30.05.2013г. ДИС - упражнения

Тейлър

$$f(x) = f(a) + \frac{x-a}{1} f'(a) + \frac{(x-a)}{2!} f''(a) + \dots + f^{(n)}(a) \frac{(x-a)^n}{n!} + \dots$$

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases} \qquad f^{(n)}(x) = \begin{cases} P\left(\frac{1}{x}\right) e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$R_n(x) \xrightarrow[n \to \infty]{} 0$$

Степенни редове

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots \qquad R = \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + \dots \qquad R = \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots \qquad R = \infty$$

$$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots \qquad R = 1$$

$$(1+x)^\alpha = \binom{\alpha}{0} + \binom{\alpha}{1} x + \binom{\alpha}{2} x^2 + \dots + \binom{\alpha}{n} x^n + \dots \qquad R = x$$

$$(x-a)^n$$

$$\frac{a_0}{2} + \sum a_n \cos nx + b_n \sin nx$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 - x^2 + x^4 - \ldots (-1)^n x^{2n} + \ldots$$

$$\int (\operatorname{arctg}' x)\,dx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

$-1 \le x \le 1$ сходящ в затв. интервал.

при $x = 1$ $\quad 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

$x = -1$ $\quad -1\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\right)$

от сти формули

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} (-x^2)^n = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} x^{2n}$$

$$\arcsin x = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} (-1)^n \frac{x^{2n+1}}{2n+1} + C$$

при $x = 0$ $\quad \arcsin 0 = 0$

$\Rightarrow C = 0$

$-1 < x < 1$

$$-\binom{-\frac{1}{2}}{1} \frac{x^3}{3} + \binom{-\frac{1}{2}}{2} \frac{x^5}{5} - \binom{-\frac{1}{2}}{3} \frac{x^7}{7}$$

$$\sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} (-1)^n \frac{2^{2n+1}}{2n+1} + C$$ (дали е сх или рх) за упражнение

$\rightsquigarrow$

$$(\ln(1+x))' = \frac{1}{1+x} = 1 - x + x^2 - x^3$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + C$$

задача

Да се развие в ред функцията $\left(\frac{x^4}{4}-\frac{1}{4}\right)\operatorname{arctg}x$

$$f(x) = \frac{x^4-1}{4}\operatorname{arctg}x - \frac{x^3}{12} + \frac{x}{4}$$

$$f'(x) = \left(\frac{\cancel{4}x^3\cdot 4}{\cancel{16}}\operatorname{arctg}x + \left(\cancel{\frac{x^4}{4}}-\frac{1}{4}\right)\frac{1}{1+x^2} - \frac{\cancel{3x^2\cdot 12}}{\cancel{4\cdot 12^2}} + \frac{\cancel{4}}{4^2} =\right)$$

има грешка

$$= x^3\operatorname{arctg}x = x^3\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+4}}{2n+1}$$

$$f(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+5}}{(2n+1)(2n+5)} + C$$

$x = 0 \qquad C = 0$

задача

$$\frac{1}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2} \qquad A = -1, \quad B = 1$$

$$Ax - 2A + Bx - B = 1$$

$$\cancel{x(A+B)}$$

$$A(x-2) + B(x-1) = 1$$

$$A = \frac{1 - B(x-1)}{x-2}$$

$B =$

$$\frac{1}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2} = -\frac{1}{x-1} + \frac{1}{x-2} = \frac{1}{1-x} - \frac{1}{2}\cdot\frac{1}{1-\frac{x}{2}} =$$

$$= 1 + x + x^2 + \ldots + x^n + \ldots \; \frac{1}{2}\left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{2^3} + \ldots\right) =$$

мий

$$= \frac{2^{2n+1}-1}{2^{2n+1}}\, x^n$$

зад.

$$\frac{1}{1+x+x^2}\cdot\frac{(1-x)}{(1-x)}=\frac{1-x}{1-x^3}=1+x+x^2+\ldots+x^n+\ldots$$