6.06.2013 ДИС-упражнение II - подготовка за контролно

1 неявни функции

$$z = x f\left(\frac{z}{y}\right)$$

$$x z'_x + y z'_y = z$$

$$F(x, y, z) = z - x f\left(\frac{z}{y}\right) = 0$$

$$\begin{cases} F'_z = 0 \\ F = 0 \end{cases}$$

$$\begin{cases} 1 - x \cdot f' \cdot \frac{1}{y} \neq 0 \\ z - x f\left(\frac{z}{y}\right) = 0 \end{cases}$$ $z'_x : f'$

f' - непр. производна

$$\begin{cases} z'_x = f\left(\frac{y}{z}\right) + x f' \frac{z'_x}{y} \\ z'_x \left(1 - \frac{x}{y} f'\right) = f\left(\frac{z}{y}\right) \end{cases}$$

$\cdot$ * условието за $\exists$ неявна

$\cdot\, x$ и за е диф трябва да $\equiv$

$$\begin{cases} z'_y = x f' \frac{z'_y y - z}{y^2} \\ \left(1 - \frac{x}{y} f'\right) z'_y = -\frac{xz}{y^2} f' \quad \cdot\, y \end{cases}$$

$$a\left(x z'_x + y z'_y\right) = x f\left(\frac{z}{y}\right) - \frac{x \cdot z}{y} f' = z\left(1 - \frac{x}{y} f'\right) = a$$

по усл $a \neq 0$

$\cancel{a(xz'_x+yz'_y)=xf\left(\frac{z}{y}\right)}$ $\quad f'\left(\frac{z}{y}\right)$ - сложна f

$$z''_{xy} = z''_{xy}a + z'_x\left(-x\,\frac{f''\frac{z'_y y - z}{y^2}\,y - f'}{y^2}\right) = f'\,\frac{z'_y \cdot y - z}{y^2}$$

$$\begin{cases} x = u+v \\ y = u^2+v^2 \\ z = u^3+v^3 \end{cases} \qquad z'_x \qquad \begin{matrix} u(x,y) \\ v(x,y) \end{matrix}$$

$$\begin{cases} z'_x = 3u^2u'_x + 3v^2v'_x \\ 1 = u'_x + v'_x \\ 0 = 2uu'_x + 2vv'_x \end{cases} \qquad u'_x = 1 - v'_x$$

$$u'_x = \frac{\begin{vmatrix} 1 & 1 \\ 0 & 2v \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2u & 2v \end{vmatrix}} = \frac{2v}{2v-2u} = \frac{v}{v-u}$$

$$F(x,y,u,v) = u+v-x = v$$
$$F(x,y,u,v) = u^2+v^2-y = u$$

$$\begin{vmatrix} F'_u & F'_v \\ G'_u & G'_v \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & 1 \\ 2u & 2v \end{vmatrix} = 2v - 2u \neq 0$$

$$v'_x = \frac{\begin{vmatrix} 1 & 1 \\ 2u & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2u & 2v \end{vmatrix}} = \frac{-2u}{2v-2u} = \frac{u}{u-v}$$

$\Rightarrow z'_x =$ заместване

II) Смяна на променливите $y(x)$

$$(1+x^2)^2 y'' + 2x(1+x^2)y' + y = 0$$

$x = f(u,v)$

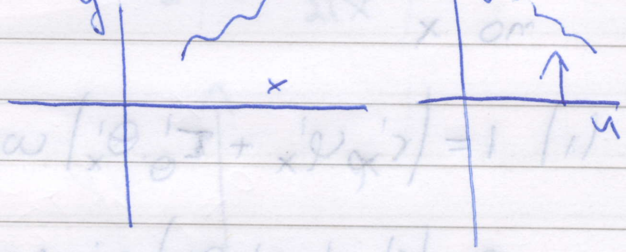
$y = g(u,v)$

(1) $x = tg\, t$ — старата $y(x)$

$y = u$ — $u(t) \leftarrow$ новата

$y(x) = u(arctg\, x)$ в (1) $t = t(x)$

$y(x) = u(t(x))$



сменяме едната равнина с другата

$y'(x) = \dot{u} = u'_t$

$y'(x) = u'_t t'$

(1)' $1 = \frac{1}{\cos^2 t} t'$ $\quad t' = \cos^2 t$

$y'(x) = \overset{\dot u}{u'_t} \cos^2 t$

$y''(x) = (u''_t \cos^2 t + u'_t 2\cos t \sin t) t'$ — получено

$y''(x) = u''_t \cos t - u'_t \cos t \sin t$

$$\frac{1}{\cos^2 t}(\ddot{u}\cos^4 t - 2\dot{u}\cos^3 t \sin t) + \frac{2\sin t}{\cos^2 t}(\dot{u}\cos^2 t) + u = 0$$

$\ddot{u} + u = 0 \qquad u(t) = A\cos t + B\sin t$

$y(x) = A\cos(arctg\, x) + B\sin(arctg\, x)$

<u>задача</u>

$y z'_x - x z'_y = 0$ в сферични координати

$z(x,y)$

(1) $x = r\cos\varphi\sin\theta$

(2) $y = r\sin\varphi\sin\theta$

$z = r\cos\theta$

$(x,y,z) \to (z,\varphi,\theta)$, нека $r(\varphi,\theta)$

$x = (r(\varphi,\theta) \ldots$ | $\varphi(x,y)$

$y = \{r\ \varphi\ \theta \ldots$ | $\theta(x,y)$

по x

(1) $1 = (r'_\varphi \varphi'_x + r'_\theta \theta'_x)\cos\varphi\sin\theta - r\sin\varphi\,\varphi'_x\sin\theta + r\cos\varphi\cos\theta\,\theta'_x$

$0 = (r'_\varphi\varphi'_x + r'_\theta\theta'_x)\sin\varphi\sin\theta + r\cos\varphi\,\varphi'_x\sin\theta + r\sin\varphi\cos\theta\,\theta'_x$

$z'_x = (r'_\varphi\varphi'_x + r'_\theta\theta'_x)\cos\theta - r\sin\theta\cdot\theta'_x$

<u>задача</u> $z'_x z''_{yy} - z'_y z''_{xy} = 0$

независими y, z

$x = w$

$y = u$

$z = v$

$w(u,v) \to u(x,y) \to v(x,y)$

III Локални екстремуми

$z'_x = 0$ точките, които удовлетв. тази система
$z'_y = 0$ са стационарни

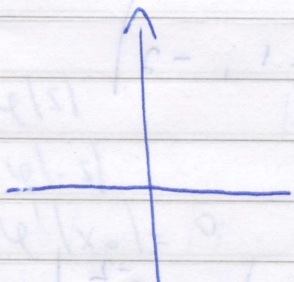
$$\begin{vmatrix} z''_{xx} & z''_{xy} \\ z''_{yx} & z''_{yy} \end{vmatrix}$$ минори

$\Delta_2 = z''_{xx} z''_{yy} - z''_{xy}{}^2$

$\Delta_2 = 0$ $\qquad$ $\text{II} = 0$ допълнителни

$\sqrt{x}$ $\qquad$ $x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

задача $\quad z(x,y) = \frac{e^y}{\sqrt{x}}(1 - x^2 - y^2)$



$$z'_x = -\frac{e^y}{x}\frac{1}{2\sqrt{x}}(1 - x^2 - y^2) + \frac{e^y}{\sqrt{x}}(-2x) =$$

$$= -\frac{e^y}{2x\sqrt{x}}(1 - x^2 - y^2) + \frac{e^y}{\sqrt{x}} 2x =$$

$$= -\frac{e^y}{\sqrt{x}}\left(\frac{1 - x^2 - y^2}{2x} + 2x\right) = -\frac{e^y}{2x\sqrt{x}}(1 - x^2 - y^2 + 4x^2) =$$

$$= -\frac{e^y}{2x\sqrt{x}}(3x^2 - y^2 + 1)$$

$$z'_y = e^y\sqrt{x}(1 - x^2 - y^2) - \frac{e^y}{\sqrt{x}}(-2y) = e^y\left(-\sqrt{x} - x^2\sqrt{x} - y^2\sqrt{x} + \frac{2y}{\sqrt{x}}\right) =$$

$$= e^y(-x - x^4 - y^2x + 2yx) = e^y x(-x^3 - 1 - y^2 + 2y)$$

$$\begin{cases} -3x^2 + y^2 = 1 \\ x^2 + y^2 + 4y = 1 \end{cases}$$

$y - 4x^2 - 2y = 0$

$2y = -2x^2$

$-3x^2 + 4x^4 - 1 = 0$

$4x^4 - 3x^2 - 1 = 0$

$D = 9 + 16 = 5^2$

$x_1^2 = \frac{3-5}{8} = -\frac{1}{4}$

$x_2^2 = \frac{3+5}{8} = 1$

$x_{1,2} = \pm 1$

$y = -2$

$(1, -2)$

$(-1, -2)$

$z''_{xx}(1,-2) = \frac{e^{-2}}{2}(-3x^2+y^2-1)'' = \frac{e^{2}}{2}(-6)$

$z''_{xy}(1,-2) = \frac{e^{-2}}{2}(-4)$

$z''_{yy}(1,-2) = \frac{e^{-2}}{1}(-2+4) = 2e^{-2}$

$\Delta < 0$

$\cancel{\frac{e^{-2}}{2}(-6) \quad \frac{e^{-2}}{2}(-4)}$

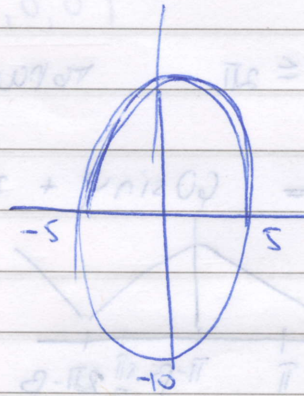
т. $(-1,-2) \notin DC$

$\Rightarrow (1,-2)$ – седловидна т.

задача

$$f(x,y) = x^2 + \frac{y^2}{4} - 12x + 8y \qquad (6,-16)$$

$$4x^2 + y^2 \le 100$$



$f'_x = 2x - 12 = 0$

$f'_y = \frac{y}{2} + 8 = 0 \qquad y = \sqrt{100-4x^2}$

$y = -16 \qquad y = \sqrt{100-4x^2}$

$f(x, \sqrt{100-4x^2})$

$-5 \le x \le 5$

$\varphi'(x) \qquad \varphi(5)$

$\varphi(-5)$

$\varphi'(x_0) = 0$

$Q(x) = f(x, -\sqrt{100-4x^2})$

$Q'(x_1) = 0$

$x = ar\cos\varphi \qquad x = 5r\cos\varphi$

$y = br\sin\varphi \qquad y = 10\sin\varphi$

$r = 1$

заместване в условието

$4x^2 + y^2 = 100$

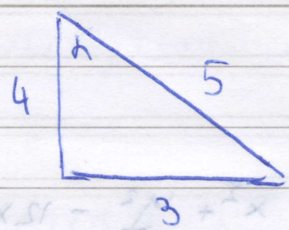
$x = 5\cos\varphi \qquad 0 \le \varphi \le 2\pi$

$y = 10\sin\varphi$

$$h(\varphi) = 25\cos^2\varphi + \frac{100\sin^2\varphi}{4} - 60\cos\varphi + 80\sin\varphi =$$

$$= -60\cos\varphi + 80\sin\varphi + 25 \qquad 0 \le \varphi \le 2\pi$$

$5|\cos\alpha\sin\varphi - \sin\alpha\cos\varphi| = 5\sin|\varphi - \alpha|$

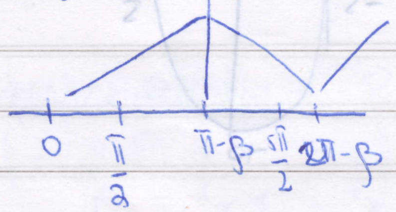


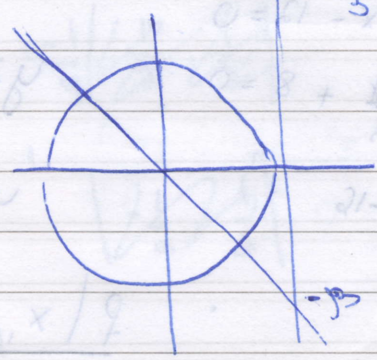
абсурдния метод →

$\rightsquigarrow$

$0 \le \varphi \le 2\pi$ търсим НГС $h(\varphi)$

$h'(\varphi) = 60\sin\varphi + 80\cos\varphi = 60\cos\varphi\left(\operatorname{tg}\varphi + \frac{4}{3}\right)$





$h(0)$
$h(2\pi)$
$h(\pi-\beta)$
$h(2\pi-\beta)$

2 начин $f(x,y) = x^2 + \frac{y^2}{4} - 12x + 8y$

$4x^2 + y^2 = 100$

$g(x,y) = f(x,y) + \lambda(4x^2 + y^2 - 100)$

$$\begin{cases} g'_x = 2x - 12 + 8x\lambda = 0 \\ g'_y = 2y + 8 + 2y\lambda = 0 \\ 4x^2 + y^2 = 100 \end{cases}$$

$x(1+4\lambda) = 6$

$y(1+4\lambda) = -16$

$x = \frac{6}{1+4\lambda}$

$y = \frac{-16}{1+4\lambda}$

$\left(\frac{1}{1+4\lambda}\right)^2 144$

$\left(\frac{1}{1+4\lambda}\right)^2 (144 + 256) = 100$

$1 + 4\lambda = \pm 2$ $\lambda = 2$ $x = 3$ $y = -8$

$\lambda =$

Задача

$u = 3x^2 + 5y^2 - 2z^2$

щом елипсата е затворена
по В-В е компактно множество

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} \le 1$ $\Rightarrow$ има НМ и НГС

$u'_x = 6x$

$u'_y = 10y$ анулират се само в $(0,0,0)$

$u'_z = -4z$ $\Rightarrow u(0,0,0) = 0$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} = 1$

$x = 3\cos\varphi \sin\theta$

$y = 5\sin\varphi \sin\theta$

$z = 2\cos\theta$

$0 \le \varphi \le 2\pi$

$0 \le \theta \le \pi$

заместваме u_x, u_y, u_z в u и се получава $h(\varphi, \theta)$

$F(x,y,z) = 3x^2 + 5y^2 - 2z^2 + \lambda\left(\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} - 1\right)$ Тм Лагранж

$$\begin{cases} F'_x = 6x + \lambda\frac{2x}{9} = 0 \\ F'_y = 10y - \lambda\frac{2y}{25} = 0 \\ F'_z = -4z + \lambda\frac{z}{2} = 0 \\ \frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} = 1 \end{cases}$$

$x(\lambda + 27) = 0$ $x = 0$ $\lambda = -27$

$y(\lambda + 125) = 0$ $y = 0$ $\lambda = -125$

$z(-8 + \lambda) = 0$ $z = 0$ $\lambda = 8$

1сл. $\lambda = -27$	2сл. $\lambda = -125$	3сл. $\lambda = 8$
$y = 0$	$x = 0$	$x = 0$
$z = 0$	$z = 0$	$y = 0$
$x = \pm 3$	$y = \pm 5$	$z = \pm 2$
$u = 27$	$u = 125$	$u = 8$