

29.05.2013

ВА - упр.ж.

Полиноми на една променлива

$$(f, g) = d = ? \quad u \text{ и } v = ?$$

безу.

$$f = x^4 - 2x^3 + 2x - 4$$

$$g = x^3 - 2x^2 + 4x - 8$$

$$\begin{array}{r} f = x^4 - 2x^3 + 2x - 4 \\ - x^4 - 2x^3 + 4x^2 - 8 \\ \hline z_1 = -4x^2 + 10x - 4 \end{array}$$

$$g = x^3 - 2x^2 + 4x - 8$$

$$q_1 = x$$

deg $q_1 <$ deg g

$$g = x^3 - 2x^2 + 4x - 8$$

$$- x^3 - 5x^2 + x$$

$$\hline 2$$

$$r_1 = -4x^2 + 10x - 4$$

$$q_2 = \frac{1}{2}x - \frac{1}{8}$$

deg $q_2 <$ deg r_1

$$\frac{1}{2}x^2 + 3x - 8$$

$$- \frac{1}{2}x^2 - \frac{5}{4}x + \frac{1}{2}$$

$$\hline z_2 = \frac{17x}{4} - \frac{17}{2}$$

$F = \mathbb{Q}$

$$z_1 = -4x^2 + 10x - 4$$

$$- -4x^2 + 8x$$

$$\hline 2x - 4$$

$$- 2x - 4$$

$$\hline 0$$

$$z_2 = \frac{17}{4}x - \frac{17}{2}$$

$$q_3 = \frac{16x}{17} + \frac{8}{17}$$

$$(f, g) = \frac{4}{17} \cdot r_2 = x - 2$$

BY - Rindorf

$$uf + vg = (f, g)$$

Polynom mit $u, v = ?$

$$r_2 = g - r_1 q_2$$

$$r_2 = g - r_1 q_2 = (f, g)$$

$$r_2 = g + \left(\frac{1}{4}x + \frac{1}{8}\right)r_1$$

$$r_1 = f - g q_1$$

$$r_2 = g + \frac{1}{4}x + \frac{1}{8} \mid \mid f - g x \mid = \left(\frac{1}{4}x + \frac{1}{8}\right) f - \left(\frac{1}{4}x^2 + \frac{1}{8}x - 1\right) g$$

$x = 1, p$

~~u, v~~

~~prob > prob~~

$$u = \frac{1}{17}x + \frac{1}{34}$$

$$u \cdot x + v = 17$$

$$v = \frac{1}{17}x^2 - \frac{1}{34}x + \frac{4}{17}$$

$$\frac{1}{8}x + \frac{1}{8} = 17$$

30g 1

17 prob > prob

$$\begin{array}{r} f = x^5 - 2x^3 + x^2 - 3x + 1 \\ - \quad x^5 - 3x^3 + x^2 \\ \hline \quad \quad x^3 - 3x + 1 \\ - \quad \quad x^3 - 3x + 1 \\ \hline \quad \quad \quad \quad \quad 0 \end{array}$$

$$\begin{array}{l} |g = x^3 - 3x + 1 \\ q = x^2 + 1 \end{array}$$

$$\begin{array}{r} F = Q - \frac{1}{5}x + \frac{1}{5} \\ \hline \frac{11}{5} - \frac{21}{5} = \frac{5}{5} \end{array}$$

$Q = 7$

$$(f, g) = g$$

$$\frac{11}{5}x + \frac{1}{5} = \frac{5}{5}$$

$$uf + vg = (f, g)$$

$$u = 0$$

$$v = 1$$

$$\frac{8}{11} + \frac{101}{11} = \frac{109}{11}$$

$$\begin{array}{r} N - x + 10x - N = \frac{5}{5} \\ \hline \quad \quad x^2 + \frac{1}{5}x - \frac{1}{5} \\ \hline \quad \quad \quad \quad \quad 0 \end{array}$$

30g. 5.1 b)

$$5 - x = \frac{1}{11}x + \frac{1}{11} = (p, f)$$

$$f = \bar{3}x^5 + x^4 + \bar{3}x^3 + \bar{u} \quad g = \bar{2}x^4 + \bar{2}x^3 + \bar{2}x + \bar{3} \quad F = \mathbb{Z}_5$$

$$\begin{array}{r} -\bar{3}x^5 + \bar{3}x^4 + \bar{3}x^2 + \bar{2}x \\ \hline \bar{3}x^4 + \bar{3}x^3 - \bar{3}x^2 + \bar{2}x + 4 \end{array}$$

$$\begin{array}{r} -\bar{3}x^4 + \bar{3}x^3 + \bar{3}x^2 + \bar{2} \\ \hline r_1 = -\bar{3}x^2 + \bar{2} \end{array}$$

$$\bar{2}x^4 + \bar{2}x^3 + \bar{2}x + \bar{3} : (-\bar{3}x^2 + \bar{2}) \quad \bar{v} = x \bar{q}_2 = -\bar{4}x^2 + \bar{u}x - \bar{1}$$

$$\begin{array}{r} -\bar{2}x^4 + \bar{3}x^2 \\ \hline \bar{2}x^3 - \bar{3}x + \bar{2}x + \bar{3} \end{array}$$

$$? \quad \begin{array}{r} -\bar{2}x^3 - \bar{3}x \\ \hline -\bar{3}x^2 + \bar{3} \end{array}$$

$$-\bar{3}x^2 + \bar{3}$$

$$-\bar{3}x^2 - \bar{2}$$

$$\bar{0}$$

$$(f, g) = r_1(-\bar{2}) = r_1 \bar{3}$$

$$u f + v g = \bar{3} r_1 \bar{3} = (p, q) x^2 \equiv \pm 1 \pmod{g}$$

$$r_1 = \bar{3} f - g(\bar{4}x + \bar{u}) \mid \bar{3} =$$

$$= \bar{3}f - g\bar{2}x + \bar{g}\bar{2} = \bar{3}f - g(\bar{2}x + \bar{2})$$

$$u = \bar{3}$$

$$g = \bar{2}x + \bar{2} \quad p = \bar{2} \cdot x - \bar{2} \cdot x \bar{5}$$

$$q = \bar{2} \cdot x - \bar{2} \cdot x \bar{5}$$

$$= \bar{u} \cdot \rho(\bar{2} - x\bar{u}) + \bar{2} \frac{\bar{v} + x\bar{v}}{\bar{u}}$$

$$\bar{2} + x\bar{v} = \bar{u}$$

$$\bar{2} + x\bar{v} = \bar{v}$$

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$$f = \begin{array}{r} x^4 + x^3 - \bar{u}x^2 + x + \bar{1} \\ \underline{-x^4 + x^3} \\ \Gamma_1 = \bar{2}x^3 - \bar{u}x^2 + \bar{4} \end{array}$$

$$g = \begin{array}{r} x^4 - x^3 + x - \bar{3} \\ \underline{-x^4 + x^3} \\ q_1 = \bar{1} \end{array}, \quad F = \mathbb{Z}_7$$

$g : \Gamma_1$ $\deg \Gamma_1 < \deg g$, $\forall \Gamma_1 \neq 0 \Rightarrow g : \Gamma_1$

$$\begin{array}{r} x^4 - x^3 + x - \bar{3} \\ \underline{-\bar{1}x^4 + \bar{2}x^3 - \bar{2}x} \\ x^3 - \bar{1}x - \bar{3} \\ \underline{-x^3 + \bar{2}x^2 + \bar{2}x} \\ x - \bar{5} \end{array}$$

$$\begin{array}{r} \bar{1} \cdot \bar{2}x^3 - \bar{u}x^2 + \bar{4} \\ \underline{-\bar{4}x + \bar{4}} \\ q_2 = \bar{4}x + \bar{4} \end{array}$$

$$\Gamma_1 = +\bar{2}x^2 - x - \bar{5} \neq 0$$

$\Gamma_1 : \Gamma_2$

$$\begin{array}{r} \bar{2}x^3 - \bar{u}x^2 + \bar{4} \\ \underline{-2x^3 - x^2 - \bar{5}x} \\ -\bar{3}x^2 + \bar{5}x + \bar{4} \\ \underline{+\bar{4}x^2 + \bar{5}x + \bar{4}} \\ \bar{u}x^2 - \bar{2}x + \bar{4} \end{array} \quad \left| \begin{array}{l} \Gamma_2 = \bar{2}x^2 - x - \bar{5} \\ q_1 = x + \bar{2} \end{array} \right.$$

$$\Gamma_3 = 0$$

$$(f, g) = (\bar{2}x^2 - x - \bar{5}) \bar{4} = x^2 - \bar{4}x + \bar{1}$$

$$u f + v g = \bar{5}x^2 - \bar{4}x - \bar{6} \neq 0$$

$$f = gq_1 + \Gamma_1 \\ \Gamma_1 = f - gq_1 = f - g$$

$$\bar{2}x^2 - x - \bar{5} = g - \Gamma_1 q_2 \\ \bar{2}x^2 - x - \bar{5} = g - (\bar{2}x^3 - \bar{u}x^2 + \bar{4}) \Gamma_1 = g - (f - g)$$

$$-\bar{u} \left(\frac{\bar{3}x + \bar{3}}{u} \right) f + \left(\frac{\bar{u}x + \bar{5}}{v} \right) g \cdot \bar{4} =$$

$$u = \bar{5}x + \bar{5}$$

$$v = \bar{2}x + \bar{6}$$

ВА - умрат.

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$$m, n, p \in \mathbb{Z} \geq 0 ; g/f$$

$$a) g = x^2 + x + 1 \quad f = x^{3m+2} + x^{3n+1} + x^{3p}$$

$$x^3 - 1 = (x^2 + x + 1)(x - 1) \Rightarrow g \mid f \quad x^2 + x + 1 \mid x^3 - 1 \Rightarrow x^3 - 1 \equiv 0 \pmod{g}$$

$$f = x^{3m} x^2 + x^{3n} x + x^{3p} \equiv 1 \cdot x^2 + 1x + 1 \pmod{g} \equiv 0 \pmod{g} \Rightarrow \begin{cases} x^3 \equiv 1 \pmod{g} \\ x^{3t} \equiv 1 \pmod{g} \quad t \in \mathbb{N} \end{cases}$$

$$\forall m, n, p \geq 0$$

$$b) g = x^2 - x + 1, \quad f = x^{3m+2} - x^{3n+1} + x^{3p} \quad (\text{char } F \neq 2)$$

$$x^3 + 1 = (x + 1)(x^2 - x + 1) \Rightarrow x^3 \equiv -1 \pmod{g}$$

$$x^{3t} \equiv \pm 1 \pmod{g}$$

$$t = 2k \Rightarrow x^{3t} \equiv 1 \pmod{g}$$

$$t = 2k+1 \Rightarrow x^{3t} \equiv -1 \pmod{g}$$

$$\text{also } m \equiv n \equiv p \pmod{2} \Rightarrow g/f$$

$$Z_1 = -4x^2 - 10x - 4$$

$$-4x^2 + 8x$$

$$2x - 4$$

$$-2x - 4$$

$$0$$

$$(f, g) = \frac{4}{17} x - 2$$