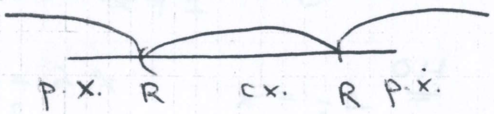
05.06.2013г.

сряда

Упражнение

$$\sum_{n=0}^{\infty} a_n x^n$$



Th $f(x) = \sum a_n x^n$

$f'(x) = \sum n a_n x^{n-1}$ за вътрешни точки

$\int f(x) = \sum a_n \frac{x^{n+1}}{n+1}$

отново сходящ в краищата

Th Абел - сумата е непрекъсната

зад. радиус сходимост

$$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n \cdot \frac{1}{2n+1}$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{2^{n+1}}}{3^{n+1}} \cdot \frac{\cancel{3^n}(2n+1)}{(2n+3)\cdot \cancel{2}} =$$

$$= \frac{2n+1}{3(2n+3)} = \frac{2n+1}{6n+9} \longrightarrow \frac{1}{3} < 1$$

$$\sqrt[n]{a_n} = \frac{2}{3} \cdot \frac{1}{\sqrt[n]{2n+1}} \longrightarrow \frac{2}{3} < 1 \text{ сходящ}$$

$$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n \cdot \frac{1}{2n+1} = \sum \left(\sqrt{\frac{2}{3}}\right)^{2n} \cdot \frac{1}{2n+1}$$

степенен ред

$$\sum \frac{x^n}{2n+1}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{2n+1}$$

$$\sqrt[n]{\frac{x^{2n}}{2n+1}} = \frac{x^2}{\sqrt[n]{2n+1}} \longrightarrow x^2 < 1$$

$x \in (-1; 1)$

$R = 1$

$$x f(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$$

$$(x f(x))' = \sum_{n=0}^{\infty} \frac{\cancel{(2n+1)} x^{2n}}{\cancel{2n+1}} = 1 + x^2 + x^4 + \dots + x^{2n} + \dots = \frac{1}{1-x^2}$$

$$x f(x) = \int \frac{1}{1-x^2}\, dx + C$$

$$\int \frac{dx}{1-x^2} = \cancel{\int \frac{1-x^2+x^2}{1-x^2}\, dx =}$$

$$\cancel{= \int dx + \int \frac{x^2}{1}}$$

1

$$= \frac{1}{1-x} - \frac{1}{2-x} =$$

$$= \frac{1}{1-x} - \frac{1}{2} \cdot \frac{1}{1-\frac{x}{2}} =$$

$$= 1 + x + x^2 + \dots + \frac{1}{2}\left(1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \dots\right)$$

$$= \sum_{n=0}^{\infty} \left(1 - \frac{1}{2^{n+1}}\right) x^n$$

около т. 0

ако е около т. 3

$t = x - 3$

$x = t + 3$

$$\frac{1}{t^2 + 6t + \cancel{9} - 3t - \cancel{9} + 2} =$$

$$= \frac{1}{t^2 + 3t + 2} = \frac{A}{t+1} + \frac{B}{t+2}$$

около т. 0

$$\sum a_n t^n$$

$t = x - 3$ и о.к.

зад. Ред на Маклорен

$$\frac{x^4 - 1}{4} \operatorname{arctg} x - \frac{x^3}{12} + \frac{x}{4}$$

$\shortparallel$ $f(x)$ ↓ даде развие

$$f'(x) \overset{\text{НО}}{=} x^3 \operatorname{arctg} x = x^3 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} =$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+4} (-1)^n}{2n+1}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+5}}{(2n+1)(2n+5)} + \underset{0}{\overset{\shortparallel}{C}}$$

arccotg x

$$-\frac{1}{1+x^2}$$

$$\operatorname{arccotg} x = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)} + C$$

$x = 0$

$C = \frac{\pi}{2}$

$$\boxed{\operatorname{arctg} x + \operatorname{arccotg} x = \frac{\pi}{2}}$$

$$\frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x}$$

$$1 = A(1+x) + B(1-x)$$

$x = 1$

$1 = 2A$ $\qquad x = -1$

$A = \frac{1}{2}$ $\qquad B = \frac{1}{2}$

$$\frac{1}{2(1-x)} + \frac{1}{2(x+1)}$$

$$\frac{1}{2}\left(\int \frac{1}{(1-x)}\,dx + \int \frac{1}{x+1}\,dx\right) =$$

$$= \frac{1}{2} \ln \frac{1+x}{1-x} + C$$

$$x f(x) = \frac{1}{2} \ln \frac{1+x}{1-x} + C$$

$x = 0$

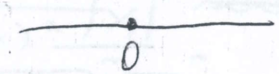
$C = 0$

$$x f(x) = \frac{1}{2} \ln \frac{1+x}{1-x} \quad \text{за } \forall x$$

$$x \neq 0 \quad f(x) = \frac{1}{2x} \ln \frac{1+x}{1-x}$$

$$\frac{1}{2x} \ln \frac{1+x}{1-x} = 1 + \frac{x^2}{3} + \dots$$

$x \neq 0$



$$f(x) = \begin{cases} \frac{1}{2x} \ln \frac{1+x}{1-x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Лопитал

$$\lim_{\substack{x\to 0 \\ x\neq 0}} f(x) \quad \frac{\frac{1}{1+x} + \frac{1}{1-x}}{2} \longrightarrow 1$$

$x \to 0$

$-1 < x < 1$

$\sqrt{\frac{2}{3}}$

$f\left(\sqrt{\frac{2}{3}}\right)$

$$\frac{1}{2}\sqrt{\frac{3}{2}} \ln \frac{1+\sqrt{\frac{2}{3}}}{1-\sqrt{\frac{2}{3}}} = \frac{1}{2}\sqrt{\frac{3}{2}} \ln \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} =$$

$$= \frac{1}{2}\sqrt{\frac{3}{2}} \ln (5+2\sqrt{6})$$

3

зад. $\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)} = f(x)$

$\frac{|x|}{\sqrt[n]{n}\cdot\sqrt[n]{n+1}}$ ≤ 1 сх. / > 1 разх.

$\frac{1}{n(n+1)} \sim \frac{1}{n^2}$

$\Rightarrow$ в кр. точки е сх.

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ - сходящ $= 1$

$xf(x) = \sum_{n=1}^{\infty} \frac{x^{n+1}}{n(n+1)} = g(x)$

$(xf(x))' = \sum_{n=1}^{\infty} \frac{\cancel{(n+1)}\, x^n}{n\cancel{(n+1)}}$

$(xf(x))'' = \sum_{n=1}^{\infty} \frac{n x^{n-1}}{n} = \sum_{n=1}^{\infty} x^{n-1} = \frac{1}{1-x}$

$(xf(x))' = \int \frac{1}{1-x}\, dx = -\ln(1-x) + C$

~~$xf(x) =$~~

~~x~~$=0$

$\Rightarrow C = 0$

$xf(x) = -\int \ln(1-x)\, dx + C =$

$= - x\ln(1-x)$

$- \int \frac{x}{1-x}\, dx + C =$

$= -x\ln(1-x) + \int \frac{1-x-1}{1-x}\, dx + C =$

$= -x\ln(1-x) + \int 1\, dx - \int \frac{1}{1-x}\, dx + C =$

$= -x\ln(1-x) + x + \ln(1-x) + C =$

$= (1-x)\ln(1-x) + x + C$

$x = 0$

$C = 0$

$\Rightarrow f(x) = \frac{(1-x)}{x}\ln(1-x) + 1$

$\frac{\ln(1-x)}{x}(1-x)$

$x \to 1$

$$\boxed{\begin{gathered} x^2 \ln x \to 0 \\ x \to 0 \end{gathered}}$$

Зад. $1+\sum_{n=1}^{\infty}\frac{1.3.5..(2n-1)}{2.4...2n}x^n$

$f'(x)$

$2f'(x)$

$-2xf'(x)$

$2f'(x)-2xf'(x)=f(x)$

$f(x)=\frac{1}{\sqrt{1-x}}$

Зад. $\sum_{n=1}^{\infty}n(n+1)x^n=f(x)$

$|x|.\sqrt[n]{n}.\sqrt[n]{n+1}$

$R=1$

$\frac{f(x)}{x}=\sum_{n=1}^{\infty}n(n+1)x^{n-1}$ $\quad\int g(x)dx=h(x)$

$\int\frac{f(x)dx}{x}=\sum_{n=1}^{\infty}\frac{\cancel{n}(n+1)x^n}{\cancel{n}}+C$

$\int\int\frac{f(x)}{x}dx=\sum_{n=1}^{\infty}\frac{\cancel{(n+1)}x^{n+1}}{\cancel{n+1}}=\frac{x^2}{1-x}$

$\Rightarrow\int h(x)dx=\frac{x^2}{1-x}+C_1+Cx$

$h(x)=\frac{2x(1-x)+x^2}{(1-x)^2}+C=$

$=\frac{2x-x^2}{(1-x)^2}+C$

$g(x)=\frac{(2-2x)(1-x)^2-(2x-x^2).2(1-x)}{(1-x)^4}$

$f(x)=xg(x)$

Зад. $\sum_{n=1}^{\infty}(-1)^{n-1}.n^2x^n=f(x)$ $\quad\sqrt[n]{n}\to 1$

$\frac{(-1)^n.(n+1)^2x^{n+1}}{}$ $\quad\boxed{R=1}$

$\frac{f(x)}{x}=\sum_{n=1}^{\infty}(-1)^{n-1}n^2x^{n-1}$

$\int\frac{f(x)}{x}=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^{2}.x^n}{\cancel{n}}$

$h(x)=\int g(x)=\sum_{n=1}^{\infty}(-1)^{n-1}nx^n$

$\int\frac{h(x)}{x}dx=\sum_{n=1}^{\infty}(-1)^{n-1}x^{n}=\frac{x}{1+x}$

$$1+\sum_{n=1}^{\infty}\frac{1.3.5\ldots(2n-1)}{2.4\ldots 2n}x^n =$$

$$\left|\frac{u_{n+1}}{u_n}\right| = \frac{\cancel{1.3.5\ldots}(2n+1)|x^{\cancel{n+1}}|}{\cancel{2.4\ldots 2n}(2n+2)}\cdot\frac{\cancel{2.4\ldots 2n}}{\cancel{1.3.5\ldots(2n-1)}\,|\cancel{x^n}|} =$$

$$= \frac{(2n+1)|x|}{(2n+2)} \longrightarrow |x| \qquad |x|<1$$

$$f^*(x) = 1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\ldots$$

$$f'(x) = \sum_{n=1}^{\infty}\frac{1.3.5\ldots(2n-1)n.\,x^{n+1}}{2.4\ldots(2n-2).2n}$$

$$f(x) = 1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\ldots$$

$$f'(x) = \frac{1}{2}+\frac{1.3.\cancel{2}}{\cancel{2}.4.}x+\frac{1.3.5.\cancel{3}.x^2}{2.4.\cancel{6}_2}+\frac{1.\cancel{3}.5.7.\cancel{4}}{2.\cancel{4}.\cancel{6}_2.8.2}x^3+\ldots$$

$$2f'(x) = 1+\frac{1.3}{2}x+\frac{1.3.5.x^2}{2.4}+\frac{1.3.5.7}{2.4.6.8.2}x^3+\ldots$$

$$2xf'(x) = x+\frac{1.3x^2}{2}+\frac{1.3.5}{2.4}x^3+\frac{1.3.5.7}{2.4.6}x^4+\ldots$$

$$2f'(x)-2xf'(x) = 1+\frac{1}{2}x+\frac{1.3x^2}{2.4}+\frac{1.3.5}{2.4.6}x^3+\ldots$$

$$2f'(x) - 2x f'(x) = f(x)$$

$$2f'(x)(1-x) = f(x)$$

$$\frac{2f'(x)}{f(x)} = \frac{1}{1-x}$$

$$2\int \frac{f'(x)dx}{f(x)} - \ln(1-x) + C$$

$$2\ln f(x) = \ln\frac{1}{1-x} + C$$

$$x = 0 \Rightarrow C = 0$$

$$\ln f(x) = \ln\frac{1}{\sqrt{1-x}}$$

$$f(x) = \frac{1}{\sqrt{1-x}}$$

<u>задача</u>

$$f(x) = \sum_{n=1}^{\infty} n(n+1)x^n$$

$$\frac{f(x)}{x} = \sum_{n=1}^{\infty} n(n+1)x^{n-1}$$

$$\int \cancel{n(n+1)x^{n-1}}$$

$$g(x) = \sum_{n=1}^{\infty} n(n+1)x^{n+1}$$

$$\int g(x)dx = \sum_{n=1}^{\infty}\int n(n+1)x^{n-1} = \sum_{n=1}(n+1)x^n + C$$

<u>Нека</u> $\int g(x)dx = h_x$

$$\int h_x(dx) = \sum_{n=1}^{\infty}\int (n+1)x^n = \sum_{n=1}^{\infty} x^{n+1} + C_1 + Cx$$

геометрична прогресия

$$\Rightarrow \sum = \frac{x^2}{1-x}$$

сумата на г.п = първия член x 1-частн.

$$h(x) = \frac{2x(1-x)+x^2}{(1-x)^2} + C = \frac{-x^2+2x}{(1-x)^2} + C$$

$$g(x) = \frac{(-2x+2)(1-x)^2 + 2(-x^2+2x)(1-x)}{(1-x)^4}$$

$$f(x) = x\,g(x)$$

<u>задача</u> $f(x) = \sum\limits_{n=1}^{\infty} (-1)^{n-1} n^2 x^n$

$$\sqrt[n]{n} \to 1 \Rightarrow R = 1$$

когато n е в числителя интегрираме

когато n е в знаменателя се диференцира

$$g(x) = \frac{f(x)}{x} = \sum_{n=1}^{\infty} n^2 x^{n-1}$$

$$h(x) = \int g(x) = \sum_{n=1}^{\infty} n x^n$$

$$\int \frac{h(x)}{x}\,dx = \sum_{n=1}^{\infty} (-1)^{n-1} x^n$$ - геометр. с $q = -x$

при $n = 1$ $a_1 = x$ $\Rightarrow = \dfrac{x}{1+x}$