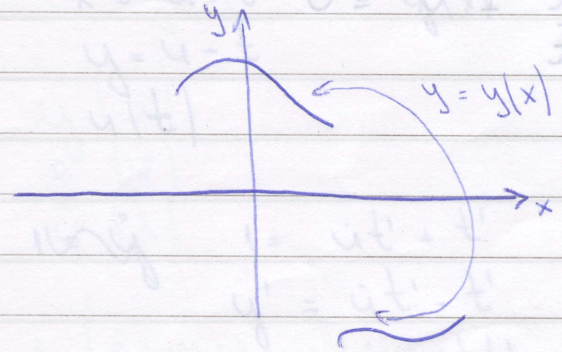
17.04.2013г. ДИС - упражнение

## Смяна на променливите

$F(x, y, u, t) = 0$
$G(x, y, u, t) = 0$ $\Rightarrow$ $x = f(u, t)$, $y = g(u, t)$ $\qquad \underline{y(f(u,t)) = g(u,t)}$




$y = y(x) \qquad u = u(t)$

$x = f(u(t), t)$
$t(x)$

$u(x, y)$
$v(x, y)$
$y(x)$
$t(x)$

задача

$(1+x^2)^2 y'' + 2x(1+x^2)y' + y = 0$

$x = \operatorname{tg} t$

$\left| \begin{array}{l} x - \operatorname{tg} t = 0 \\ y = u \end{array} \right| \quad u(t) \qquad \begin{array}{l} x = \operatorname{tg} t \\ y = u \end{array} \quad t(x)$ замест. и получаваме тъждествата

$x - \operatorname{tg}(t(x)) \equiv 0$

$y(x) \equiv u(t(x))$

$1 - \dfrac{1}{\cos^2 t} t' = 0 \Rightarrow t' = \cos^2 t$

$\boxed{\dot u(t)} = u'(t)$ знак за улеснение

$y' = \dot u t' = \dot u \cos^2 t$

$y'' = (\ddot u \cos^2 t - 2\dot u \cos t \sin t)t' = \ddot u \cos^4 t - 2\dot u \cos^3 t \sin t$

$\dot u(t)$

заместваме с $y$ и $t$ в $(1+x^2)y'' + 2x(1+x^2)y' + y = 0$

$~~1+tg^2 t~~$

$$\frac{1}{\cos^4 t}\left(\ddot{u}\cos^4 t - 2\dot{u}\cos^3 t \sin t\right) + \frac{2\sin t}{\cos t \cdot \cos^2 t}\cdot \dot{u}\cos^2 t + u = 0$$

$$\ddot{u}\,\cancel{\cos^4 t} - 2\dot{u}\,\cancel{\cos}\frac{\sin t}{\cos t} + \frac{2\dot{u}\sin t}{\cos t} + u = 0$$

$$\cancel{\ddot{u}\cos^2 t - \dot{u}\sin t + \dot{u}\frac{2\sin t}{\cos t} = 0}$$

$$\ddot{u} - 2\dot{u}\frac{\sin t}{\cos t} + 2\dot{u}\frac{\sin t}{\cos t} + u = 0$$

$$\ddot{u} + u = 0$$

задача

$$y'' + (e^x - x)y'^3 = 0$$

смяна на променливите, като и вземе независимата променлива за зависимата и обратно

$x = u$

$y = t$

$y(x) \quad u(t) \qquad x = u(t) \qquad t(x)$

$x = u(t(x)) \xrightarrow{\text{по } x} 1 = \dot{u}t'$

$y = t(x) \qquad y' = t' = \frac{1}{\dot{u}(t)}$

$$y'' = -\frac{1}{\dot{u}^2}\cdot t' = -\frac{1}{\dot{u}^2}\cdot\frac{1}{\dot{u}} = -\frac{1}{\dot{u}^3} = -\frac{\ddot{u}}{\dot{u}^2}t' = -\frac{\ddot{u}}{\dot{u}^3}$$

$$-\frac{\ddot{u}}{\dot{u}^3} + (e^u - u)\cdot\frac{1}{\dot{u}^3} = 0$$

$$-\ddot{u} + e^u - u = 0$$

~~интеграл~~

задача

$$y' \cdot y''' - 3y''^2 = 0$$

$x = u$

$y = t$

... от миналата задача

$$y''' = -\frac{(\dddot{u}\dot{u}^3 - \ddot{u}.3\dot{u}^2\ddot{u})}{\dot{u}^6} . t' = -\frac{\dddot{u}\dot{u}^3 - \ddot{u}3\dot{u}^2\ddot{u}}{\dot{u}^6} \cdot \frac{1}{\dot{u}} =$$

$$\frac{1}{\dot{u}} \cdot \frac{3\ddot{u}^2 - \dot{u}.\dddot{u}}{\dot{u}^5} - 3\frac{\ddot{u}^2}{\dot{u}^6} = 0$$

$\dot{u}.\dddot{u} = 0 \Rightarrow$ едно от двете $= 0$ (') $\dot{u} = 0$

$\Rightarrow u$ - const

('') $\dddot{u} = 0$

$\Rightarrow \ddot{u}$ - const

$\Rightarrow u = At^2 + Bt + C$

$u = At^2 + Bt + C$

$$x = Ay^2 + By + C$$

$y(x)$

задача

$y'' + (x+y)(1+y')^3 = 0$

$x = u + t \qquad y(x) \qquad t(x)$

$y = u - t$

$y\ u(t)$

$1 \neq \dot{y} \qquad 1 = \dot{u}t' + t' \qquad 1 = t'(\dot{u}+1) \qquad t' = \frac{1}{\dot{u}+1}$

$y' = \dot{u}t' - t'$

$y' = (\dot{u}-1)\,t' =$

$= \frac{(\dot{u}-1)}{\dot{u}+1}$

$y'' = \frac{\ddot{u}(\dot{u}+1) - (\dot{u}-1)\ddot{u}}{(\dot{u}+1)^2} \cdot t' = \frac{1}{\dot{u}+1}$

$y'' = \frac{\ddot{u}(\dot{u}+1 - \dot{u}+1)}{(\dot{u}+1)^2(\dot{u}+1)} = \frac{2\ddot{u}}{(\dot{u}+1)^3}$

$\frac{2\ddot{u}}{(\dot{u}+1)^3} + 2u\left(1 + \frac{\dot{u}-1}{\dot{u}+1}\right)^3 = 0$

$\frac{2\ddot{u}}{(\dot{u}+1)^3} + 2u\left(\frac{\dot{u}+1+\dot{u}-1}{\dot{u}+1}\right)^3 = 0$

$\frac{2\ddot{u}}{(\dot{u}+1)^3} + \frac{2u \cdot (2\dot{u})^3}{(\dot{u}+1)^3} = 0$

$2\ddot{u} + 2u \cdot 8\dot{u}^3 = 0$

$2\ddot{u} + 16u\dot{u}^3 = 0$

$\ddot{u} + 8u\dot{u}^3 = 0$

$\lambda \; x$

$\rho \; \varphi \; \rho \; \varphi \; \rho \; \varphi$

задача

$$y' = \frac{x+y}{x-y}$$

$x = \rho \cos\varphi \qquad \rho(\varphi)$

$y = \rho \sin\varphi$

$y(x)$

$\rho(\varphi)$

$\varphi(\lambda)$

$1 = (\dot\rho \cos\varphi - \varphi \sin\varphi)\varphi'$

$y' = (\dot\rho \sin\varphi + \varphi\cos\varphi)\varphi'$

$$y' = \frac{\dot\rho \sin\varphi + \rho\cos\varphi}{\dot\rho\cos\varphi - \rho\sin\varphi} = -\frac{\cos\varphi + \sin\varphi}{\cos\varphi - \sin\varphi}.$$

$\cancel{\dot\rho \sin\varphi\cos\varphi - \rho\sin^2\varphi + \rho\cos^2\varphi \ldots - \rho\cos\varphi\sin\varphi - \dot\rho\cos\varphi}$

$$-\frac{\cos\varphi + \sin\varphi}{\cos\varphi - \sin\varphi} = \frac{\rho\cos\varphi + \rho\sin\varphi}{\rho\cos\varphi - \rho\sin\varphi}$$

$-(\cos\varphi + \sin\varphi)(\rho\cos\varphi - \rho\sin\varphi) = (\rho\cos\varphi + \rho\sin\varphi)(\cos\varphi - \sin\varphi)$

в пространството

$$F(x, y, z, u, v, w) = 0 \qquad\qquad x = f(u, v, w)$$
$$G(x, y, z, u, v, w) = 0 \quad \Leftrightarrow \quad y = g(u, v, w)$$
$$H(x, y, z, u, v, w) = 0 \qquad\qquad z = h(u, v, w)$$

$$x = f(u, v, w(u, v))$$
$$y = g(u, v, w(u, v))$$
$$u(x, y)$$
$$v(x, y)$$

$$F(x, y, z(x, y), u(x, y), v(x, y), w(u(x, y), v(x, y))) \equiv 0$$

$$x = f(u, v)$$
$$y = g(u, v)$$
$$z(x, y) \qquad z = w$$

<u>задача</u>

$$y z'_x - x z'_y = 0 \quad (1)$$
$$u = x \qquad\qquad z(x, y)$$
$$v = x^2 + y^2 \qquad\qquad w = z$$
$$z(x, y) = w(u(x, y), v(x, y))$$

$$z'_x = w'_u \cdot u'_x + w'_v \cdot v'_x = w'_u \cdot 1 + 2 w'_v x \quad (2)$$
$$\left.\begin{array}{l} u'_x = 1 \\ v'_x = 2x \end{array}\right\} \nearrow$$

заместване $(2) + (3) \to (1)$

$$z'_y = w'_v \cdot 2y \quad (3)$$
$$u'_y = 0$$
$$v'_y = 2y$$

$$y(w'_u + 2 w'_v x) - x(w'_v \cdot 2y) = 0$$
$$y \cdot w'_u = 0$$
$$w'_u = 0$$

~ ДИС ~ 18.04.13г.

Зад.1 $\sum_{n=1}^{\infty} \underbrace{\frac{(3n)!}{27^n (n!)^3}}_{a_n}$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(3n+3)!}{27^{n+1}((n+1)!)^3}}{\frac{(3n)!}{27^n (n!)^3}} = \frac{(3n+3)!\,27^n}{(3n)!\,(27)^{n+1}}\left(\frac{n!}{(n+1)!}\right)^3 =$$

$(3n)!(3n+3)(3n+2)(3n+1)$

$$= \frac{1}{\cancel{27}\,9}\,\frac{(3n+1)(3n+2)\cancel{(3n+3)}}{(n+1)^3} = \frac{1}{9}\,\frac{(3n+1)(3n+2)}{(n+1)^2} \xrightarrow[n\to\infty]{} 1$$

$\Rightarrow$ Даламбер не дава отговор

$$n\left(\frac{a_n}{a_{n+1}} - 1\right) = n\left(\frac{9n^2+18n+9}{9n^2+9n+2} - 1\right) = n\left(\frac{9n^2+18n+9-9n^2-9n-2}{9n^2+9n+2}\right) =$$

$$= \frac{9n^2 + 7n}{9n^2+9n+2} \xrightarrow[n\to\infty]{} 1 \Rightarrow \text{Дюамел} \to \text{не}$$

$\Rightarrow$ с критерий на Гаус:

## Критерий на Гаус

Нека $\frac{a_n}{a_{n+1}} = \alpha + \frac{\beta}{n} + \frac{\gamma_n}{n^{1+\varepsilon}}$, където $|\gamma_n| \le c$

Тогава: 1) ако $\alpha > 1$ редът е сходящ; $\alpha < 1$ - разход.

2) $\alpha = 1$, $\beta > 1$ - сходящ; $\alpha = 1$, $\beta < 1$ - разход.

3) $\alpha = 1$, $\beta = 1$ - разходящ

Д-во:

1. $\alpha > 1$ $\quad \frac{a_{n+1}}{a_n} \to \frac{1}{\alpha} < 1 \Rightarrow$ сх. по Даламбер

$\alpha < 1$ $\quad \frac{1}{\alpha} > 1$ - рах. по Даламбер

2\. $n\left(\frac{a_n}{a_{n+1}}-1\right)=\beta+\frac{\gamma_n}{n^{\varepsilon}} \Rightarrow \beta:$ $\beta>1$ – сх. по Р-Дюамел

$\beta<1$ – разх. по Р-Д.

~*~*~*~

3\. <u>Критерий на Кумер</u> → не влиза в изпита!

Нека $c_n>0$ и $\sum\limits_{n\geq 1}^{\infty}\frac{1}{c_n}$ е разходящ. Образ. $K_n=c_n\frac{a_n}{a_{n+1}}-c_{n+1}$

Ако $\exists\ \delta>0$: $K_n\geq\delta$, редът е сход. Ако $K_n\leq 0$, редът е разход.

Ще док. частта за разходимост:

$$K_n\leq 0$$

$$c_n\cdot\frac{a_n}{a_{n+1}}\leq c_{n+1}$$

$$\frac{a_{n+1}}{a_n}\geq\frac{c_n}{c_{n+1}}$$

$$\frac{a_{n+1}}{a_1}=\frac{a_{n+1}}{a_n}\cdot\frac{a_n}{a_{n-1}}\cdots\frac{a_2}{a_1}\geq\frac{c_n}{c_{n+1}}\cdot\frac{c_{n-1}}{c_n}\cdots\frac{c_1}{c_2}=\frac{c_1}{c_{n+1}}$$

$$a_{n+1}\geq\frac{a_1.c_1}{c_{n+1}}=a_1.c_1.\frac{1}{c_{n+1}}$$

$\boxed{a_n\geq a_1.c_1.\frac{1}{c_n}}$ ⇒ редът със сума $a_n$ е разход., защото $\sum\frac{1}{c_n}$ е разход.

Ако $c_n=1$ → $\frac{a_n}{a_{n+1}}-1\geq\delta$

$\frac{a_{n+1}}{a_n}\leq\frac{1}{1+\delta}$ → крит. на Даламбер

$c_n=n$ → Раабе-Дюамел

$c_n=\frac{1}{n\ln n}$ → $\sum\limits_{n=1}^{\infty}\frac{1}{n\ln n}$ – разх.

## Критерий на Бертран

за разходимост:

$$K_n = \frac{1}{n\ln n} \cdot \frac{a_n}{a_{n+1}} - \frac{1}{(n+1)\ln(n+1)}$$

$$B_n = \ln n \cdot \left[n\left(\frac{a_n}{a_{n+1}} - 1\right) - 1\right] \longrightarrow 0$$

$$K_n - n.\ln.n + \ln n.n + \ln n - \ln n = \ln n\left[n\left(\frac{a_n}{a_{n+1}} - 1\right) \pm \frac{1}{n}\right] -$$

$$-(n+1)\left[\ln(n+1) - \ln n\right] =$$

$$= B_n - (n+1)\ln\left(1 + \frac{1}{n}\right) = B_n - \ln\left(1 + \frac{1}{n}\right)^{n+1} \longrightarrow B_n - 1$$

$K_n = B_n - 1$

$K_n \leq 0$, $B_n \leq 1$ - разход.

$B_n \geq 1 + \delta$ - сход.

$$B_n = \ln n\left[n\left(\frac{a_n}{a_{n+1}} - 1\right) - 1\right] \to 0$$

Ако $B_n \geq 1 + \delta$ - сход

$B_n \leq 1$ - разход

(3) $$\frac{B_n}{n\ln n} = \frac{a_n}{a_{n+1}} - 1 - \frac{1}{n} = \frac{\gamma^n}{n^{1+\varepsilon}} \longrightarrow$$

$$B_n = \frac{\ln n}{n.\varepsilon} \cdot \gamma^n \longrightarrow 0 \quad \Rightarrow B_n \to 0$$

огран.

$B_n \leq 1$, т.е. редът е разходящ

! за изпит - само формулировка на крит. на Гаус продължение на задачата:

$$\frac{9n^2-7n}{9n^2+9n+2} \xrightarrow[n\to\infty]{} 1$$

~ * ~ * ~ * ~

3. $$\cancel{\frac{a_n}{a_{n+1}} = 1+\frac{1}{n}+\frac{9n-1}{9n^2+9n+2} = 1+\frac{1}{n}+\frac{\overbrace{9n-1+7}^{3(3n+2)}-7}{(3n+1)(3n+2)} = 1+\frac{1}{n}+\frac{3}{3n+1}+\frac{7}{(3n+1)(3n+2)}}$$

$$\cancel{= 1+\frac{1}{n}+\frac{3}{3n+1}+\text{ч. (малко е)} =}$$

$$\cancel{= 1+\frac{1}{n}+\frac{1}{n}\cdot\frac{3n}{3n+1}+\dots}$$

$$\frac{a_n}{a_{n+1}} = 1+\frac{1}{n}+\frac{9n+7}{(3n+1)(3n+2)} = 1+\frac{9n+6}{(3n+1)(3n+2)}+\underbrace{\frac{1}{(3n+1)(3n+2)}}_{\text{малко число}} =$$

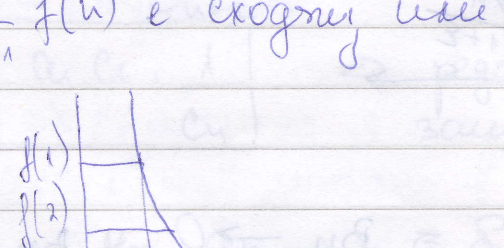
$$= 1+\frac{3}{3n+1}+\text{ч.} = 1+\frac{3}{3n+1}-\frac{1}{n}+\frac{1}{n}+\text{ч.} = 1+\frac{1}{n}+\frac{3n-3n-1}{n(3n+1)}+\text{ч.} \quad -1+\frac{1}{n}+\frac{C}{n^2}$$

разход. по Гаус

## Интегрален критерий на Коши (К!)

**Th** Нека $f(x)$ е неотриц., монот. намаляваща $[1,\infty)$. Тогава редът $\sum_{n=1}^{\infty} f(n)$ е сходящ или разход. едновр. с $\int_1^{\infty} f(x)\,dx$.

Д-во:




$$f(n+1)(n+1-n) \le \int_n^{n+1} f(x)\,dx \le f(n)(n+1-n)$$

сум. по $n = \overline{1, N-1}$

$$\sum_{n=1}^{N-1} f(n+1) \le \int_1^N f(x)\,dx \le \sum_{n=1}^{N-1} f(n) = f(1)+f(2)+\dots+f(N-1)$$

$$f(2)+f(3)+\dots+f(N) = S_N - f(1) \qquad f(1)+f(2)+\dots+f(N-1) = S_{N-1}$$

$$\boxed{S_N - f(1) \le \int_1^N f(x)\,dx \le S_{N-1}}$$

$\Rightarrow$ Нека е сх. редът $\Rightarrow$ Парциалните суми са огр. с $L$ $\Rightarrow$ $\int_1^N f(x)\,dx \le L$. Означ. с $F(A) = \int_1^A f(x)\,dx$

$\Rightarrow$ $F(A) \le F(N) \le L$ $\Rightarrow$ ф-ията $F(A)$ е монот. растяща и огран. отгоре $\Rightarrow$ $\exists \lim\limits_{A\to\infty} F(A)$, т.е. $\int_1^\infty f(x)\,dx$ е сходящ.

$\Leftarrow$ Нека $\int_1^\infty f(x)\,dx$ е сходящ. $\Rightarrow$ $\int_1^N f(x)\,dx$ е огран. $\Rightarrow$ $S_N \le f(1) + \int_1^N f(x)\,dx$ е огран. $\Rightarrow$ редът е сходящ

Приложения на Th-та:

1. Обобщен хармоничен ред

$$\sum_{n=1}^{\infty} \frac{1}{n^s} \qquad f(n)$$

$$f(x) = \frac{1}{x^s} \qquad \int_1^\infty \frac{dx}{x^s}$$

$s > 1$ – сх.

$s \le 1$ – разх.

• $c_n = \frac{1}{n \cdot \ln n}$, $\sum c_n$ е разх.

$$\int_2^\infty \frac{dx}{x \cdot \ln x} = \int_2^\infty \frac{d(\ln x)}{\ln x}$$

$$\int_2^A \frac{d(\ln x)}{\ln x} = \ln|\ln(x)|\Big|_2^A = \ln(\ln A) - \ln(\ln 2) \xrightarrow[A\to\infty]{} \infty$$

разх.

• $\sum_{n=2}^\infty \frac{1}{n \cdot \ln^p n}$

$$\int_2^\infty \frac{dx}{x \ln^p x} \qquad \int_2^A \frac{d \ln x}{\ln^p x}, \quad p \neq 1 =$$

$$= \int_2^A \frac{(\ln x)^{1-p}}{1-p}\Bigg|_2^A = \frac{(\ln A)^{1-p}}{1-p} - C$$

за $p > 1$, $\infty^{\text{отр. степен}} \to$ сх.

$p < 1$, $\infty^{\text{полож. степ.}} \to$ разх.

Валидно е,

Ако $f(x) = \frac{1}{x \ln^p x}$ е мон. намал.

$$f'(x) = \frac{-(\ln^p x + x \cdot p-1 \cdot \ln^{p-1} x \cdot \frac{1}{x})}{x^2 \ln^{2p} x} = \frac{-\ln^{p-1} x (\ln x + p)}{(x)^2}$$

✓

(*) $\sum a_n$ е сх. $\Leftrightarrow \sum_{n=1}^\infty 2^n a_{2^n}$ — гр. крит. на Коши

ДИС 24.04.2013 г. упражнение;

Домашно

1зад.

$z_x z''_{xy} - z'_y z''_{xx} = 0$

$y, z$ - независими

да се направи смяна на променливите

$x = w$

$y = u \qquad w(u,v)$

$z = v$

2зад. $\alpha = ?$ $f(x,y) = \begin{cases} (x^2+y^2)^\alpha \sin\frac{1}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0 & x^2+y^2 = 0 \end{cases}$

а е непрекъсната и в е диференцуема в $(0,0)$

( 2 случая $\alpha = 1$ $\alpha = \frac{1}{2}$ )

3зад. Нека $\varphi$ е диферер. функция на две променливи

$f(x,y,z) = \varphi(xy, \frac{y}{z}) \Rightarrow -x f'_x + y f'_y + z f'_z = 0$

4зад. $z = x^y$

Да се док. че $\frac{x}{y} \cdot z'_x + \frac{1}{\ln x} \cdot z'_y = 2z$

5зад. диф. на неявни функции

$f(x,y,u,v) = 0$

$g(x,y,u,v) = 0$

$u(x,y)$

$v(x,y) \qquad u'_x \quad v'_y$

24. 04. 2013г. ДИС - упражнение

I тип задачи

$y = f(x)$

1) лок. екстремум

2) абс. екстремум

$f' = 0$ 20 $x_0, x_1, \ldots$ вътрешните точки,

$f'$ не съществува;

$f'$ да си смени знака

$f''$

II тип задачи

$f(x_0), f(x_1) \ldots$

конт.

$F(x, y)$

$F'_x = 0$ - вътр. точки

$F'_y = 0$

M

Правилото на Силвестър

$$\Delta \begin{vmatrix} F''_{xx} & F''_{xy} \\ F''_{yx} & F''_{yy} \end{vmatrix}$$

$F''_{xx} > 0$ $\Delta > 0$ - имаме лок. min

$F''_{xx} < 0$ $\Delta > 0$ лок max

$\Delta < 0$ - нямаме екстр.

(седловидна точка)

$\Delta = 0$

1 зад.

$$f(x,y) = \int_{5x}^{5x+3y} (4t^2-3)\,dt$$

$\rightsquigarrow$ от Лайбниц - Нютон

$$\left( \int_a^x f(t)\,dt \right)' = f(x)$$

$$\left( \int_a^{\varphi(x)} f(t)\,dt \right)' = f(\varphi(x))\,\varphi'(x)$$

$$\left( \int_{\psi(x)}^{\varphi(x)} f \right) = f(\varphi)\varphi' - f(\psi)\psi'$$

$\rightsquigarrow$

$$f(x,y) = \int_{5x}^{5x+3y} (4t^2-3)\,dt$$

$$f'_x = \overbrace{(4(5x+3y)^2-3)}^{=0} \cdot 5 - (4(5x)^2-3)5 = 0$$

$$f'_y = \underbrace{(4(5x+3y)^2-3)}_{=0} 3 = 0$$

$$(5x+3y)^2 = \frac{3}{4}$$

$$5x+3y = \pm\frac{\sqrt{3}}{2}$$

1сл.) $5x = \frac{\sqrt{3}}{2}$ $\quad 3y = \pm\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

2сл.) $5x = -\frac{\sqrt{3}}{2}$ $\quad 3y = \pm\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

$$25x^2 - 3 = 0$$

$$25x^2 = 3$$

$$x^2 = \frac{3}{25}$$

$x_1 = \frac{\sqrt{3}}{10} \Rightarrow y = 0$, $y = -\sqrt{3}$

$x_2 = -\frac{\sqrt{3}}{10} \Rightarrow y = 0$, $y = \sqrt{3}$

$$(5x)^2 = \frac{3}{4}$$

$$x^2 = \pm\frac{\sqrt{3}}{10}$$

отговор:

$\left(\frac{\sqrt{3}}{10}; 0\right)$

$\left(\frac{\sqrt{3}}{10}; -\sqrt{3}\right)$

$\left(-\frac{\sqrt{3}}{10}; 0\right)$

$\left(-\frac{\sqrt{3}}{10}; \sqrt{3}\right)$

$$\begin{vmatrix} z''_{xx} & z''_{xy} \\ z''_{yx} & z''_{yy} \end{vmatrix} \qquad \Delta = z''_{xx} z''_{yy} - z''^{2}_{xy}$$

$f''_{xx} =$

$f'_x = 20(5x+3y)^2 - 20(5x)^2 = 0$

$f'_y = 12(5x+3y)^2 - 9 = 0$

$f''_{xx} = 40(5x+3y).5 - 40.5 = 0$

$f''_{xy} = 40(5x+3y).3$

$f''_{yy} = 24(5x+3y)3$

$f''_{xx}(p_1) = 0$

$f''_{xy}(p_1) = 40\sqrt{3}$

$\Delta(p_1) < 0 \Rightarrow$ седловидна точка

$\Delta(p_2)$

$\Delta(p_3) < 0$ 〃

$p_1\left(\frac{\sqrt{3}}{10}; 0\right)$

$p_2\left(\frac{\sqrt{3}}{10}; -\sqrt{3}\right)$

$p_3\left(-\frac{\sqrt{3}}{10}; 0\right)$

$p_4\left(-\frac{\sqrt{3}}{10}; \sqrt{3}\right)$

$\Delta = z''_{xx} z''_{yy} - z''^{2}_{xy}$

$f''_{xx}(p_2) = -600\sqrt{3}$

$f''_{xy}(p_2) = 120\sqrt{3}\left(\frac{1}{2} - 3\right) = 300\sqrt{3}$

$f''_{yy}(p_2) = -\frac{24}{40} \cdot 300\sqrt{3} = -180$

$600.180.3 - 300^2.3 = 3.300(2.180 - 300) =$

$= 900(360 - 300) = 54000 (+)$

$\Rightarrow P_3 - max$

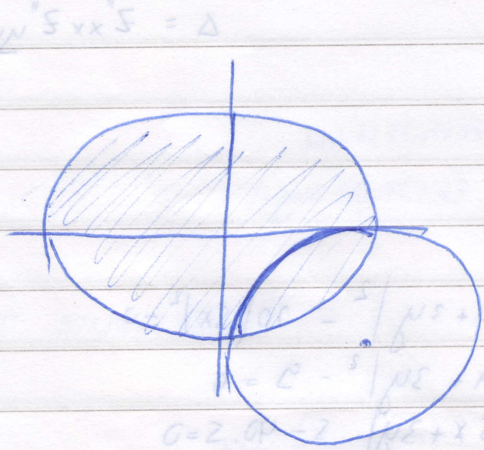
задача

$f(x,y) = xy$

$x^2 + y^2 \le 1$

$x^2 - 2x + y^2 + 2y + 1 \ge 0 \quad |+1$

$(x-1)^2 + (y+1)^2 \ge 1$



$f'_x = y$

$f'_y = x \qquad т. A(0,0)$

$fA(0,0) = 0$

$f\,B\left(\frac{1}{\sqrt2};\frac{1}{\sqrt2}\right) = \frac{1}{2}$

$f(C)\left(-\frac{1}{\sqrt2};\frac{1}{\sqrt2}\right) = -\frac{1}{2}$

$x = \cos\varphi \qquad \varphi \in \left[0;\frac{3\pi}{2}\right]$

$y = \sin\varphi$

$h(\varphi) = \sin\varphi\cos\varphi = \frac{1}{2}\sin 2\varphi$

$2\varphi \in [0; 3\pi]$

тB

$x = 1 + \cos\varphi$

$y = -1 + \sin\varphi$

$\varphi \in \left[\frac{\pi}{2};\pi\right]$

$g(\varphi) = xy = -1 + \sin\varphi - \cos\varphi + \sin\varphi\cos\varphi$

$(\sin\varphi - \cos\varphi)^2 = 1 - \sin 2\varphi$

$\overset{\shortparallel}{t}$

$t - 1 = -\sin 2\varphi$

$= -1 + t + \frac{1-t^2}{2} = \frac{-2+2t+1-t^2}{2} = -\frac{t^2}{2} + t - \frac{1}{2}$