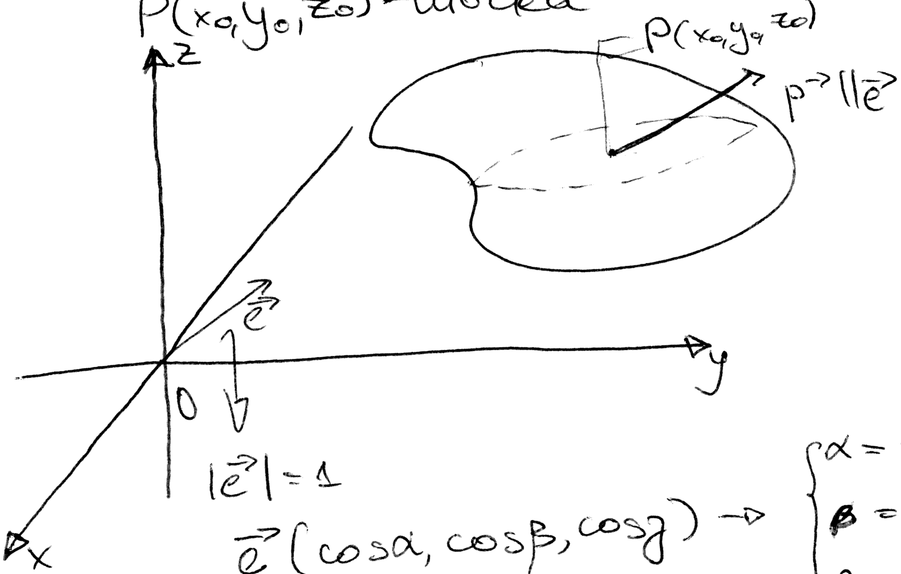
13 Производна по посока. Градиент

Нека $f(x,y,z)$ е дефинирана в $U(x_0,y_0,z_0)$

$P(x_0,y_0,z_0)$ - точка



$|\vec{e}|=1$

$\vec{e}(\cos\alpha, \cos\beta, \cos\gamma) \to \begin{cases} \alpha = \sphericalangle(\vec{Ox}, \vec{e}) \\ \beta = \sphericalangle(\vec{Oy}, \vec{e}) \\ \gamma = \sphericalangle(\vec{Oz}, \vec{e}) \end{cases}$

$M(x,y,z) \in p^{\to} \longrightarrow \begin{vmatrix} x = x_0 + t\cos\alpha \\ y = y_0 + t\cos\beta \\ z = z_0 + t\cos\gamma \end{vmatrix}, \ t \in [0, +\infty)$

Дефинираме: $h(t) = f(x_0 + t\cos\alpha, y_0 + t\cos\beta, z_0 + t\cos\gamma), \ t \geq 0$

$\boxed{D}$ $h'_t(0)$ се нарича ПРОИЗВОДНА НА f В (x_0,y_0,z_0) ПО ПОСОКА НА $\vec{e}$,

т.е. $h'_t(0) = \lim\limits_{t \to 0^+} \dfrac{h(t)-h(0)}{t} = \lim\limits_{t\to 0^+} \dfrac{f(x_0+t\cos\alpha, y_0+t\cos\beta, z_0+t\cos\gamma) - f(x_0,y_0,z_0)}{t} =$

$= \dfrac{\partial f(x_0,y_0,z_0)}{\partial e}$

$\boxed{D}$ $\left(\dfrac{\partial f(x_0,y_0,z_0)}{\partial x}, \dfrac{\partial f(x_0,y_0,z_0)}{\partial y}, \dfrac{\partial f(x_0,y_0,z_0)}{\partial z}\right) = \operatorname{grad} f(x_0,y_0,z_0)$

ГРАДИЕНТА НА $f(x,y,z)$ В (x_0,y_0,z_0)

$\boxed{T}$ $\dfrac{\partial f(x_0,y_0,z_0)}{\partial e} = \dfrac{\partial f(x_0,y_0,z_0)}{\partial x}\cos\alpha + \dfrac{\partial f(x_0,y_0,z_0)}{\partial y}\cos\beta + \dfrac{\partial f(x_0,y_0,z_0)}{\partial z}\cos\gamma =$

$= \operatorname{grad} f(x_0,y_0,z_0) . \vec{e} = |\operatorname{grad} f(x_0,y_0,z_0)| |\vec{e}| \cos\sphericalangle(\operatorname{grad} f(x_0,y_0,z_0), \vec{e})$