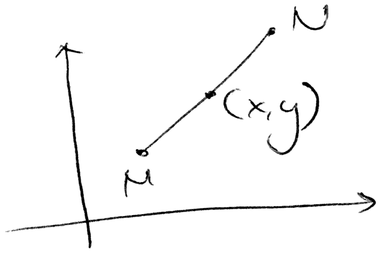
12 Теорема за крайните нараствания

D Множеството $G \subset \mathbb{R}^2$ се нарича **изпъкнало**, ако $\forall M, N \in G$ $\Rightarrow [M,N] \subset G$, т.е. ако

$M(x_1, y_1) \in G$, $N(x_2, y_2) \in G$

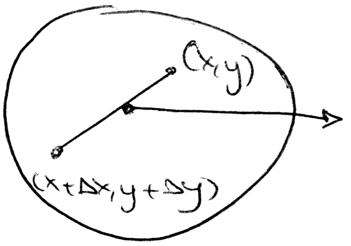
$$\left.\begin{aligned} x &= x_1 + (x_2 - x_1)t \\ y &= y_1 + (y_2 - y_1)t \end{aligned}, \ t \in [0,1] \right\} \Rightarrow (x,y) \in G$$



T Нека $z = f(x,y)$ е диференцируема върху изпъкналата област $G \subset \mathbb{R}^2$. Тогава $\forall (x,y), (x+\Delta x, y+\Delta y) \in G$
$\exists\ 0 < \theta < 1$:

$$f(x+\Delta x, y+\Delta y) - f(x,y) = f'_x(x+\theta\Delta x, y+\theta\Delta y)\Delta x + f'_y(x+\theta\Delta x, y+\theta\Delta y)\Delta y$$

Доказателство:



$\to (x+t\Delta x, y+t\Delta y),\ t \in [0,1]$

Образуваме функцията $\varphi(t) = f(x+t\Delta x, y+t\Delta y) \to$
$\to$ диференцируема върху $[0,1] \to$ 1) непр. в $[0,1]$
2) $\exists\ \varphi'(t)$ в $(0,1)$

$\Rightarrow$ от Т-мата на Лагранж:

$$\exists\ 0 < \theta < 1: \varphi(1) - \varphi(0) = \varphi'(\theta(1-0)).(1-0) = \varphi'(\theta)$$

$$\Delta f = f(x+\Delta x, y+\Delta y) - f(x,y) = \varphi(1) - \varphi(0) = \varphi'(\theta) =$$

$$= f'_x(x+\theta\Delta x, y+\theta\Delta y)\Delta x + f'_y(x+\theta\Delta x, y+\theta\Delta y)\Delta y$$