $\qquad \bar{2}x^2 - x - 5 = g - (\bar{2}x^3 - \bar{4}x^2 + \bar{4})r_1 = g - (f - g)$

$\overset{\shortparallel}{\bar{1}}$

$-\bar{4}\underbrace{(\bar{3}x + \bar{3})}_{u}f + \underbrace{(\bar{4}x + \bar{5})}_{v}g \cdot \bar{4} =$

$u = \bar{5}x + \bar{5}$

$v = \bar{2}x + \bar{6}$

зад

$m, n, p \in \mathbb{Z}_{\geq 0}$; g / f

а) $g = x^2 + x + 1 \qquad f = x^{3m+2} + x^{3n+1} + x^{3p}$

$x^3 - 1 = (x^2+x+1)(x-1) \Rightarrow \qquad g \rightsquigarrow f \qquad x^2+x+1 \mid x^3-1 \Rightarrow x^3 - 1 \equiv 0 (\mathrm{mod}\, g)$

$f = \underbrace{x^{3m}} x^2 + \underbrace{x^{3n}} x + \underbrace{x^{3p}} \equiv 1 . x^2 + 1x + 1 (\mathrm{mod}\, g) \equiv \qquad \Rightarrow x^3 \equiv 1 (\mathrm{mod}\, g)$

$\equiv 0 (\mathrm{mod}\, g) \qquad x^{3t} \equiv 1 (\mathrm{mod}\, g) \quad t \in \mathbb{N}$

$\forall\, m, n, p \geq 0$

б) $g = x^2 - x + 1 \quad , \quad f = x^{3m+2} - x^{3n+1} + x^{3p} \quad (\mathrm{char}\, F \neq 2)$

$x^3 + 1 = (x+1)(x^2 - x + 1) \Rightarrow x^3 \equiv -1 (\mathrm{mod}\, g)$

$x^{3t} \equiv \pm 1 (\mathrm{mod}\, g)$

$t = 2k \Rightarrow x^{3t} \equiv 1 (\mathrm{mod}\, g)$

$t = 2k+1 \Rightarrow x^{3t} \equiv -1 (\mathrm{mod}\, g)$

ако $m \equiv n \equiv p (\mathrm{mod}\, 2) \Rightarrow g / f$