Answers To Selected Problems

Chapter 1

1-2. a. Not possible, $\boldsymbol{A}$ and $\boldsymbol{B}$ are of different order

b. $\begin{bmatrix} 4 & 1 & 6 \\ -1 & 4 & -10 \end{bmatrix}$

c. Not possible, the product $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}$ is not compatible with $\boldsymbol{A}^{\mathrm{T}}$
d. **ones(3)**
e. Not possible, $\boldsymbol{A}^{\mathrm{T}}$ and $\boldsymbol{B}$ are not of the same order

1-5. **m = [pi, -2; 3, exp(1)];**

1-7. b. $\boldsymbol{PM}$, row order inverted; $\boldsymbol{MP}$, column order inverted

1-11. Representation of systems of linear algebraic equations

Chapter 2

2-1. a. $(111110)_2$, $(175)_8$, $(7D)_{16}$
b. $(0.0001)_2$, $(0.04)_8$, $(0.1)_{16}$
c. $(101011.1010111100001.....)_2$, $(53.534121....)_8$, $(2B.AE14....)_{16}$

2-4. 0 10000000011 1100110000 0000000000 0000000000 0000000000 0000000000 00

2-6. a. $-14 \leq \text{exp} \leq 15$, reserving extreme values, b. 2047, c. three digits, d. $+784.5$,
e. 0 11000 1000100010

Chapter 3

3-1. a. $T_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$
b. $R_4(x) = \frac{1}{5!}\frac{105}{c^{9/2}}(x-1)5$
c. $R_4(1.5) \leq 0.0008545$

3-3. a. $T_3(x) = x - \frac{1}{3}x^3$, b. $R_3(x) = \frac{(c-c^3)}{(1+c^2)^4}\,x^4$, c. $T_3(0.3) = 0.291$

3-6. a. $T_{10}(x) = 1 - x^2 + x^4/2! - x^6/3! + x^8/4! - x^{10}/5!$

b. $|\,f(1) - T_8(1)\,| = 0.00712 \leq 1/5! = 0.0083$

3-8. The maximum value of $f^{(n+1)}(x)$ on $|\,x-3\,| \leq 2$ is one. $n = 11$

Chapter 4

4-1. a. $0.5625,\ 0.0625$, b. 17

4-3 a. **ezplot('1+x^2'), hold on, ezplot('tan(x)')**
b. For example $[\,0.8,\ 1.2\,],\ 1.175$
c. 0.025
d. 16

4-6. a. 1.4118, b. 0.56714

4-9. a. $1.1723,\ 2.3219 \cdot 10^{-4}$, b. $1.1818,\ 9.3892 \cdot 10^{-3}$, c. $4,\ 5$

4-11. a. 1.4404, b. 1.2629

4-14. Set dt/dx equal to zero. 0.29972

4-15. Sphere sinks if $\delta = 70$ lb/ft^2.

4-17. 4

4-19. a. $k=2,\ 0.0$, b. $k=3,\ -1.1$, c. $k=3,\ \pm\sqrt{2}$, d. $k=3,\ -1.4113$

Chapter 5

5-1. a. $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, b. $\begin{bmatrix} 2 \\ 1/2 \\ 3 \end{bmatrix}$

5-2. See 5-1.

5-4. a. $\boldsymbol{LU} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3/2 & 3/2 \\ 0 & 0 & 1 \end{bmatrix}$, b. $\boldsymbol{x} = \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}$

5-7. a. $\boldsymbol{LU} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/3 & 1 & 0 & 0 \\ 0 & -6/5 & 1 & 0 \\ 0 & 0 & 5/16 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 & 0 \\ 0 & -5/3 & 1 & 0 \\ 0 & 0 & 16/5 & 2 \\ 0 & 0 & 0 & 19/8 \end{bmatrix}$, b. $\boldsymbol{x} = \begin{bmatrix} 2 \\ -1 \\ 1 \\ -3 \end{bmatrix}$

5-8. $\boldsymbol{LU} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$

5-9. $\boldsymbol{P} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \boldsymbol{L} = \begin{bmatrix} 1 & 0 \\ 1/3 & 1 \end{bmatrix}, \boldsymbol{U} = \begin{bmatrix} 3 & 4 \\ 0 & -1/3 \end{bmatrix}$

5-11. $3n - 2$

5-12. $\widehat{\boldsymbol{x}}_1 = \begin{bmatrix} 7/3 \\ 3/5 \end{bmatrix}, \widehat{\boldsymbol{x}}_2 = \begin{bmatrix} 29/15 \\ 16/15 \end{bmatrix}$

5-15. a. $\widehat{\boldsymbol{x}}_1 = \begin{bmatrix} 1/3 \\ -1 \\ -3/4 \end{bmatrix}, \widehat{\boldsymbol{x}}_2 = \begin{bmatrix} 5/4 \\ -2/3 \\ -2/3 \end{bmatrix}$, b. $\widehat{\boldsymbol{x}}_1 = \begin{bmatrix} -6/5 \\ -3/4 \\ 1 \end{bmatrix}, \widehat{\boldsymbol{x}}_2 = \begin{bmatrix} .9 \\ .35 \\ .225 \end{bmatrix}$

5-16. a. $\widehat{\boldsymbol{x}}_1 = \begin{bmatrix} 1/3 \\ -2/3 \\ 5/6 \end{bmatrix}, \widehat{\boldsymbol{x}}_2 = \begin{bmatrix} 1.0556 \\ .0556 \\ -2.0972 \end{bmatrix}$, b. $\widehat{\boldsymbol{x}}_1 = \begin{bmatrix} -1.2 \\ .35 \\ -.3250 \end{bmatrix}, \widehat{\boldsymbol{x}}_2 = \begin{bmatrix} .1100 \\ .3575 \\ -.6537 \end{bmatrix}$

5-17. 12

5-20. Jacobi: 2 decimal places, Gauss-Seidel: 4 decimal places

Chapter 6

6-1. $\begin{bmatrix} (-1)^2 & e^{-1} \\ 2^2 & e^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

6-3. b. $P_2(x) = \frac{(x-4)(x-7)}{(2-4)(2-7)}(-1) + \frac{(x-2)(x-7)}{(4-2)(4-7)}(5) + \frac{(x-2)(x-4)}{(7-2)(7-4)}(2)$

c. $P_2(x) = \frac{(x-3)(x-4)}{(1-3)(1-4)}(-2) + \frac{(x-1)(x-4)}{(3-1)(3-4)}(0) + \frac{(x-1)(x-3)}{(4-1)(4-3)}(3)$

6-4. a. Use $\tan \pi/4 = 1$, $\tan \pi/3 = \sqrt{3}$.
$P_1(x) = \frac{x-\pi/3}{\pi/4-\pi/3}(1) + \frac{x-\pi/4}{\pi/3-\pi/4}(\sqrt{3})$

b. $P_2(x) = \frac{(x-3)(x-4)}{(1-3)(1-4)}(1) + \frac{(x-1)(x-4)}{(3-1)(3-4)}(\frac{1}{3}) + \frac{(x-1)(x-3)}{(4-1)(4-3)}(\frac{1}{4})$

6-5. b.

x_i	y_i	First DD	Second DD
2	-1		
		$\frac{5-(-1)}{4-2} = 3$	
4	5		$\frac{-1-3}{7-2} = -\frac{4}{5}$
		$\frac{2-5}{7-4} = -1$	
7	2		

$P_2(x) = -1 + 3(x-2) - \frac{4}{5}(x-2)(x-4)$

6-9. a. Use the inequality $|\tan x - P_1(x)| \le \frac{1}{2}|(x-\pi/4)(x-\pi/3)| \max_{\frac{\pi}{4} \le c \le \frac{\pi}{3}} |f''(c)|$

Note: $|f''(x)| = |2\sec^2 x \tan x| \le 8\sqrt{3} < 14$ on the interval $\frac{\pi}{4} \le x \le \frac{\pi}{3}$
$|\tan x - P_1(x)| \le 0.12$ on the interval $\frac{\pi}{4} \le x \le \frac{\pi}{3}$

b. Use $\left|\frac{1}{x} - P_2(x)\right| \le \frac{1}{3!}|(x-1)(x-3)(x-4)| \max_{1 \le c \le 4} |f'''(c)|$.

Note: $f'''(x) = 6/x^4$ so on $[1,4]$, it follows that $|f'''| \le 6$. A graph of $(x-1)(x-3)(x-4)$ will show that it is less than 2.2 on $[1,4]$. So $\left|\frac{1}{x} - P_2(x)\right| \le \frac{1}{6} \cdot 2.2 \cdot 6 = 2.2$.

6-12.
```
» coef = polyfit(x,y,length(x)-1);
» valueatpi = polyval(coef,pi)
```

6-13. a. yes, no; b. no; c. yes, no; d. no

6-15. a. $P_7(4.25) = 17.3508$ $\quad P_7(6.7) = 28.4005$
$S(4.25) = 17.3016$ $\quad S(6.7) = 25.8441$

b. $P_7'(4.25) = 12.1605$ $\quad P_7'(6.7) = -1.3496$
$S'(4.25) = 12.3775$ $\quad S'(6.7) = 1.3537$

6-18. $\begin{bmatrix} 6 & 0 & 0 \\ 1 & 4 & 1 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} b_2 \\ b_3 \\ b_4 \end{bmatrix} = \frac{3}{4} \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$. For example $a_1 = 0.19271,\ b_1 = -1.65625,$

$c_1 = 3.4517$ and $d_1 = 2$.

6-22. $h_{j-1}b_{j-1} + 2(h_{j-1} + h_j)b_j + h_j b_j = 3(\frac{1}{h_{j-1}})d_{j-1} - 3(\frac{1}{h_{j-1}} + \frac{1}{h_j})d_j + 3(\frac{1}{h_j})d_{j+1}$

Chapter 7

7-2. a. $E(c_1,\, c_2) = (c_2 - 1)^2 + (c_1 + c_2 - 1)^2 + (2c_1 + c_2 - 2)^2 + (4c_1 + c_2 - 5)^2$.

b. $42c_1 + 14c_2 = 50$
$14c_1 + 4c_2 = 18$

c. Better be the same!

7-5. a. **lscoef2 = polyfit(T,V,2)**
c. at 66° $2.5073,\ -0.0371$; at 76° $2.1732,\ -0.0297$

7-8
$$\begin{bmatrix} \sum_{i=1}^{n}(x_i^2)^2 & \sum_{i=1}^{n}x_i^2e^{-x_i} \\ \sum_{i=1}^{n}x_i^2e^{-x_i} & \sum_{i=1}^{n}(e^{-x_i})^2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}y_i^2x_i^2 \\ \sum_{i=1}^{n}y_i^2e^{-x_i} \end{bmatrix}$$

7-9. a.
```
» cquad = polyfit(x,y,2)
cquad =
  7.8125e-001 -1.0757e+000  5.4935e-001
» cexp = polyfit(x,log(y),1)
cexp =
  1.2957e+000 -2.3732e+000
» cpow = polyfit(log(x),log(y),1)
cpow =
  1.1489e+000 -9.3735e-001
» crec = polyfit(x,1./y,1)
crec =
 -3.2962e+000  6.6597e+000
```

b. Reciprocal followed by, in order, quadratic, exponential, and power

7-10. **lscoef = polyfit(t,1./y,2)**

Chapter 8

8-2. a. Derivative values:

	Forward	Forward	Forward	Central	Backward	Backward
h	Two term	Three Term	Four Term	Two Term	Two Term	Three Term
0.1	-0.2453	-0.2676	-0.2702	-0.2725	-0.2996	-0.2665
0.05	-0.2576	-0.2698	-0.2706	-0.2711	-0.2847	-0.2679
0.025	-0.2640	-0.2705	-0.2707	-0.2708	-0.2776	-0.2704

a. Errors:

	Forward	Forward	Forward	Central	Backward	Backward
h	Two term	Three Term	Four Term	Two Term	Two Term	Three Term
0.1	-0.0253	-0.0031	-0.0004	0.0018	0.0290	-0.0042
0.05	-0.0131	-0.0008	-0.00006	0.00045	0.0140	-0.00097
0.025	-0.0067	-0.0002	$-8\cdot 10^{-6}$	0.00011	0.0069	-0.00023

c. Derivative values:

	Forward	Forward	Forward	Central	Backward	Backward
h	Two term	Three Term	Four Term	Two Term	Two Term	Three Term
0.1	-0.1995	0.0947	0.0854	0.0644	0.3283	0.1085
0.05	-0.0586	0.0824	0.0780	0.0738	0.2062	0.0841
0.025	0.0100	0.0785	0.0771	0.0762	0.1425	0.0787

c. Errors:

	Forward	Forward	Forward	Central	Backward	Backward
h	Two term	Three Term	Four Term	Two Term	Two Term	Three Term
0.1	0.2765	-0.0177	-0.0084	0.0126	-0.2513	-0.0315
0.05	0.1356	-0.0054	-0.00097	0.0032	-0.1292	-0.0071
0.025	0.0670	-0.0015	-0.00012	0.0008	-0.0655	-0.0017

8-3. a. $h > 0$ gives forward differences of 1
$h < 0$ gives forward differences of -1
b. All centered differences are zero
c. Slope of "left half" of $|x|$ is -1 while "right half" is $+1$
d. Numerical differentiation at points of non differentiability might be misleading

8-4. $f''(x_i) = \frac{1}{h^2}[f(x_i - 2f(x_i - h) + f(x_i - 2h)] + \mathcal{O}(h)$

8-9. $f''(x_i) = \frac{1}{h^2}[-f(x_i + 3h) + 4f(x_i + 2h) - 5f(x_i + h) + 2f(x_i)] + \mathcal{O}(h^2)$

8-10.
```
» coef3 = polyfit(x,y,3);
» coef3p = polyder(coef3);coef3pp = polyder(coef3p);
» d1 = polyval(coef3p,x(1)), d2 = polyval(coef3pp,x(1))
```

8-11. a. 1.0462
b. Use the formula $E_4^T \leq \frac{(b-a)^3}{12n^2} \max_{[2.5,\ 3]} |f''(x)|$. Graph f'' to see that $|f''(x)| \leq 1.7$ for $2.5 \leq x \leq 4$. So $E_4^T \leq \frac{(4-2.5)^3}{12 \cdot 4^2} \cdot 1.7 = .0298828$

8-12. a. 1.0688
b. Use the formula $E_4^S \leq \frac{(b-a)^5}{180n^4} \max_{[2.5,\ 3]} \left|f^{(4)}(x)\right|$. Graph $f^{(4)}$ to see that $\left|f^{(4)}(x)\right| \leq 6.7$ for $2.5 \leq x \leq 4$. So $E_4 \leq \frac{(4-2.5)^5}{180\, 4^4} \cdot 6.7 = .00110413$

8-13. a. Solve $\frac{(4-2.5)^3}{12 \cdot n^2} \cdot 1.7 \leq .0001$ to get $n \geq 69.1466$, so $n = 70$ will work.
b. Solve $\frac{(4-2.5)^5}{180\, n^4} \cdot 6.7 \leq .0001$ to get $n \geq 7.29146$, so $n = 8$ will work.

8-15. a.
```
» syms x
» int(sin(sqrt(x)),1,4)
ans =
2*sin(2)-4*cos(2)-2*sin(1)+2*cos(1)
```

b. $398,\ 24$
c. $238,\ 14$

8-20. First do a change of variables: $x = \frac{4-2.5}{2}t + \frac{4+2.5}{2} = \frac{1.5}{2}t + \frac{6.5}{2}$. Note $dx = \frac{1.5}{2}dt$. So $\int_{2.5}^{4} f(x)dx = \frac{1.5}{2}\int_{-1}^{1} f(\frac{1.5}{2}t + \frac{6.5}{2})dt = \int_{-1}^{1} g(t)dt$, where $g(t) = \frac{1.5}{2} f(\frac{1.5}{2}t + \frac{6.5}{2}) = 0.75 \sin((\frac{1.5}{2}t + \frac{6.5}{2})^2/5)$.

$G_1 = 2\ g(0) = 1.28525$
$G_2 = 1 \cdot g(\sqrt{3}/3) + 1 \cdot g(-\sqrt{3}/3) = 1.06165$
$G_3 = \frac{5}{9}g(\sqrt{3/5}) + \frac{8}{9}g(0) + \frac{5}{9}g(-\sqrt{3/5}) = 1.06883$

(Actual value: 1.06880)

8-24. 6.6825

8-26. $10.4888,\ 10.5184,\ 10.5166$

8-28. Use the code in the text or see Problem 8-27.
$n = 5,\ 3.7 \cdot 10^{-4}; n = 10, 4.7 \cdot 10^{-6}; n = 15,\ 1 \cdot 10^{-6}$

Chapter 9

9-1. a. Values

x	Euler	Improved	MidPt	R-K	Exact
0	1.0000	1.0000	1.0000	1.0000	1.0000
.1	1.1000	1.1065	1.1058	1.1062	1.1062
.2	1.2130	1.2303	1.2288	1.2298	1.2298
.3	1.3463	1.3796	1.3771	1.3790	1.3790

b. Errors

x	Euler	Improved	MidPt	R-K
0	0.0000	0.0000	0.0000	0.0000
.1	0.0062	−0.0003	0.0004	0.0000
.2	0.0168	−0.0005	0.0010	0.0000
.3	0.0327	−0.0006	0.0019	0.0000

9-2. In impeuler.m modifiy the contents of the for-end statement computing y_{i+1} as follows:

a. Midpoint

```
for i = 1:n
  k1 = ydot(t(i),       y(i)            );
  k2 = ydot(t(i)+h/2, y(i)+h*k1/2);
  y(i+1) = y(i)+h*k2;
end
```

b. Runge-Kutta

```
for i = 1:n
  k1 = ydot(t(i),       y(i)            );
  k2 = ydot(t(i)+h/2, y(i)+h*k1/2);
  k3 = ydot(t(i)+h/2, y(i)+h*k2/2);
  k4 = ydot(t(i)+h,    y(i)+h*k 3  );
  y(i+1) = y(i)+h*(k1+2*k2+2*k3+k4)/6;
end
```

9-3. a. Values

x	Euler	Improved	MidPt	R-K	Exact
0	1.0000	1.0000	1.0000	1.0000	1.0000
.05	1.0500	1.0514	1.0513	1.0514	1.0514
.1	1.1029	1.1063	1.1061	1.1062	1.1062
.15	1.1595	1.1654	1.1651	1.1653	1.1653
.2	1.2209	1.2299	1.2295	1.2298	1.2298
.25	1.2879	1.3008	1.3003	1.3007	1.3007
.3	1.3617	1.3792	1.3785	1.3790	1.3790

b. Errors

x	Euler	Improved	MidPt	R-K
0	0.0000	0.0000	0.0000	0.0000
.05	0.0014	0.0000	0.0001	0.0000
.1	0.0033	0.0001	0.0001	0.0000
.15	0.0058	0.0001	0.0002	0.0000
.2	0.0089	0.0001	0.0003	0.0000
.25	0.0128	0.0001	0.0004	0.0000
.3	0.173	0.0001	0.0005	0.0000

9-6. At $t = 0.75$ the error in the midpoint method drops below the improved Euler method for the remainder of the interval.

9-7. $1.3607 \cdot 10^{-3}$ at $t = 2.1456$

9-9. a. 26.061, 10.776 at $t = 0.2$
b. 24.772, 1.4409 at $t = 0.13$